"13th JAEE CONGRESS IN OKAYAMA "

(2009/03/29 D3 非線形動学 10:00-12:00)

Coevolution and Diversity in Evolutionary Game Theory
(進化ゲーム理論における共進化と多様性)
：Stochastic Environment
(−確率的環境の場合−)

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EVO LUT ION

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Coevolution (共進化), Diversity （多様性)

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(進化経済学会 ニューズレターNo.25, P3 より)
進化ゲーム理論 においては,
「Replicator 方程式」が該当.

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平衡点:「純粋戦略(pure strategy)」
or 「混合戦略(mixed strategy)」

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多様性あり

SECTION: NONLINEAR DYNAMICS

非線形動力学というと一般的にカオス・微分方程式についての研究

進化ゲーム理論：REPLICATOR EQ.

- $$\dot{x}_i = x_i\left((Ax)_i - x\cdot Ax\right), i = 1,\cdots,n.$$

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例）戦略が2つの場合(3次関数)

$$\dot{x} = x(1-x)\{b-(a+b)x\}$$

→平衡点: x^*=0(pure), 1(pure), $b/(a+b)$ (mixed)

→進化ゲーム理論では、この**均衡選択 (equilibrium selection)** がメインテーマ.

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もちろん進化ゲーム理論の中にもカオスの議論は存在する(戦略の数が3つ以上の場合).

OUTLINE

1. INTRODUCTION （Motivation, Purpose）
2. RELATED LITERATURES and PRELIMINARIES
3. OUR MODEL
   3-1. HARSANYI TYPE
   3-2. SELTEN TYPE
4. EXTENSION (Global Game)
5. APPLICATION (FINANCE : Black-Sholes eq.)
6. SUMMARY and FUTURE WORKS

1. INTRODUCTION

OUR PROBLEM

- Q How does each player choose the action in stochastic environment ?

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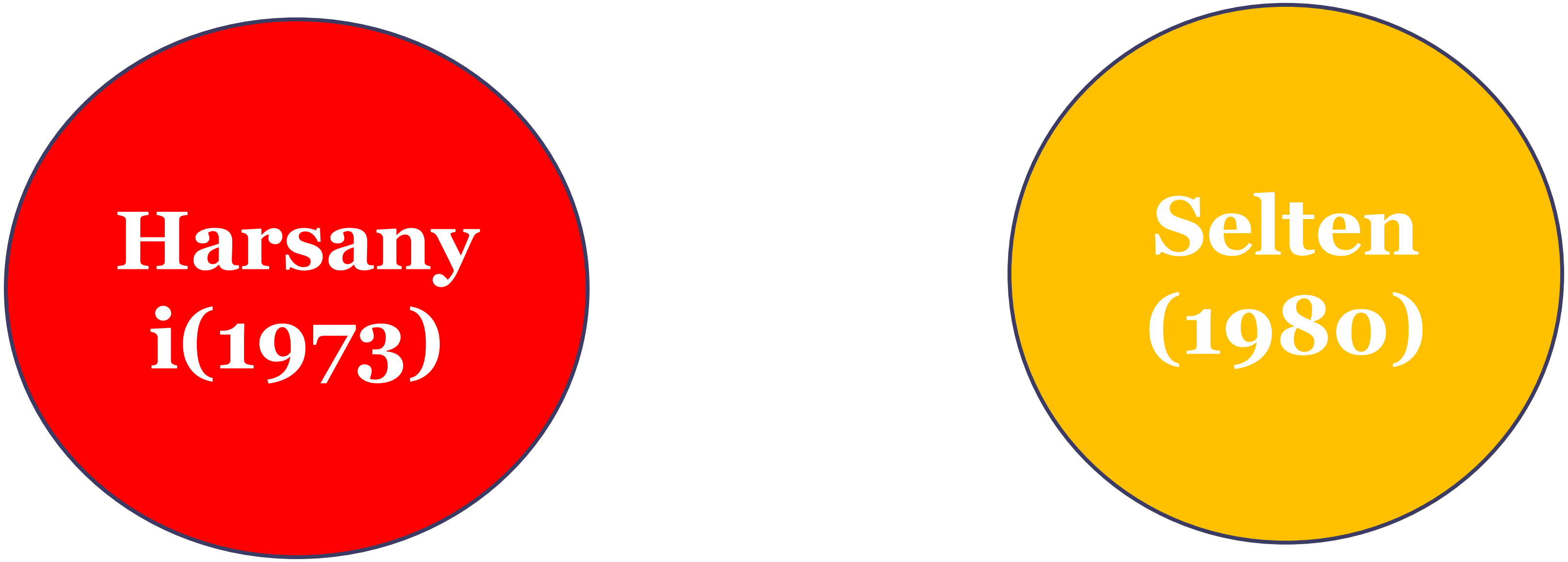
- Q How does each player choose the action in stochastic environment ?

- A.1：Each player randomly chooses the action. (mixed strategy) (Harsanyi , 1973)

- A.2：Each player chooses the better action. (pure strategy) (Selten, 1980)

Research Fields (this study)



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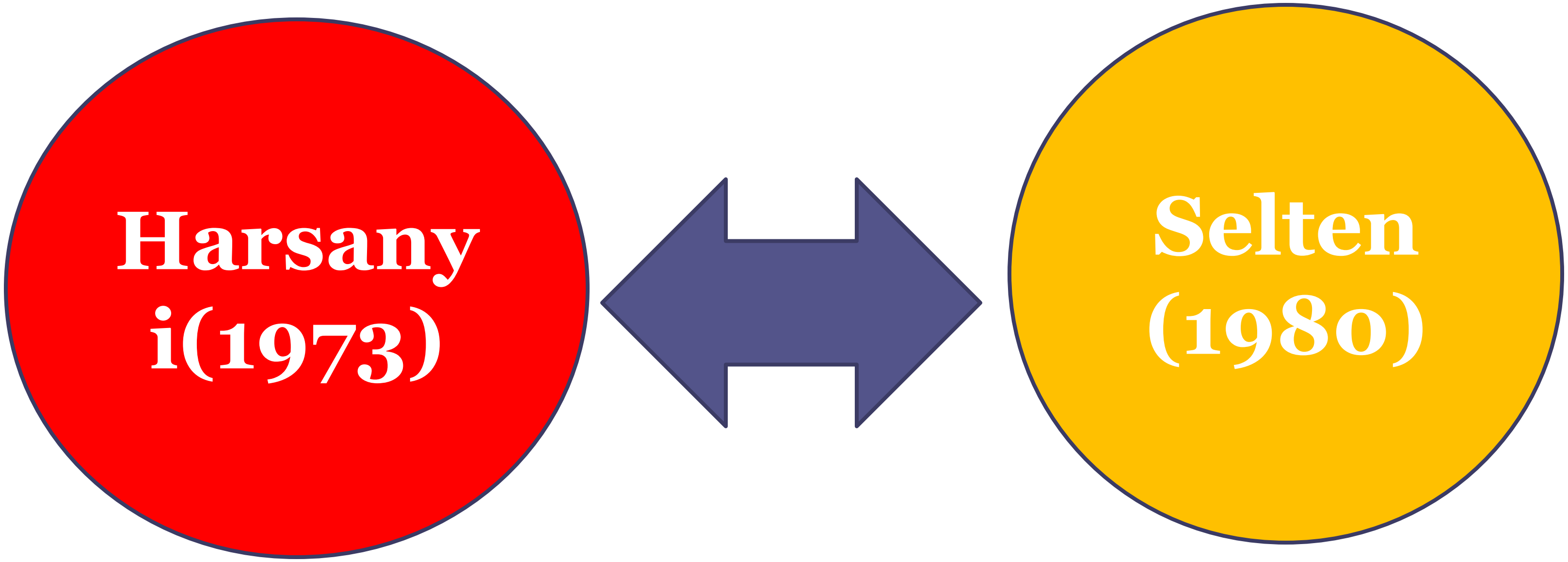

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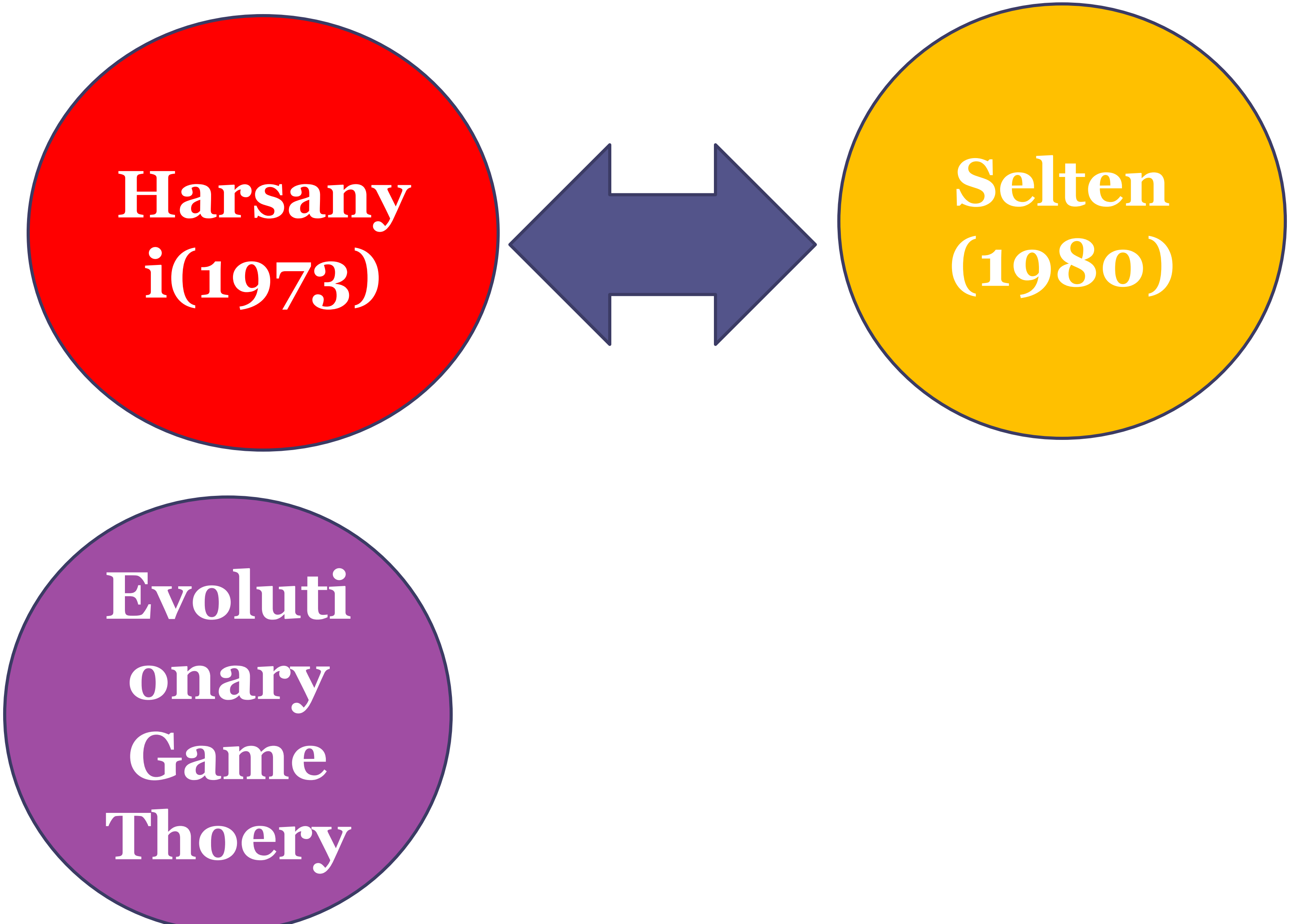

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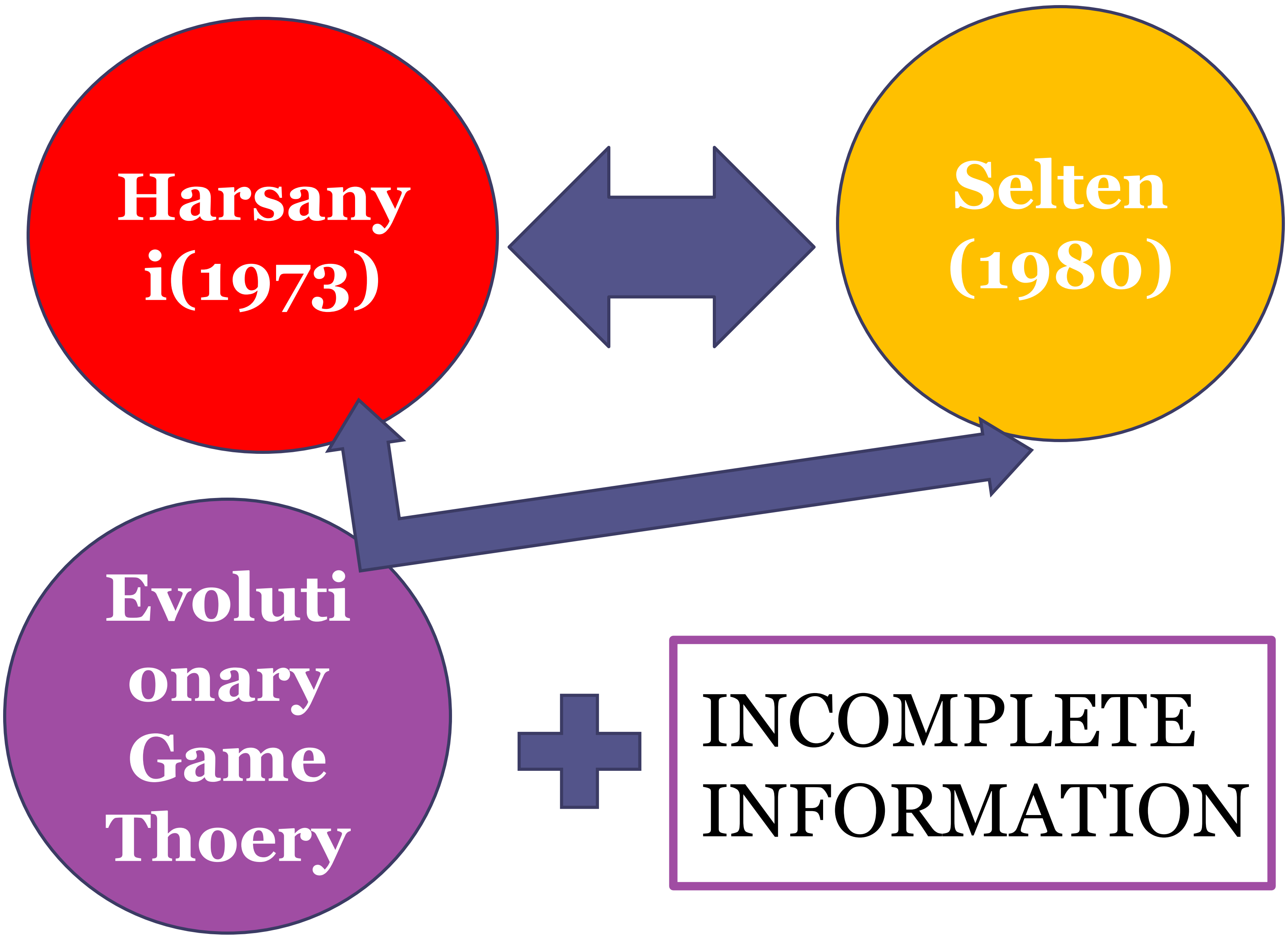


2. RELATED LITERATURES and PRELIMINARIES

Metcalfe

- 「多様性が変化を促進する」

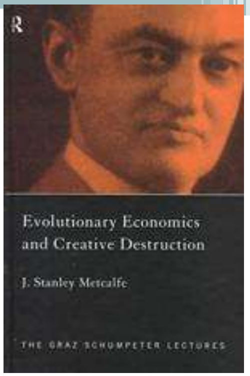
Prop. 2　平均利得の上昇率は, 各プレイヤーの戦略の分散に等しい. また平均利得は単調増加する.

→ Fisher's fundamental theorem of natural selection.

Proof.

Step 1) 平均利得を時間微分.

Step 2) 式変形.



Metcalfe (1998)

INOUE (1999)

MATHEMATICAL BIOLOGY

- Stochastic Environment
- Bet-Hedging Strategy (=Mixed Strategy)

IWASA, Y. (1998)

IMAI and OKADA (2002)

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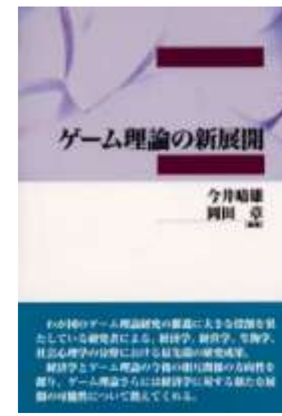

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Game Theory

- The fitness(utility) function is a von-Neumann-Morgenstern utility function .
- → **No** Bet-Hedging Strategy ?

Harsanyi (1973)

- Harsanyi, J. C. (1973): ”Games with Randomly Distributed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points,” *International Journal of Game Theory*, Vol.**2**, pp.1-23. [HP]

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- **Th.** Fix a set of I players and strategy spaces S_i. For a set of payoffs $\{u_i(s)\}_{i\in F, s\in S}$ of Lebesgue measure 1, for all independent, twice-differentiable distributions p_i on $\Theta_i=[-1,1]^{\#S}$, any equilibrium of the payoffs u_i is the limit as $\varepsilon \to 0$ of a sequence of pure-strategy equilibria of the perturbed payoffs $\tilde{u}_i$.

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→The probability distributions over strategies induced by the pure-strategy equilibria of the perturbed game converge to the distribution of the equilibrium of the unperturbed game.

EX. Battle of Sexes (BoS)

- Consider two-player games in which each player i has two pure strategies, a_i and b_i. Let δ_i for i=1,2 be independent random variables, each uniformly distributed on [-1, 1], and let the random variables $\varepsilon_i(a)$ for i=1,2 and $a \in A$ have the property that $\varepsilon_1(a_1,x)-\varepsilon_1(b_1,x)= \delta_1$ for $x=a_2,b_2$ and $\varepsilon_2(x,a_2)-\varepsilon_2(x,b_2)= \delta_2$ for $x=a_1,b_1$.
- All the equilibrium of BoS are approachable under ε.

	a2	b2
a1	$2+\gamma\delta_1$, $1+\gamma\delta_2$	$\gamma\delta_1$,0
b1	0, $\gamma\delta_2$	1,2

Proof Outline

(1) The pure equilibria are trivially approachable.

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(2) We consider the strictly mixed equilibrium.

For $i = 1, 2$ let pi be the probability that player i's type is one for which he chooses a_i in some Nash equilibrium of $G(\gamma\varepsilon)$.

(i) it is optimal for player 1 to choose a_1 if $(2 + \gamma\delta_1)p_2 \geqq (1 - \gamma\delta_1)(1 - p_2)$.

(ii) $-1 \leqq \delta_1 \leqq 1$

(i) + (ii) : $p_1 = 1/2(1 - (1 - 3p_2)/\gamma)$.

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(3) By a symmetric argument about p_2.

Solving for p_1 and p_2 we find that $p_1 = (2 + \gamma)/(3 + 2\gamma)$ and $p_2 = (1 + \gamma)/(3 + 2\gamma)$ satisfies these conditions. Since $(p_1, p_2) \rightarrow (2/3, 1/3)$ as $\gamma \rightarrow 0$ the mixed strategy equilibrium is approachable.

Selten (1980)

- Selten, R. (1980): ”A Note on Evolutionary Stable Strategies in Asymmetric Animal Conflicts,” *Journal of Theoretical Biology*, Vol.**84**, pp.93-101. [HP]

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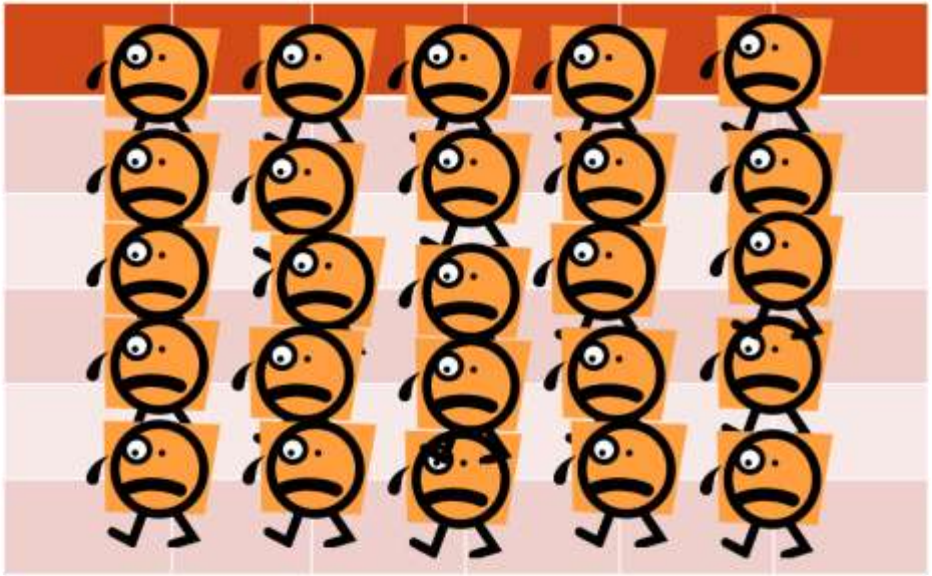
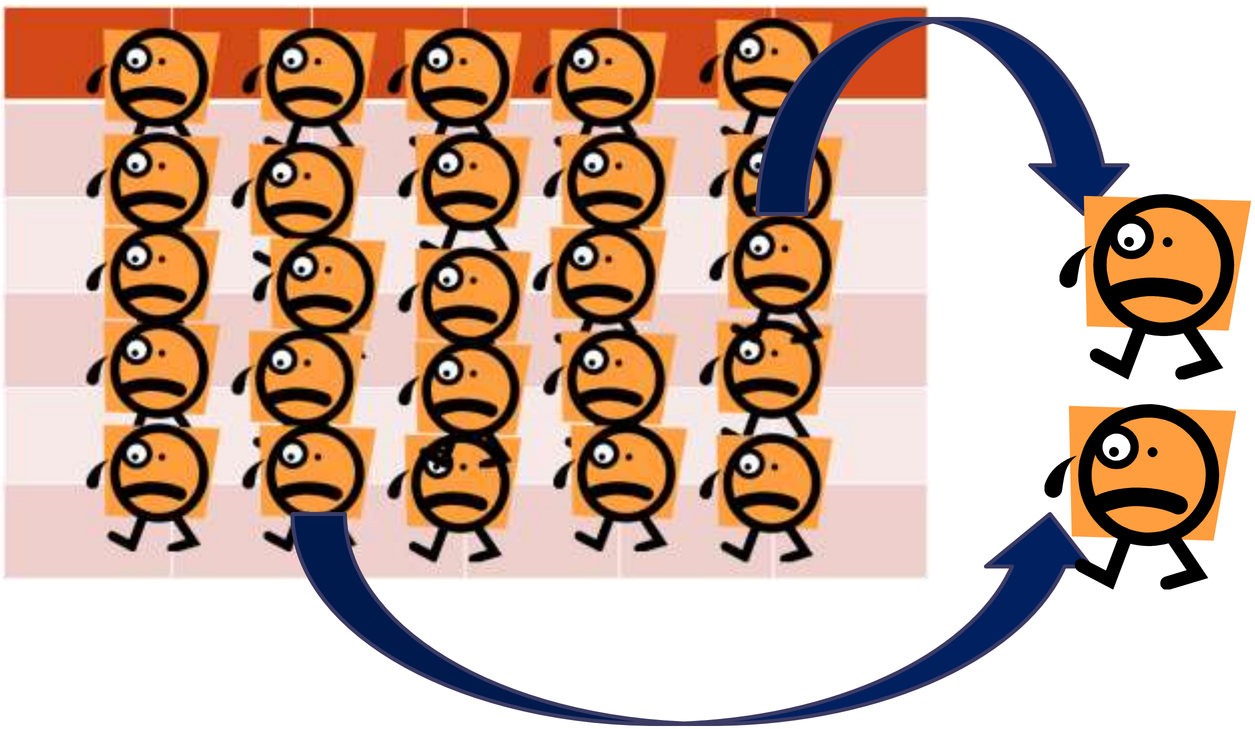
→No mixed equilibria are evolutionary stable when players can condition their strategies on their roles in a game.

Situation (Role Completed Game)



Situation (Role Completed Game)

At Random



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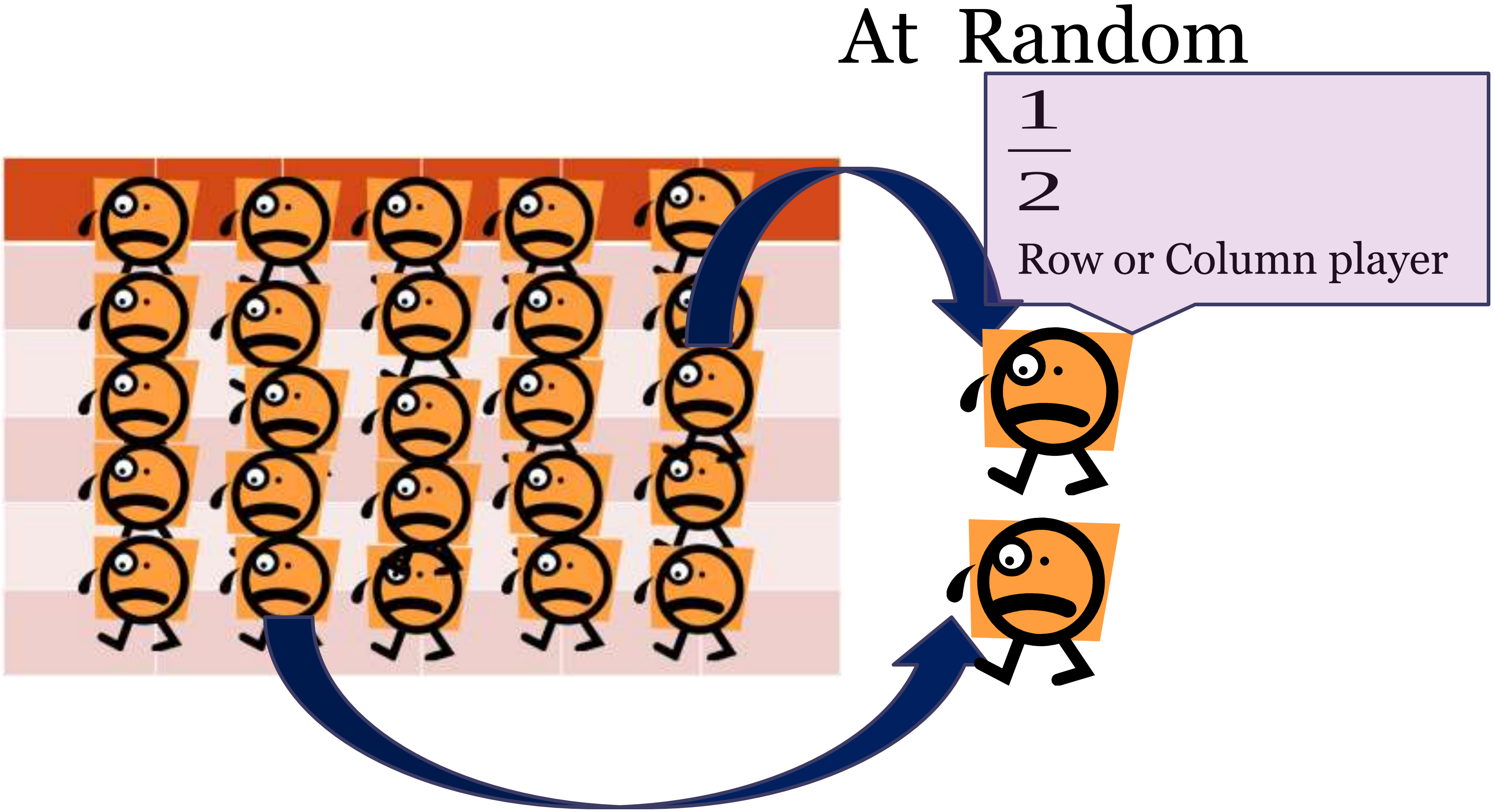

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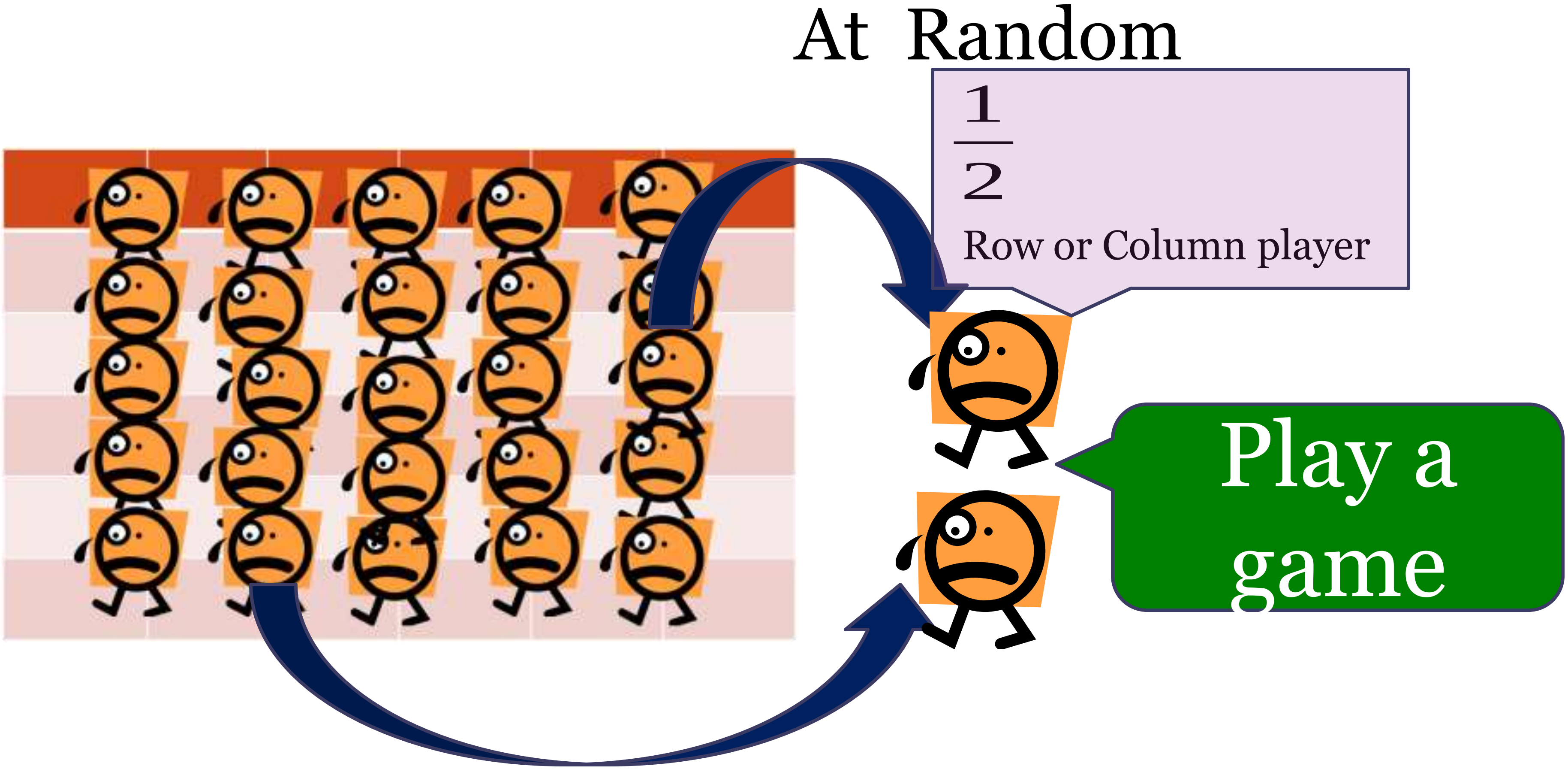
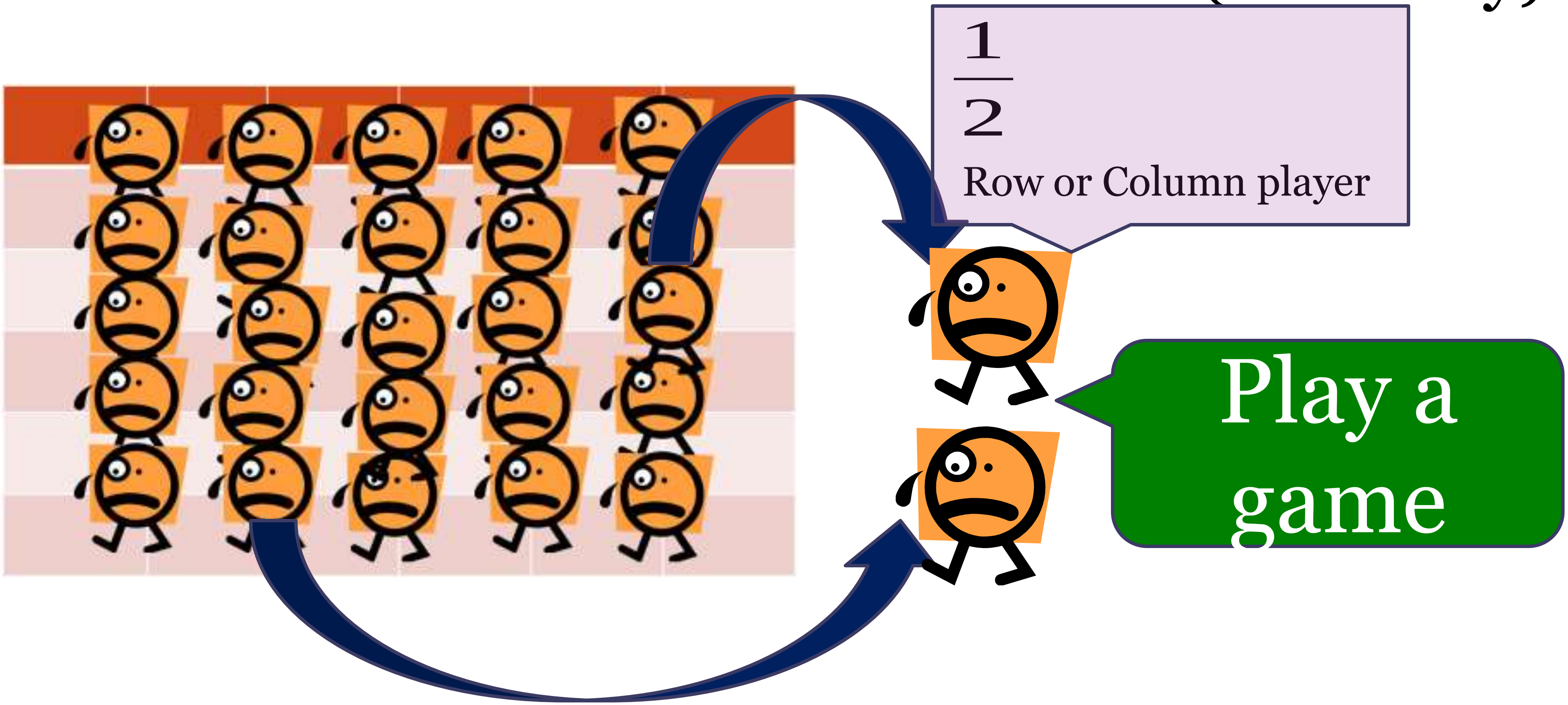


Situation (Role Completed Game)

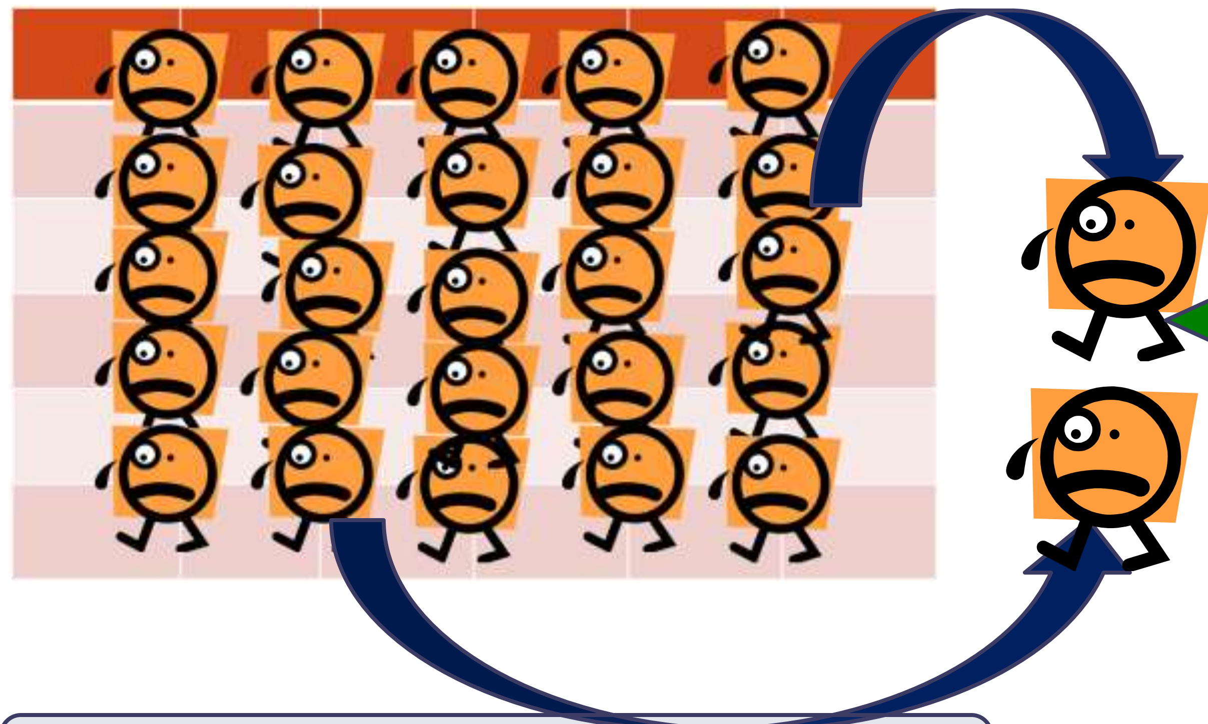
At Random (infinitely)



Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



Play a game

Another players look at the game.

Replicator Equation

REVIEW: Replicator Equation

REPLICATOR EQ. $\dot{x}_i = x_i\left((Ax)_i - x \cdot Ax\right), i = 1, \cdots, n.$

If the player's payoff from the outcome *i* is greater than the expected utility $x\,Ax$, the probability of the action *i* is higher than before.

REVIEW: Replicator Equation

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If the player's payoff from the outcome i is greater than the expected utility $x\,Ax$, the probability of the action i is higher than before. And this equation shows that the probability of the action i chosen by another players is also higher than before (**externality**).

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Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \quad \cdots (*)$$

Classification

(I) Non-dilemma: a > 0. b < 0, ESS : one
(II) Prisoner's dilemma : a < 0. b > 0, ESS :one
(III) Coordination : a>0,b>0, ESS two
(IV) Hawk-Dove : a<0,b < 0, ESS one (mixed strategy)

1 \ 2	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

Ex. Hawk-Dove Game

	D	H
D	V/2, V/2	0,V
H	V, 0	V/2-C,V/2-C

H-D Game

- **Classical H-D Game** :
- **Strategy** : {Dove, Hawk}
- **Payoff** : $V>0$, $V<C$
- **Nash Eq.** : Pure {(H,D), (D, H)} + Mixed
-
-
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- **ESS :** Mixed

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- **Nash Eq.** : Pure {(H,D), (D, H)} + Mixed
- **ESS :** Mixed (× strict Nash)
- **Stability** : Limit Cycle, Structurally Unstable.
- **Replicator Eq.**

$$\dot{x} = x(1-x)\{V/2 - C + Cx\}$$

- **Role Completed H-D Game**
- **Pure Strategy : {DD}, {DH}, {HD}, {HH}**
- **{DH} means play Dove if chosen to be a row player in the surface game and Hawk if chosen to be a column player.**
-

	DD	DH	HD	HH
DD	V/2,V/2	V/4,3V/4	3V/4,V/4	0,V
DH	3V/4,V/4	V/2,V/2	(V-C)/2, (V-C)/2	V/4-C/2, 3V/4-C
HD	V/4,3V/4	(V-C)/2, (V-C)/2	V/2,V/2	V/4-C/2, 3V/4-C
HH	V,0	3V/4-C,V/4-C/2	3V/4-C,V/4-C/2	V/2-C,V/2-C

Role Completed H-D Game

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- **ESS : (DH,DH) , (HD, HD)**

	DD	DH	HD	HH
DD	V/2,V/2	V/4,3V/4	3V/4,V/4	0,V
DH	3V/4,V/4	**V/2,V/2**	(V-C)/2, (V-C)/2	V/4-C/2, 3V/4-C
HD	V/4,3V/4	(V-C)/2, (V-C)/2	**V/2,V/2**	V/4-C/2, 3V/4-C
HH	V,0	3V/4-C,V/4-C/2	3V/4-C,V/4-C/2	V/2-C,V/2-C

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- **ESS : (**DH,DH) , (HD, HD) (○ strict Nash)

	DD	DH	HD	HH
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DH	3V/4,V/4	V/2,V/2	(V-C)/2, (V-C)/2	V/4-C/2, 3V/4-C
HD	V/4,3V/4	(V-C)/2, (V-C)/2	V/2,V/2	V/4-C/2, 3V/4-C
HH	V,0	3V/4-C,V/4-C/2	3V/4-C,V/4-C/2	V/2-C,V/2-C

Role Completed H-D Game

3. OUR MODEL

3-1. HARSANYI TYPE

Stochastic Environment (Harsanyi (1973) + Dynamics)

- Stochastic Environment = payoff variation
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- Stochastic Environment = payoff variation
- Replicator Eq.

Brownian Motion

$$\dot{x}_i(t) = x_i(t)\{g_i(t) - g(t)\},\ g_i(t) = g_i + \zeta(t)$$

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Brownian Motion

$$\dot{x}_i(t) = x_i(t)\{g_i(t) - g(t)\},\ g_i(t) = g_i + \zeta(t)$$

- **Pro.** Let x be a strategy distribution. It satisfies :

$$P(x,t)dx = (2\pi\sigma^2 t)^{-1/2} \exp\left[-\frac{\left(\log x - \log x^*(t)\right)^2}{2\sigma^2 t} \right] \frac{dx}{x}.$$

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→Approachable under variance (σ^2)

PROOF OUTLINE

- Teramoto (1997)

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- (i) transformation

$$\dot{x}_i(t) = x_i(t)\{g_i(t) - g(t)\},\ g_i(t) = g_i + \zeta(t)$$

$$\log \frac{x_i(t)}{x(0)} - g_i t + \int_0^t g(t)dt = \sum_{k=1}^{n} \xi_k$$

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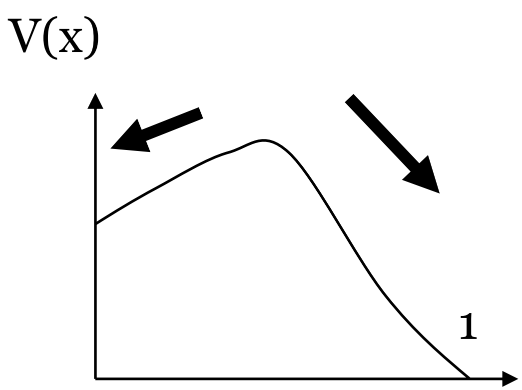
- (ii) apply central limit theorem
- (iii) transformation

EX-1.

	a_2	b_2
a_1	$2+\gamma\delta_1$, $2+\gamma\delta_1$	$\gamma\delta_1$,0
b_1	0, $\gamma\delta_1$	1, 1

- **Coordination Game**
- Replicator Eq. : $\dot{x} = x(1-x)\{3x-1+\gamma\delta_1\}$
- Equilibrium : 0, 1, $\frac{1-\gamma\delta_1}{3}$
- Potential Func. :

$$V(x) = \frac{3}{4}x^4 - \frac{4-\gamma\delta_1}{3}x^3 + \frac{1-\gamma\delta_1}{2}x^2 + C$$



- The equilibrium of the mixed strategy is **unstable**.

EX-2.

	a_2	b_2
a_1	$2+\gamma\delta_1$, $1+\gamma\delta_1$	$\gamma\delta_1$,0
b_1	0, $\gamma\delta_2$	1, 2

- Battle of Sexes (BoS)
- Replicator Eq. :

$$\dot{x} = x(1-x)\{2-\gamma\delta_2-3y\}, \qquad \dot{y} = y(1-y)\{2+\gamma\delta_1-3x\}$$

- Equilibrium point : (y^*,x^*)=(0,0), (1,0), (0,1), (1,1),

$$\left(\frac{2-\gamma\delta_2}{3}, \frac{2+\gamma\delta_1}{3}\right)$$

- The stability of the Mixed Strategy is saddle point.

3. OUR MODEL

3-2. SELTEN TYPE

Stochastic Environment (Selten (1980) + Dynamics)

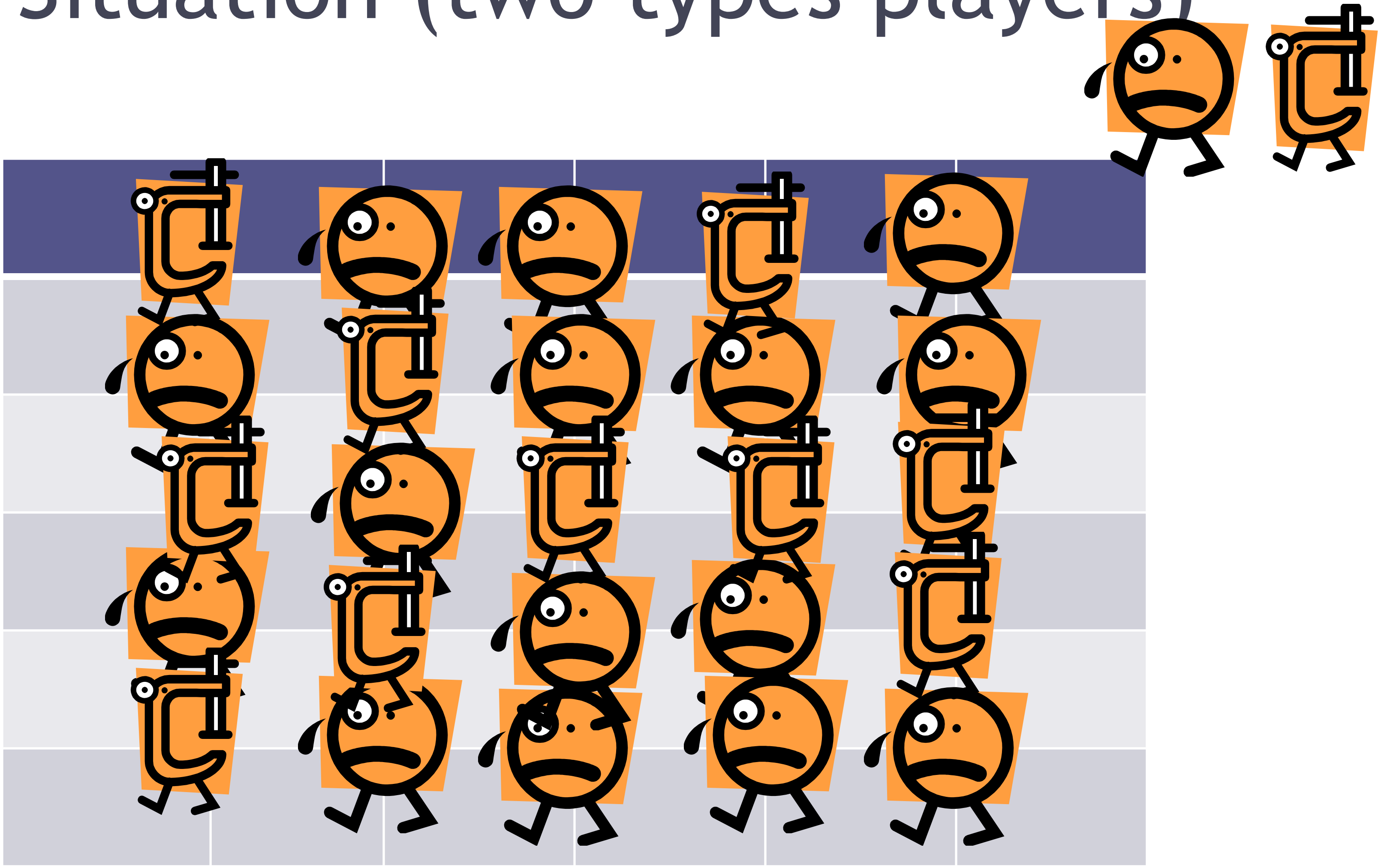
- “Role” = “Group”

Stochastic Environment (Selten (1980) + Dynamics)

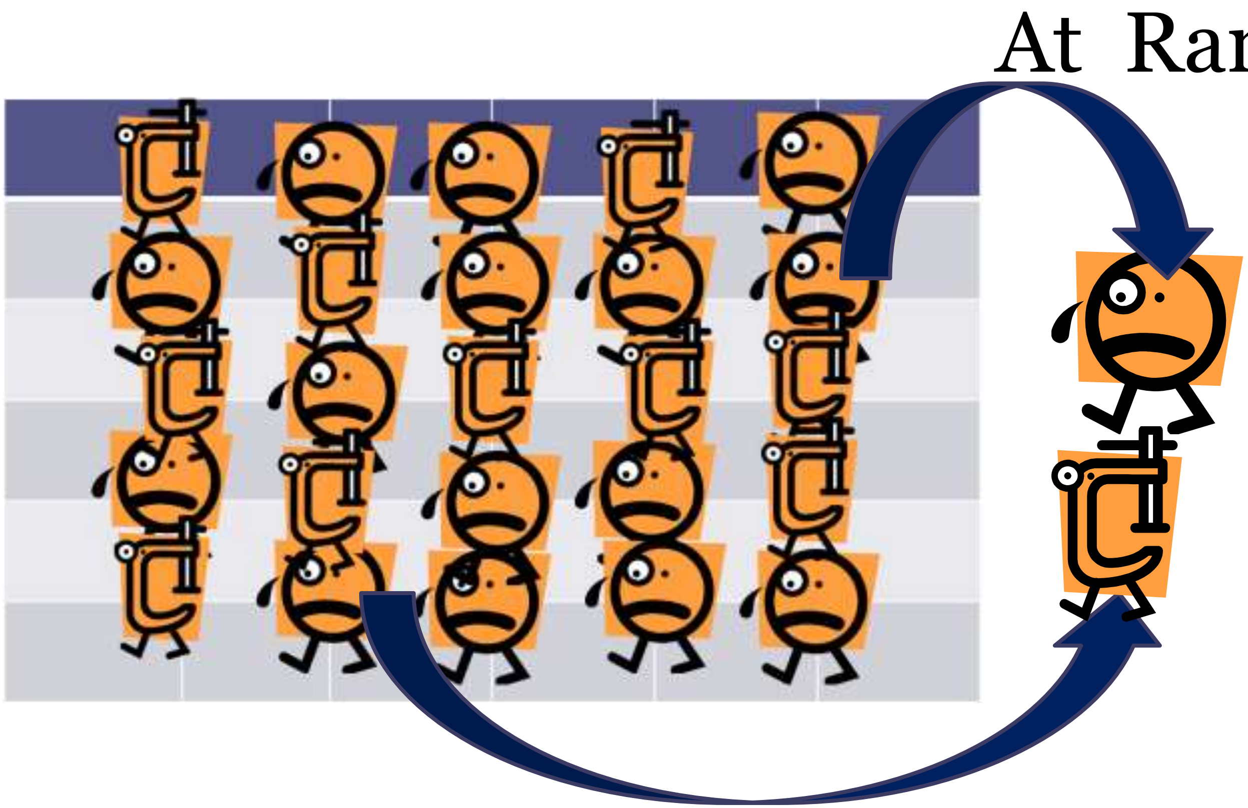
- “Role” = “Group”

- **Situation** : see next slide.

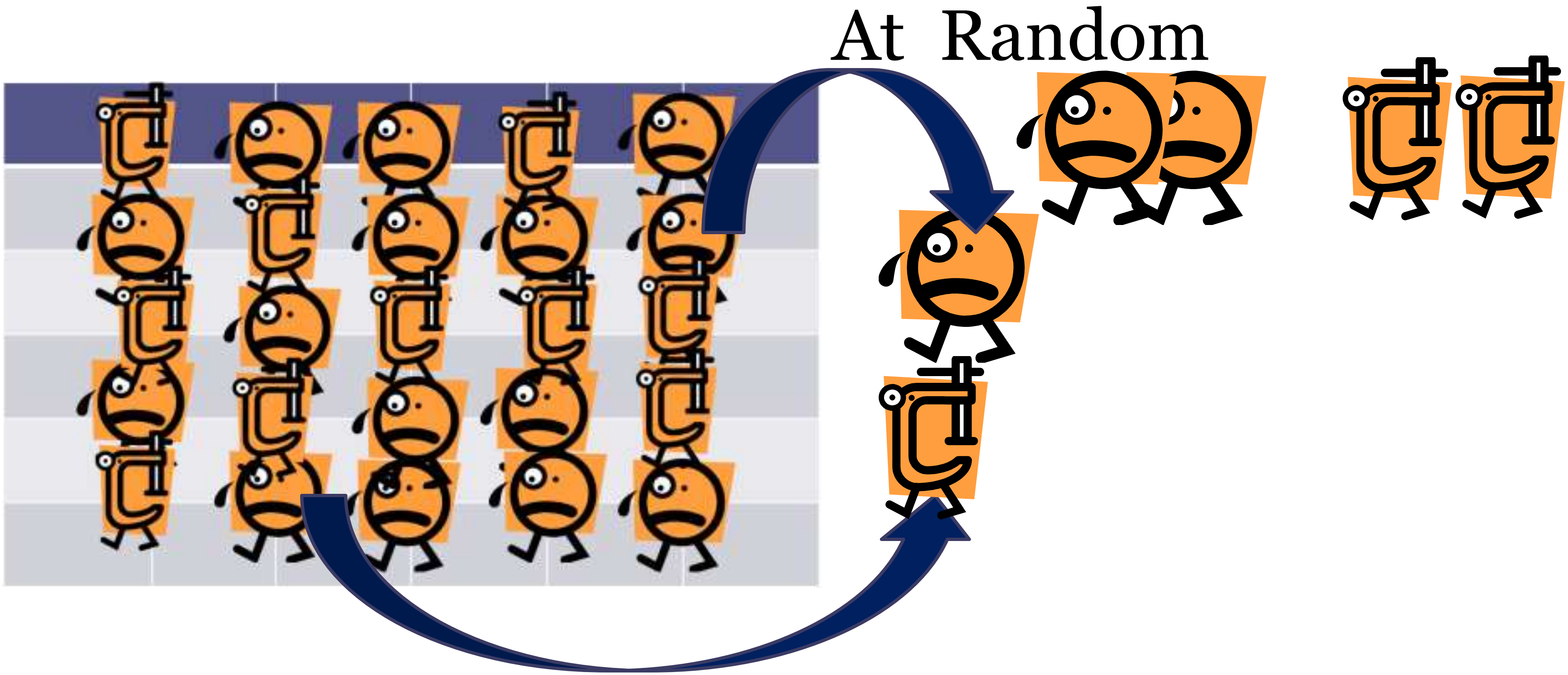
Situation (two types players)



Situation (Evolutionary Game Theory with Group Structure)



Situation (Evolutionary Game Theory with Group Structure)



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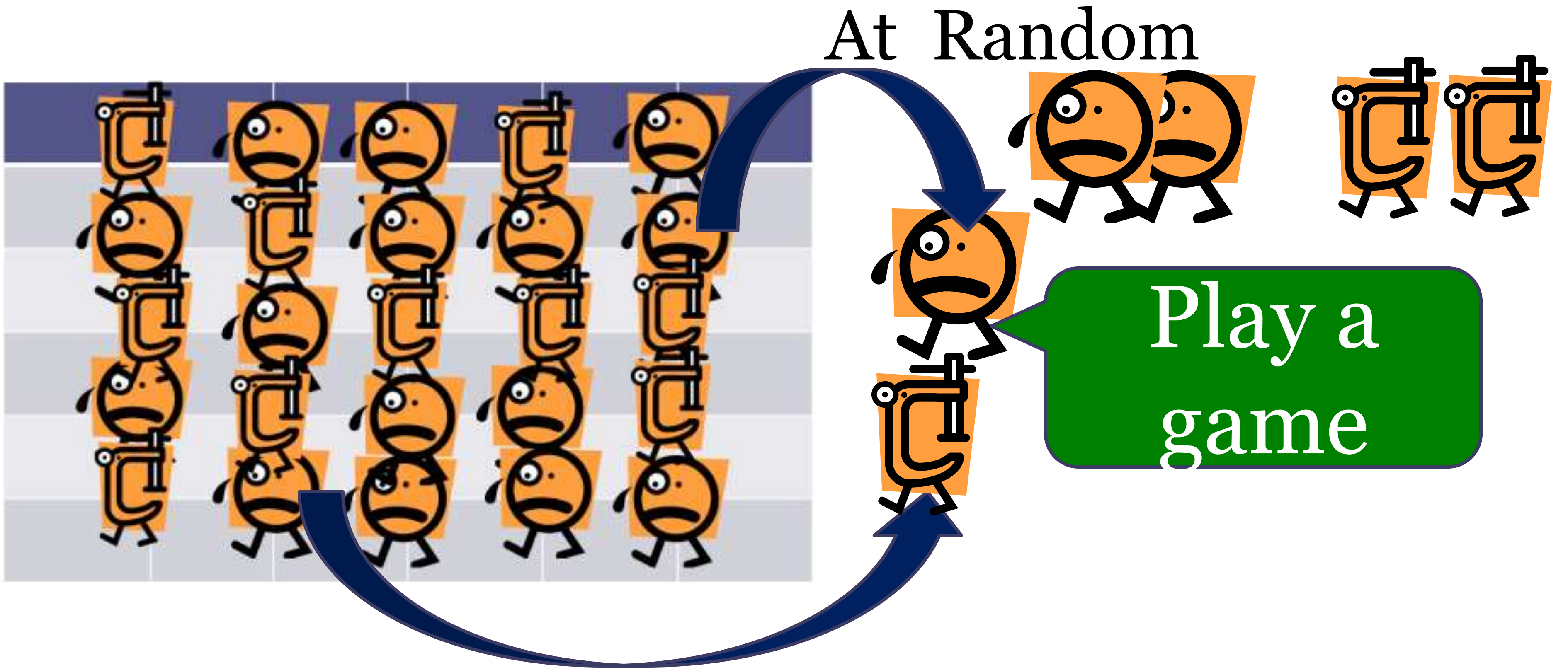
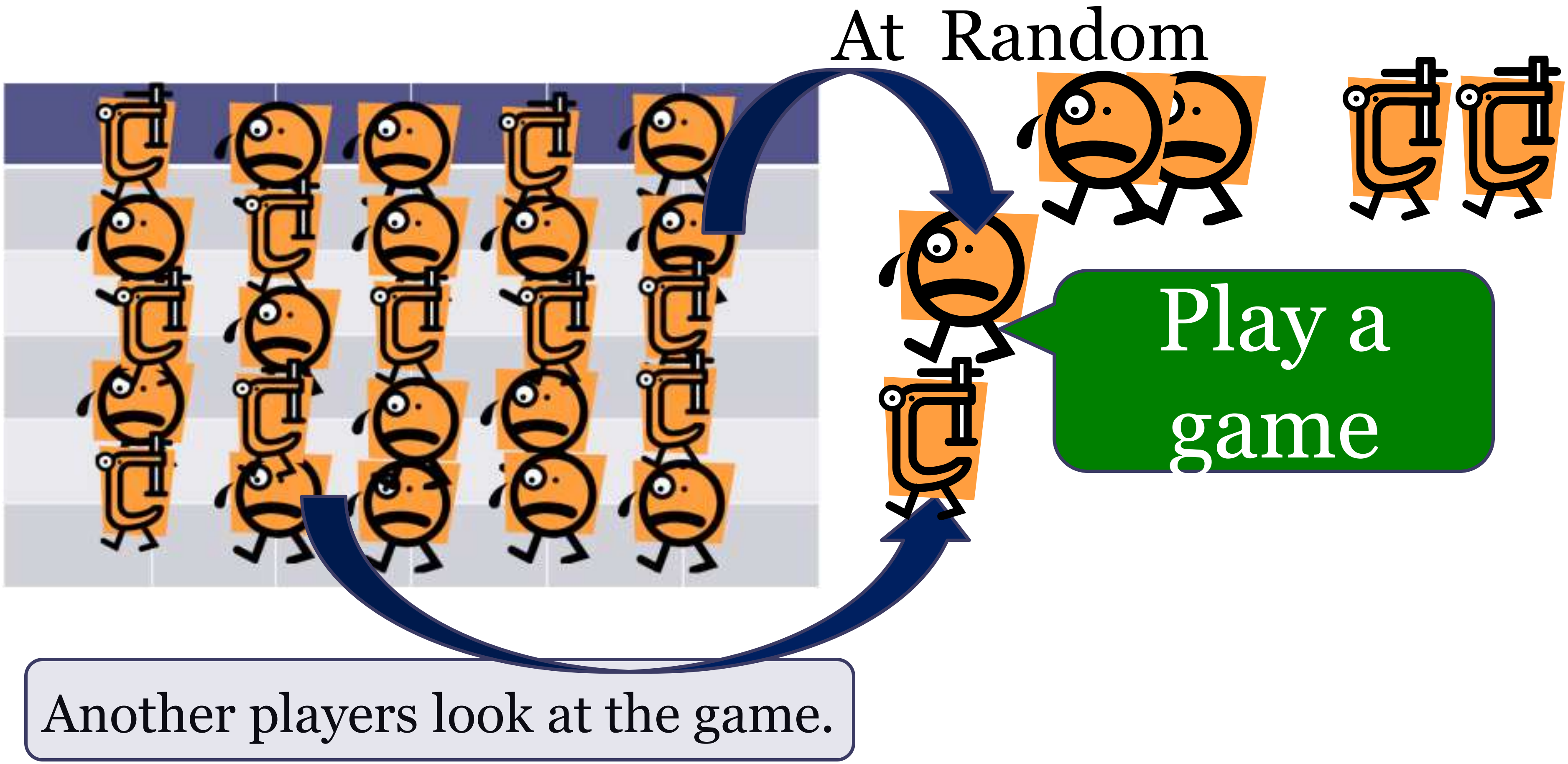
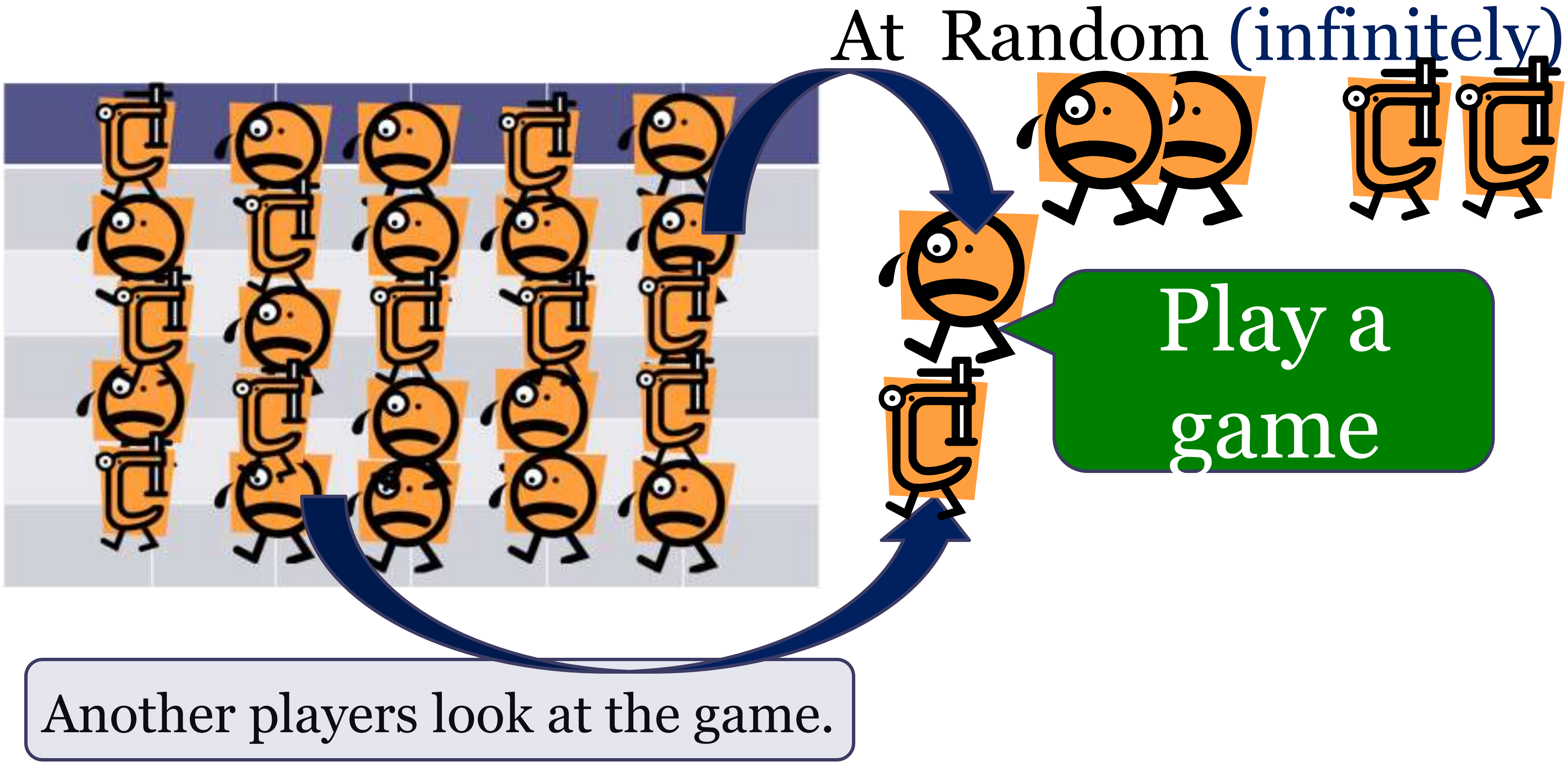


Situation (Evolutionary Game Theory with Group Structure)



Situation (Evolutionary Game Theory with Group Structure)



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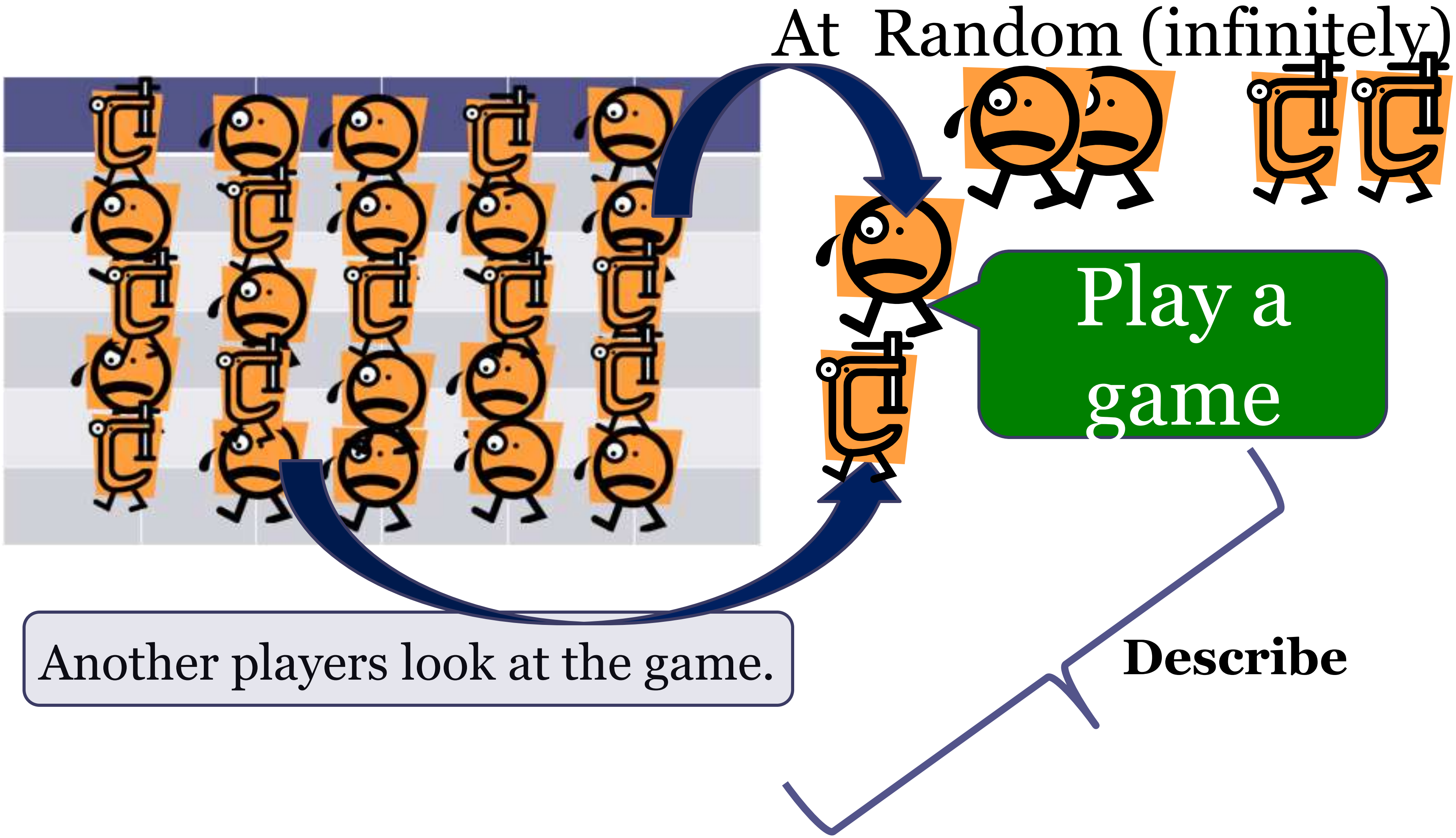
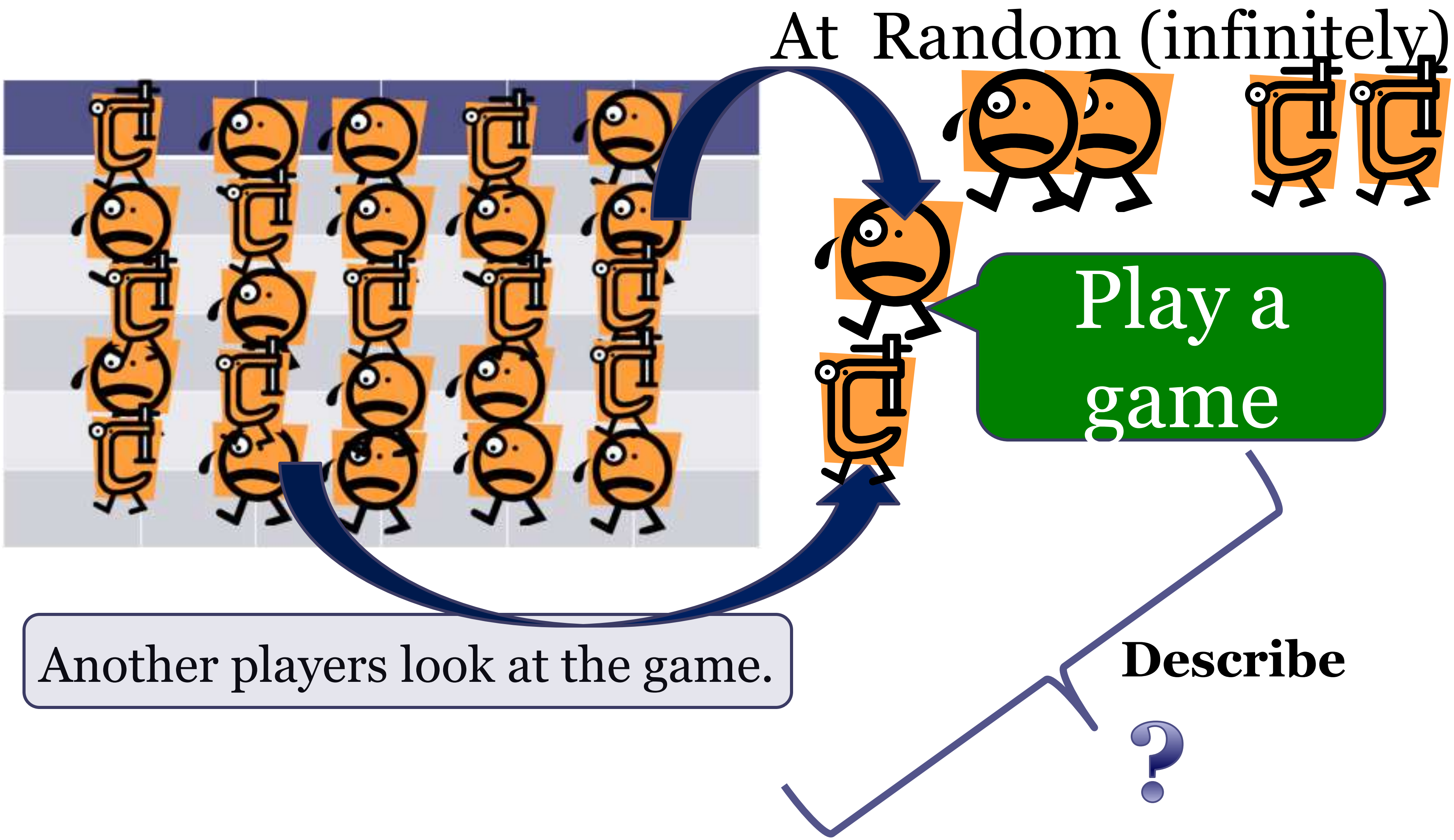


Situation (Evolutionary Game Theory with Group Structure)

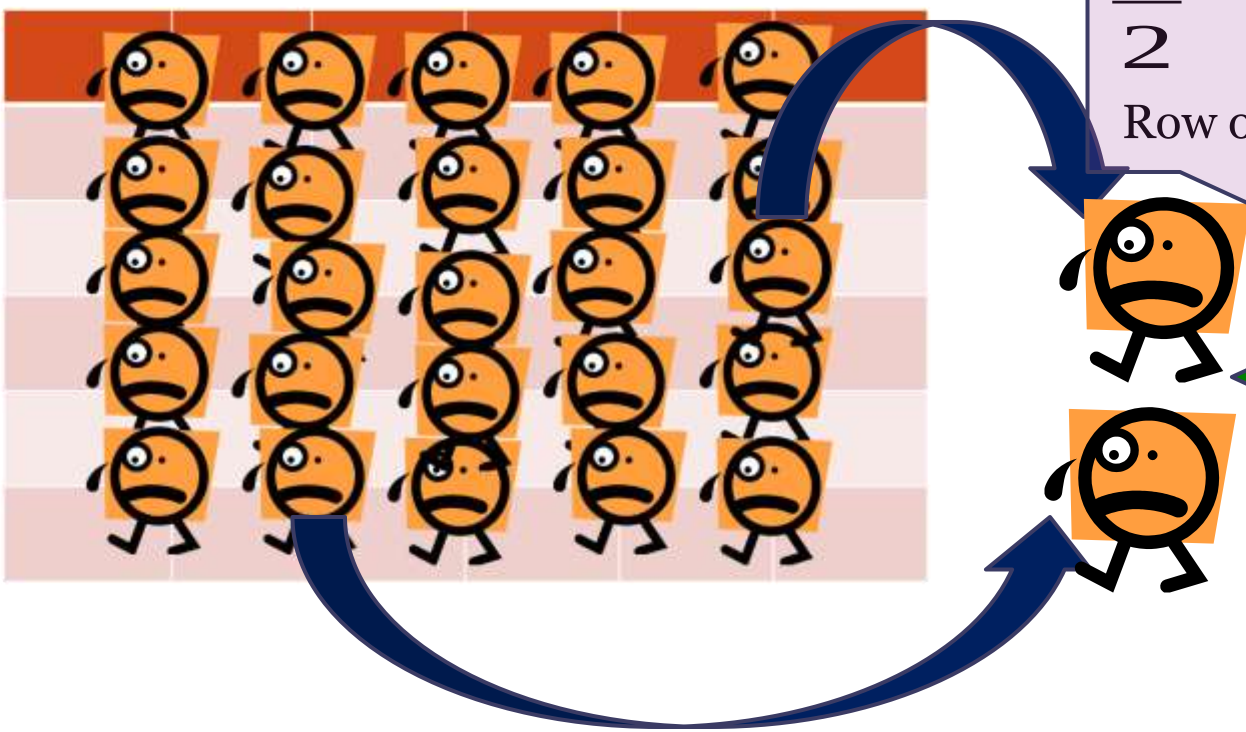


Situation (Role Completed Game)

At Random (infinitely)

$\frac{1}{2}$

Row or Column player

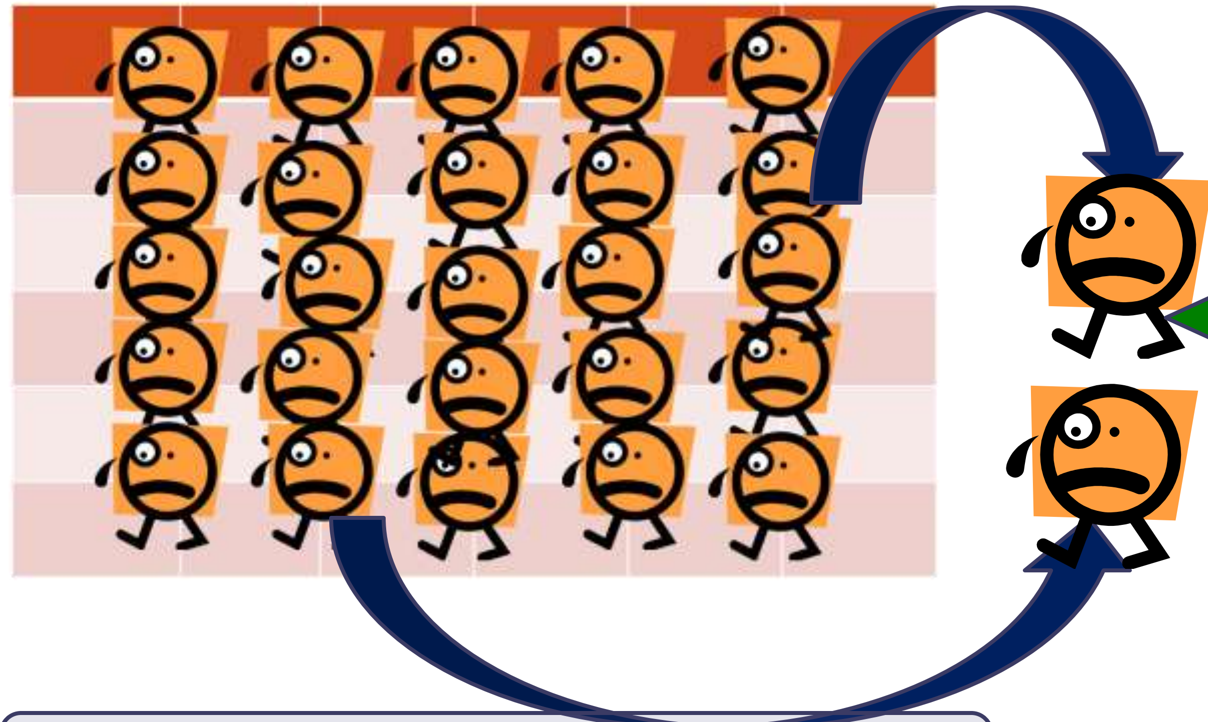


Play a game

Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



Play a game

Another players look at the game.

Replicator Equation

Stochastic Environment (Selten (1980) + Dynamics)

- “Role” = “Group”

- **Pro.** Group size and it’s fitness in a game with group structure are as follows :

  Price equation $\dot{E}(p) = Cov(f, p) + E\left(\dot{p}\right).$

PROOF OUTLINE

1) transformation

$$\bar{x}' - \bar{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\bar{\pi}} \right) x_i - \sum_i f_i \frac{\pi_i}{\bar{\pi}} \Delta x_i$$

where $\Delta x_i = x'_i - x_i$

PROOF OUTLINE

1) transformation

$$\bar{x}' - \bar{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\bar{\pi}} \right) x_i - \sum_i f_i \frac{\pi_i}{\bar{\pi}} \Delta x_i$$

where $\Delta x_i = x'_i - x_i$

2) $\bar{\pi} \Delta \bar{x} = \sum_i f_i (\pi - \bar{\pi}) x_i + \sum_i f_j \pi_i \Delta x_i$ where $\Delta \bar{x} = \bar{x}' - \bar{x}$

PROOF OUTLINE

1) transformation

$$\bar{x}'-\bar{x}=\sum_i f'_i\cdot x'_i-\sum_i f_i\cdot x_i=\cdots=\sum_i f'_i\left(\frac{\pi_i}{\bar{\pi}}\right)x_i-\sum_i f_i\frac{\pi_i}{\bar{\pi}}\Delta x_i$$

where $\Delta x_i = x'_i - x_i$

2) $\bar{\pi}\Delta\bar{x}=\sum_i f_i(\pi-\bar{\pi})x_i+\sum_i f_j\pi_i\Delta x_i$ where $\Delta\bar{x}=\bar{x}'-\bar{x}$

3) Definition

$$Cov[\pi,x]=\sum_i f_i(\pi_i-\bar{\pi})(x_i-\bar{x}),\ \sum_i f_i(\pi_i-\bar{\pi})\bar{x}=0$$

$$\bar{\pi}\Delta\bar{x}=Cov[\pi,x]+E[\pi\Delta x].$$

PROOF OUTLINE

1) transformation

$$\overline{x}'-\overline{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\overline{\pi}}\right) x_i - \sum_i f_i \frac{\pi_i}{\overline{\pi}} \Delta x_i$$

where $\Delta x_i = x'_i - x_i$

2) $\overline{\pi}\Delta\overline{x} = \sum_i f_i(\pi - \overline{\pi}) x_i + \sum_i f_j \pi_i \Delta x_i$ where $\Delta\overline{x} = \overline{x}' - \overline{x}$

3) Definition

$$Cov[\pi, x] = \sum_i f_i(\pi_i - \overline{\pi})(x_i - \overline{x}), \sum_i f_i(\pi_i - \overline{\pi})\overline{x} = 0$$

$$\overline{\pi}\Delta\overline{x} = Cov[\pi, x] + E[\pi\Delta x].$$

- **Remark** : Price equation is equivalent to Replicator equation.

	H	D
H	a, a	0,0
D	0, 0	b, b

Payoff matrix

EX.

- Two type agent : {S,A}
- Random Matching : {SS}, {SA}, {AA}

$$Cov[\pi, x] = \sum_{i \in \{AA, AS, SS\}} f_i \left(\pi_i - \overline{\pi}\right)\left(x_i - \overline{x}\right) = f(1-f)\{f(a+b) - b\}.$$

- Price Eq. = Replicator Eq.

- H-D game (a, b < 0)
- $Cov[\pi,x]=0 \Leftrightarrow f=0,1, b/(a+b)$.

4. EXTENSION

GLOBAL GAME

	C	D
C	x, x	x,0
D	0, x	a, a

a>0

Global Game

(1) Complete information about x

(i) unique Nash eq.

$x<0$: strategy "D", $x > a$: strategy "C"

(ii) Multiple eq. $x \in [0,a]$: strategy "C" and "D"

•

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Global Game

(1) Complete information about x

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$x<0$: strategy "D", $x > a$: strategy "C"

(ii) Multiple eq. $x \in [0,a]$: strategy "C" and "D"

(2) Incomplete information about x

- Player i observes a private signal $s=x+\varepsilon_i$.

Pro. (Carlsson and van Damme, 1993) Let $\gamma \in \{\alpha, \beta\}$. If x lies on a continuous curve C such that $C \subseteq \Theta$, $g(C) \subseteq R^{\gamma}$, and $g(C) \cap D^{\gamma} \neq \emptyset$, then γ is iteratively dominant at x in Γ^{ε} if ε is sufficiently small.

	C	D
C	x, x	x,0
D	0, x	a, a

a>0

Global Game

(1) Complete information about x

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→ unique equilibrium : $x \in [0,a]$

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)
(2) Group Structure : {S,A}

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)
(2) Group Structure : {S,A}
(3) Price eq.

$$Cov[\pi, x] = f(1-f)\{af-(a-x)+r(x-af)\}$$

$$Cov[\pi,x] = 0 \Leftrightarrow f=0,\ 1\ ,\ \frac{a\{f(r-1)+1\}}{1+r}.$$

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)
(2) Group Structure : {S,A}
(3) Price eq.

$$Cov[\pi, x] = f(1-f)\{af-(a-x)+r(x-af)\}$$

$Cov[\pi,x] = 0 \Leftrightarrow f=0, 1,\ \dfrac{a\{f(r-1)+1\}}{1+r}.$

$r \to 1$: $x>a/2$, $Cov[\pi,x]>0$, $x<a/2$, $Cov[\pi,x]<0$
→ ESS Unique.

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Global Game

(1) Complete information about x

(i) unique Nash eq.

$x<0$: strategy "D", $x > a$: strategy "C"

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→ unique equilibrium : $x \in [0,a]$

5. Application

利得が確率的に変動 →ファイナンス理論へ応用

売り手と買い手の行動から考えるとどのようなBlack-Sholes (ヨーロピアンコールオプション)の公式が導かれるのか？

OPTION

- **オプション(option)** とは売買を行う権利のことであり、買い付ける権利を**コール(call)**、売り付ける権利を**プット(put)**と言います。
-
-

OPTION

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- 日本で取引されている株価指数オプションには、**日経225オプション、日経300オプション**(大阪証券取引所)、**TOPIXオプション**(東京証券取引所)。

[DATA] 大阪証券取引所日報

http://www.nippo.ose.or.jp/pdf.html

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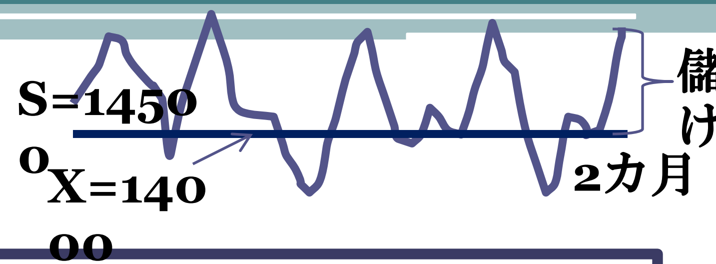
http://www.nippo.ose.or.jp/pdf.html

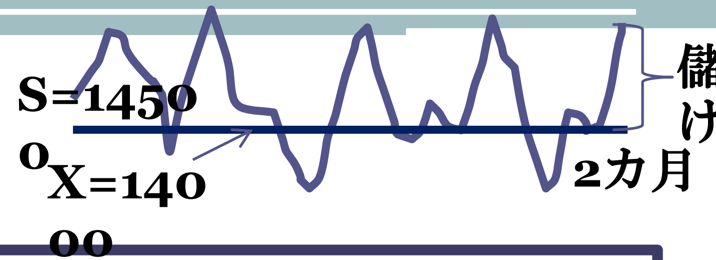
- **Black-Sholesの公式・・・ヨーロピアン・オプションの価格評価公式**

PROBLEM



- 次のヨーロピアン・コールオプションの価格を求めよ。
  現在の株価 S=14500円, 権利行使価格 K=14000円, オプションの期間=2ヵ月, ボラティリティ σ =38%, 非危険利子率 r=6%

PROBLEM



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→あなたは上記の条件でいくら支払い、購入する権利を得るのか？

SOLUTION.

- **Black-Sholes Formula**

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{x}} + \sigma\sqrt{x}\right) - K \cdot \exp(-rx) \cdot N\left(\frac{u}{\sigma\sqrt{x}}\right).$$

SOLUTION.

- **Black-Sholes Formula**

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{x}} + \sigma\sqrt{x}\right) - K \cdot \exp(-rx) \cdot N\left(\frac{u}{\sigma\sqrt{x}}\right).$$

- オプションの期間 T-t=2/12=0.1667
- u=log(S/K)+(r-σ²/2)(T-t)=0.0331
- u/σ√x+σ√x=0.3685, u/σ√x=0.2133.
- 標準正規分布の数表から
- N(u/σ√x+σ√x)=0.6437, N(u/σ√x)=0.5845.
- 以上から, ヨーロピアンコールオプション価格は f(S,t)=14500×0.6437-14000×exp(-0.06×0.1667)×0.5845 = **1232.0884**円

SOLUTION.

- **Black-Sholes Formula**

$$f(S,t)=S\cdot N\left(\frac{u}{\sigma\sqrt{x}}+\sigma\sqrt{x}\right)-K\cdot\exp(-rx)\cdot N\left(\frac{u}{\sigma\sqrt{x}}\right).$$

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⇒2ヶ月後の株価 ＞ **15232**円「儲け」
＜ 「損」

Model

- **連続時間**
- 財：安全資産(金利r)、危険資産: 幾何ブラウン運動
- 主体：売り手と買い手 (非対称2人ゲーム)
- δt の間に, 現在の株価を見て、自分と相手の利得を勘定し、自らの戦略を決定するということを行っている。
- 戦略：2つ。例：{bear, bull} など
- 利得：売り手：K(t)-S(t), 買い手: S(t)-K(t)
- →ゼロ＝サム型
- **+仮定**：無裁定条件

- 利得表

	戦略1	戦略2
戦略1	a(t), -a(t)	0,0
戦略2	0, 0	b(t),-b(t)

- ↑の利得表は相対的な「利得差」を表している。

- 利得表

	戦略1	戦略2
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戦略2	0, 0	b(t),-b(t)

- ↑の利得表は相対的な「利得差」を表している。

このときのReplicator 方程式

$$\dot{s}_1 = s_1(1-s_1)\{a(t)-(a(t)+b(t))s_2\},$$

$$\dot{s}_2 = s_2(1-s_2)\{-a(t)+(a(t)+b(t))s_1\},$$

s_1を主体1が戦略1を採用する確率, s_2 を主体2が戦略2を採用する確率

平衡点とその安定性

- このときの平衡点は純粋戦略の組み4つと、内点解(=混合戦略)の5つ存在する.

-

平衡点とその安定性

- このときの平衡点は純粋戦略の組み4つと、内点解(=混合戦略)の5つ存在する.
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平衡点とその安定性

- このときの平衡点は純粋戦略の組み4つと、内点解(=混合戦略)の5つ存在する.

- ESSは内点解。

補題 ノイズがない場合の内点の安定性はリミットサイクルであり, またノイズがある場合もリミットサイクルである.

この場合のBlack-Sholesの公式

- 前に取り上げたBlack-Sholesモデルにおいて、行使価格の影響があるのは、境界条件を使用するとき。
- よって、K:= $\overline{K}$ とすればよい. つまり

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{\tau}} + \sigma\sqrt{\tau}\right) - \overline{K} \cdot e^{-r\tau} \cdot N\left(\frac{u}{\sigma\sqrt{\tau}}\right).$$

ただし$\overline{K}$ ＝ 平衡時の戦略1における行使価格 $\cdot s_1^*$
＋ 平衡時の戦略2における行使価格 $\cdot (1 - s_1^*)$
s_1^* は混合戦略を採用する場合の確率.

注)　離散時間の場合は確率的進化ゲーム理論を応用させることによって、同様にBlack-Sholesの公式を導出することができる.

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ただし$\overline{K}$ = 平衡時の戦略1における行使価格 $\cdot s_1^*$

\+ 平衡時の戦略2における行使価格 $\cdot (1 - s_1^*)$

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6. SUMMARY and FUTURE WORKS

OUR PROBLEM

- Q How does each player choose the action in stochastic environment ?

- A.1：Each player randomly chooses the action. (mixed strategy) (Harsanyi , 1973)

- A.2：Each player chooses the better action. (pure strategy) (Selten, 1980)

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→ incomplete information

Summary

1. Harsanyi(1973)+ Dynamics :

2. Selten (1980)+ Dynamics :

3. Global Game + Dynamics :

Summary

1. Harsanyi(1973)+ Dynamics :

$\rightarrow$ log-normal distribution (central limit theorem)

2. Selten (1980)+ Dynamics :

3. Global Game + Dynamics :

Summary

1. Harsanyi(1973)+ Dynamics :

$\rightarrow$ log-normal distribution (central limit theorem)

$\rightarrow$Approachable under variance (σ^2)

2. Selten (1980)+ Dynamics :

3. Global Game + Dynamics :

Summary

1. Harsanyi(1973)+ Dynamics :

→ log-normal distribution (central limit theorem)

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→ Bayesian Game = Game with Group Structure

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Summary

1. Harsanyi(1973)+ Dynamics :

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→ Price eq. = Replicator eq.

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Summary

1. Harsanyi(1973)+ Dynamics :

→ log-normal distribution (central limit theorem)

→Approachable under variance (σ^2)

2. Selten (1980)+ Dynamics :

→ Bayesian Game = Game with Group Structure

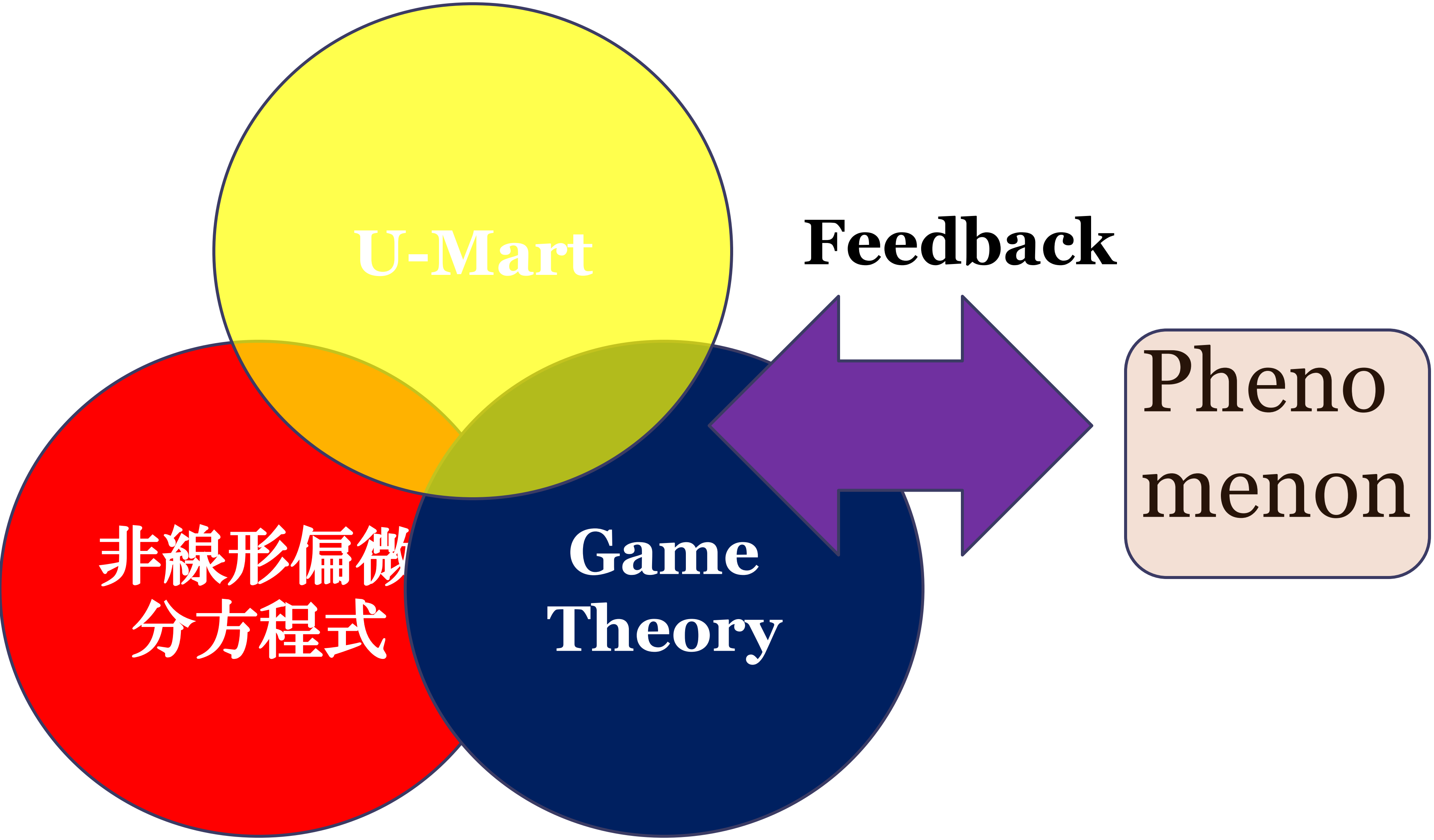
→ Price eq. = Replicator eq.

3. Global Game + Dynamics :

FUTURE WORKS

- U-Mart (http://www.u-mart.org/html/)
- Option Market の人工市場

Future Works.



Thank you for your attention.

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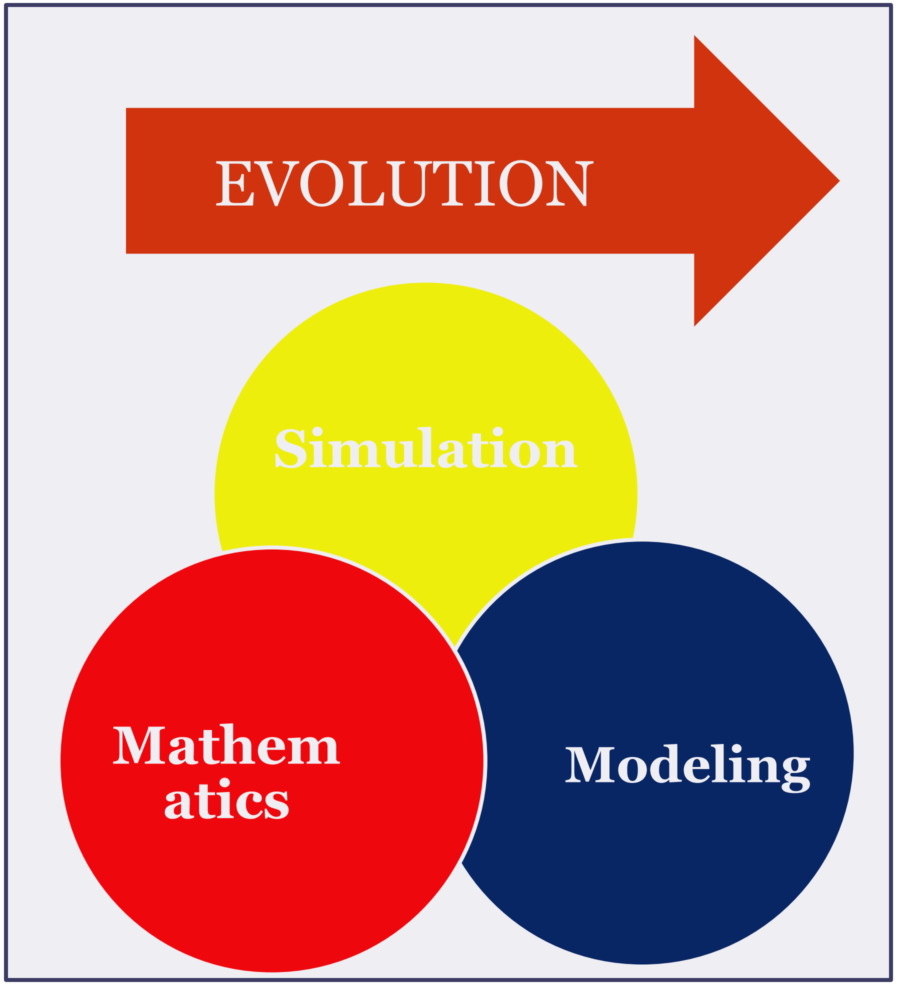

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明治大学
グローバルCOEプログラム
現象数理学の形成と発展

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おまけ

進化ゲーム理論におけるトラバース（移行過程）理論

[15] 吉川満:「進化ゲーム理論の数理」『第4回数学総合若手研究集会 - 他分野との学際的交流を目指して - (北海道大学)』, 2008年2月13日.
【Abstract】,【Slide】,【Full Paper】,
【Photo】

進化ゲーム理論におけるトラバース理論 (要旨)

- 進化ゲーム理論においては、「**均衡選択**」が重要（最初のスライド）。
- 平衡状態ではどの均衡に収束するのか？

トラバース理論

→複数の均衡を行き来する。

「外生的なショックにより、均衡が変化する」というのは、よくありそうな議論。

ここでは、「内生的に均衡が変化していく」。

→**Milnor Attractor**.

仮定3.1, 3.2.

- **仮定**: 純粋戦略は無限集合であり、その実現可能集合Uは有界閉集合（コンパクト）であるとする。
- **仮定**: 利得関数 $F\left(q_i, q_j\right)$ は q_i, q_j 共に2回微分可能である。

このときのReplicator方程式は、次のようなものとなる (Bomze(1990))。

$$\frac{dP}{dt}\left(B\right)=\int_B\left(\pi\left(x,P,\mu(S)\right)-\pi\left(P,P,\mu(S)\right)\right)P(dx)$$

定義3.3. CSS

- **定義**: Eshel(1983) 戦略 q_u **が連続的に安定な戦略**(Continuously Stable Strategy, CSS) であるとは, (1) ESS である. (2) 任意の q_v について $|q_u - q_v| < \varepsilon$ を満たすような $\varepsilon > 0$ が存在し、任意の q_i について $|q_v - q_i| < \eta$ を満たすような $\eta > 0$ が存在し、次の関係を満たすときをいう。

(3.1) $F(q_v, q_i) > F(q_i, q_i)$ if and only if $|q_v - q_u| < |q_i - q_u|$

平衡点 q_i^* からずれているとき q_i^* より q_i^* に近い突然変異戦略が必ず進入できる。したがって戦略は突然変異戦略の進入と置換の繰り返しによって平衡点 q_i^* に近づく。

命題3.4.

- **命題**: Eshel(1983)　$\hat{q}_i$ がESS であるための必要条件は、次の条件を満たすときである。

(3.2)　(i)
$$\left.\frac{\partial}{\partial q_j} F\left(q_j, \hat{q}_i\right)\right|_{q_j=\hat{q}_i} = 0,$$

(ii)
$$\left.\frac{\partial^2}{\partial q_j^2} F\left(q_j, \hat{q}_i\right)\right|_{q_j=\hat{q}_i} \leq 0.$$

⇒この条件は利得関数が極大であるための必要条件である。

命題3.5.

- **命題**: Eshel (1983)

(i) ESS q_i が $q_i = q_j = \hat{q}_i$ において、CSS となる必要条件は次の条件を満たすときである。

(3.3) $$\frac{\partial^2 F}{\partial q_i \partial q_j} + \frac{\partial^2 F}{\partial q_i^2} \leq 0.$$

(ii) ESS $\hat{q}_i$ がCSS であるための十分条件は, (3.2)-(ii), (3.3) の等号を除いたものが成り立つことである。

ESS と　CSS の関係

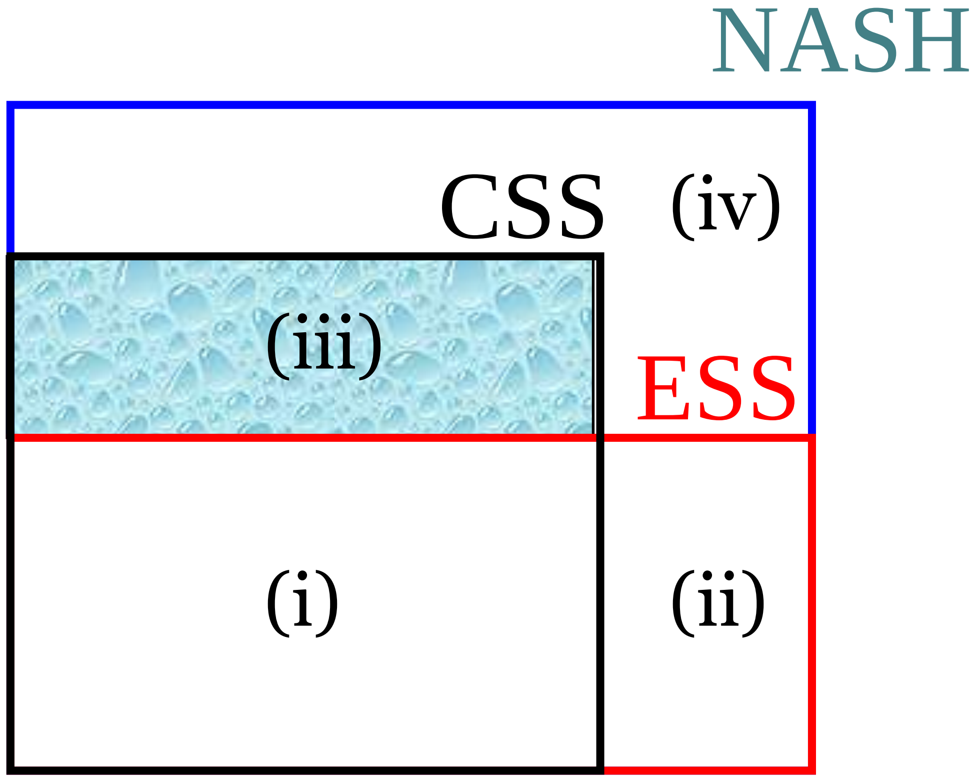
	(3.3)	Not (3,3)
(3.2)-(ii)	(i) 漸近安定	(ii) Lyapunov 安定
Not (3.2)-(ii)	(iii) ?	(iv) 漸近不安定

(i) 漸近安定な平衡点、(ii) Lyapunov安定

(iii) ESSでないがCSS(戦略の2極分化など興味深い Sasaki and Ellner(1995))

(iv) 不安定

NASH , ESS, CSS



定義3.6 MILNOR ATTRACTOR(Milnor 1985)

定義: A closed subset $A \subset M$ will be called an ***(Milnor) attractor*** if it satisfies two conditons:

(1) the *realm of attraction* ρ(A), consisting of all points $\chi \in M$ for which $\varpi(x) \subset A$, must have strictly positive measure; and

(2) there is no strictly smaller closed set $A' \subset A$ so that ρ(A’) coincides with ρ(A) up to a set of measure zero.

この定義には、その近傍の**全て**の軌道がそこに吸収されるという条件が含まれて**いない**。よってアトラクターの近傍から離れていく軌道が存在する。

一般に社会現象は定常的な概念ではなく、**常に変化**している。

→これに対応する概念として、**Milnor Attractor**.

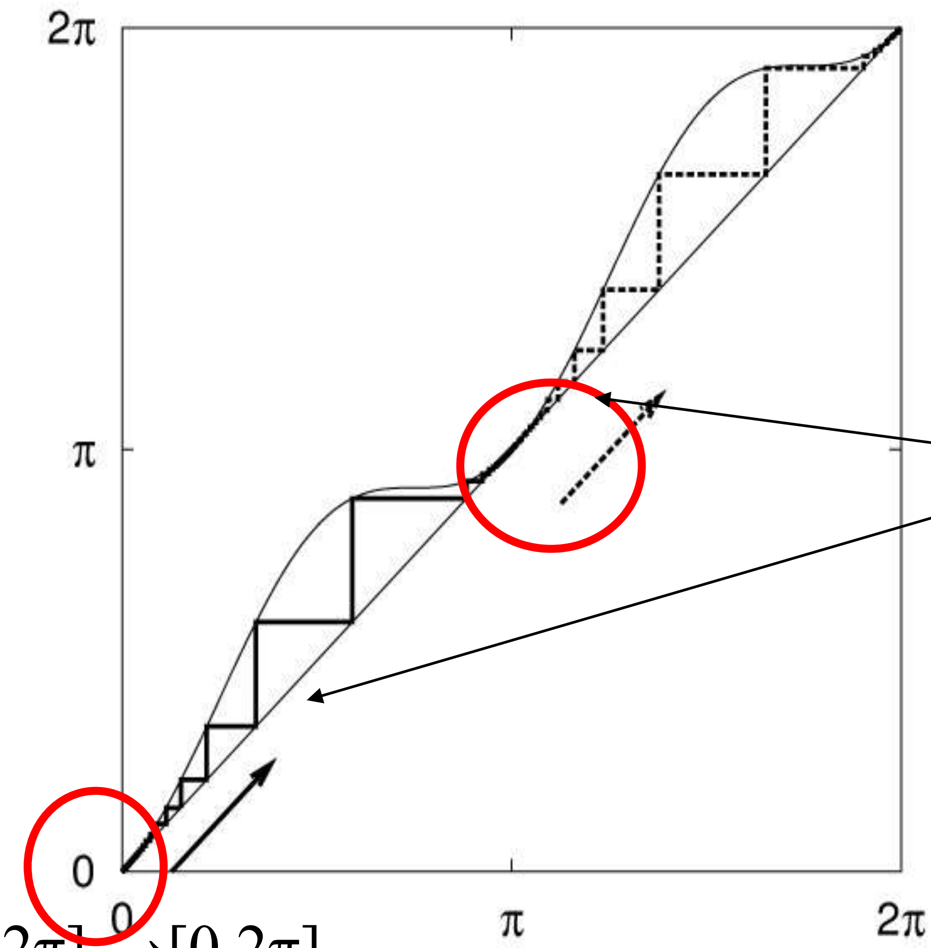
定義: ATTRACTOR

- 従来の意味でのアトラクター

定義: 力学系 f の閉不変集合Λが**アトラクター**(attractor) であるとは、Λの近傍 U で、 $f(U) \subset U$ かつ $\Lambda = \bigcap_{n \geq 0} f^n(U)$ となるものが存在することをいう.

よって従来の意味でのアトラクターとは、Λの近傍U 内の点 f で写像された後もΛの近傍内に留まり続けることを意味している。

EXAMPLE 3.7.:



Milnor Attractor

x = 0, π

f:[0,2π]→[0,2π]

$$x_{n+1} = f(x_n) = x_n + \sin^2 x_n \quad (\mathrm{mod} \quad 2\pi)$$

命題3.8.

- 命題: ある均衡 $\hat{q}_i$ が Milnor Attractor であるための必要条件は次を満たすことをいう.

(3.4) (i)

$$\left.\frac{\partial^2}{\partial q_j^2} F\left(q_j, \hat{q}_i\right)\right|_{q_j=\hat{q}_i} > 0.$$

(3.5) (ii)

$$\left.\frac{\partial^2 F}{\partial \hat{q}_i \partial q_j} + \frac{\partial^2 F}{\partial \hat{q}_i^2}\right|_{q_i=q_j=\hat{q}_i} \leq 0.$$

⇒均衡は不安定であるが、均衡に近づいている軌道の集合が存在することを意味している。

PRELIMINARIES (EVOLUTIONARY GAME THEORY)

EVOLUTIONARY STABLE STRATEGY (ESS)

DEF.： Weibull(1995): $x \in \Delta$ is an ***evolutionary stable strategy (ESS)*** if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$.

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

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$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

INTERPRETATION：incumbent payoff (fitness) is higher than that of the post-entry strategy

（ESS : ①the solution of the Replicator equation + ② asymptotic stable.）

PROPOSITION

PRO.(Bishop and Cannings (1978)): $x \in \Delta$ is evolutionary stable strategy if and only if it meets these first-order and second-order best-reply :

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Nash Eq.

$$(2.5) \quad \begin{array}{l} u(y,x) = u(x,x) \\ \Rightarrow u(y,y) < u(x,y), \end{array} \quad \forall y \neq x,$$

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Asymptotic Stable Conditon

Acknowledgements

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2009年3 月　　　吉川　満。

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