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Statistical Mechanics of Games ： Evolutionary Game Theory

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OUTLINE

1. Introduction （Motivation, Purpose）
2. Related Literatures and Preliminaries
3. Our Model

   3-1. Nearest neighbor （Ising TYPE）

   3-2. Random Matching （SK MODEL）

   Annealed System, Quenched System
4. Implication : Cont- Bouchaud's Model
5. Summary and Future Works

1． INTRODUCTION

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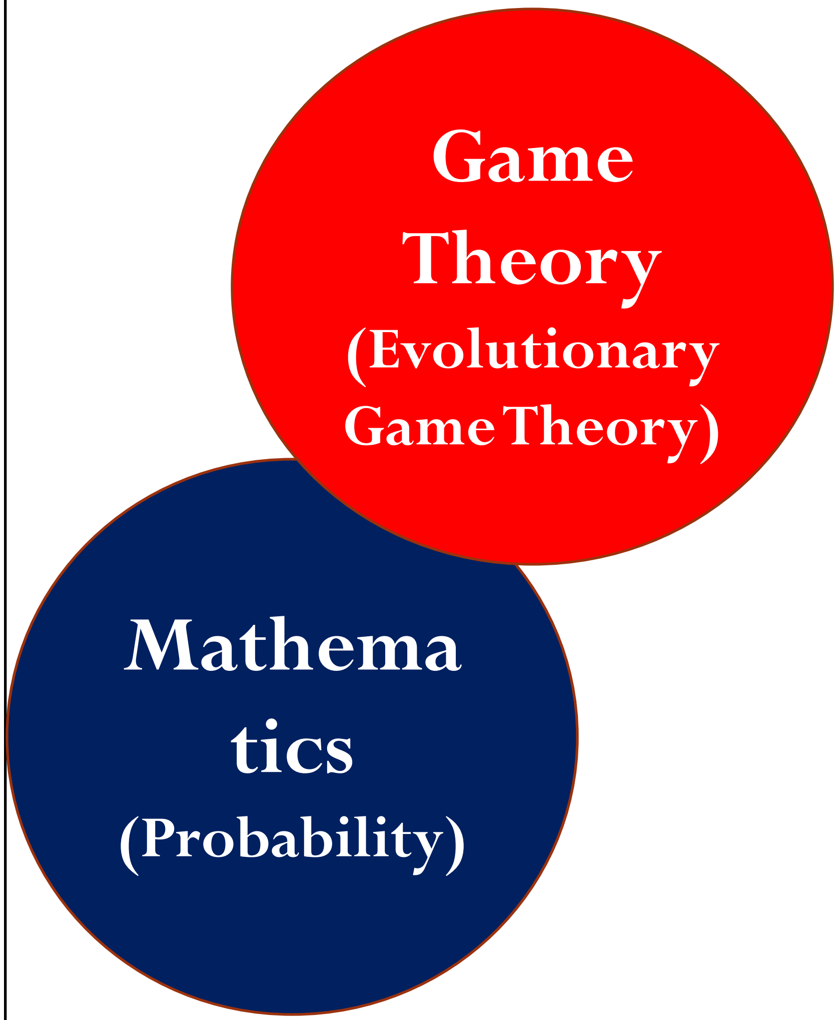
- The emergence of the equilibrium using “**Phase Transition(相転移)**“

Research Fields (this study)

Game Theory (Evolutionary Game Theory)

Research Fields (this study)



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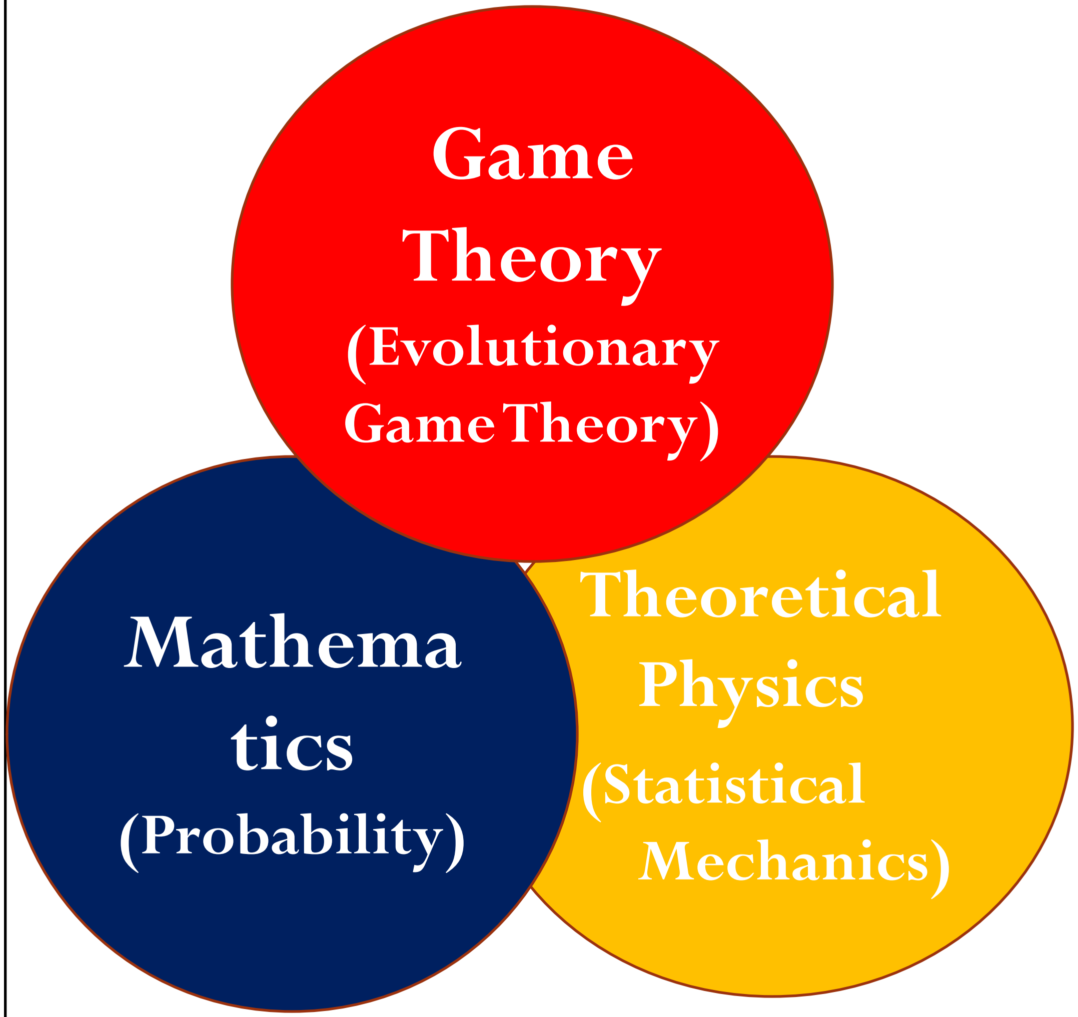

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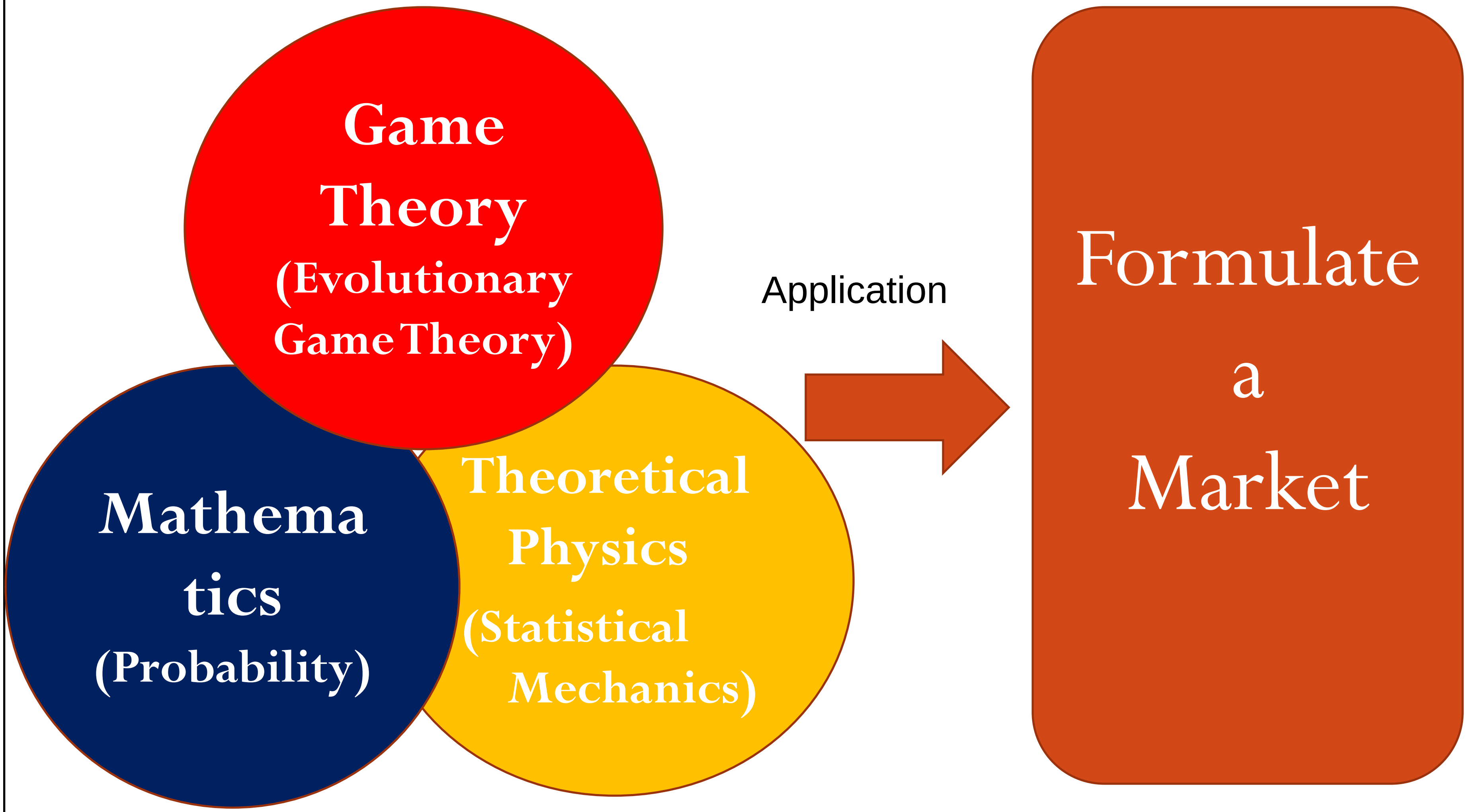
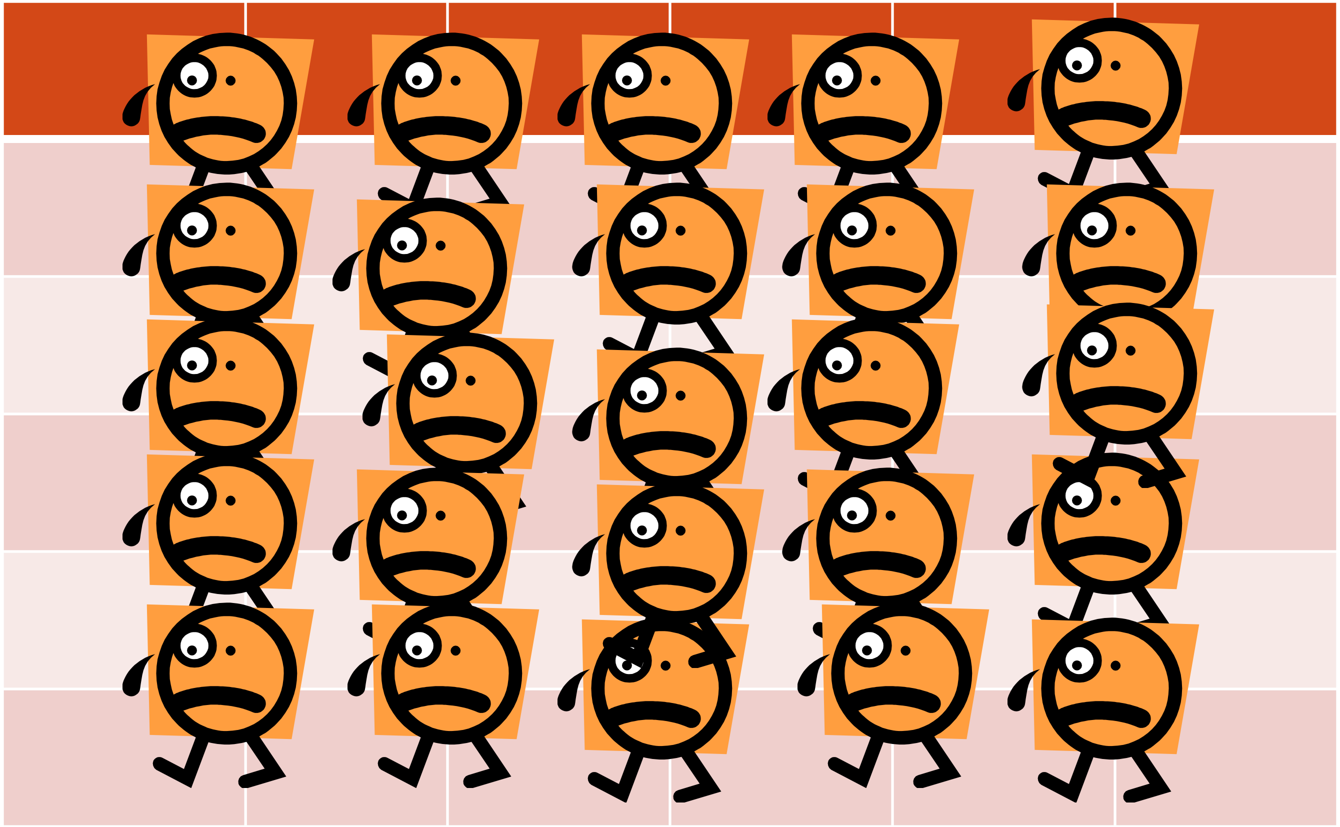


Situation



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例） 1. **Micro** ： → **Debreu and Scarf (IER,1963)**

→Replica Economy

2. **Macro** (Micro-foundation):

→Representative man.

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統計力学では、分布を考える。

MOVITATION

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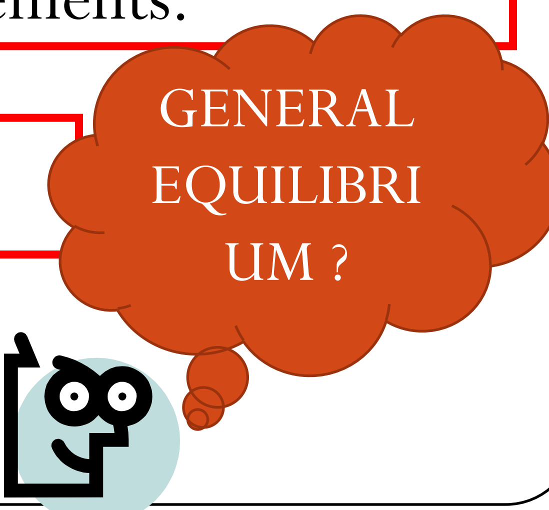
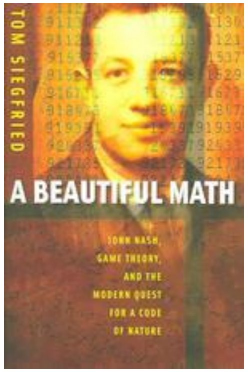


2 .　Related Literatures and Preliminaries

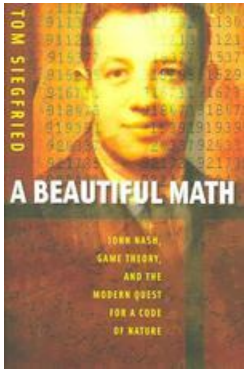
Related Literatures



Tom Siegfried **A Beautiful Math: John Nash, Game Theory, And the Modern Quest for a Code of Nature** (**「世界で最も美しい数学」**) , Joseph Henry Press, 2006/09/25.

→ Ch. 11

Related Literatures



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- **Blume (GEB, 1993) , McKelvey and Palfrey (GEB, 1995, JER, 1996)**

→ Ising model.

- **Diederich and Opper(PRA,1989)**

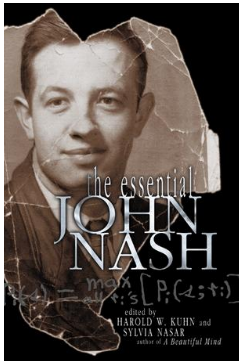
→ SK model (Spin Glass)

Contribution:

SK model : Lyapunov function (fitness function)

Interpretation of Nash Equilibrium (J.F.Nash's Ph D. Thesis)

- 1. "**Rationality**" ・・・ the players are perceived as rational and they have complete information about the structure of the game, including all of the players' preferences regarding possible outcomes, where this information about each other's strategic alternatives and preferences, they can also compute each other's optimal choice of strategy for each set of expectations. If all of the players expect the same Nash equilibrium, then there are no incentives for anyone to change his strategy.



Nash has received a grant from the National Science Foundation to develop a new "**evolutionary**" solution concept for **cooperative games**.(SOURCE: *the essential John Nash*)

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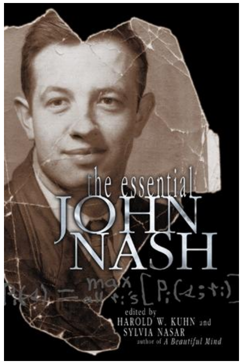

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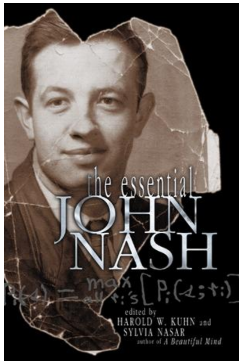

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- Fitness Function: $f = -H = \frac{1}{2} \sum_{\nu\mu} x_\nu c_{\nu\mu} x_\mu ,$

where, $f_\nu = \frac{\partial f}{\partial x_\nu}, \quad c_{\nu\mu} = c_{\mu\nu} (\mu \neq \nu)$ This is a element of the Random Matrix , it is Gauss Distribution, Average is 0, Variance is 1/N.

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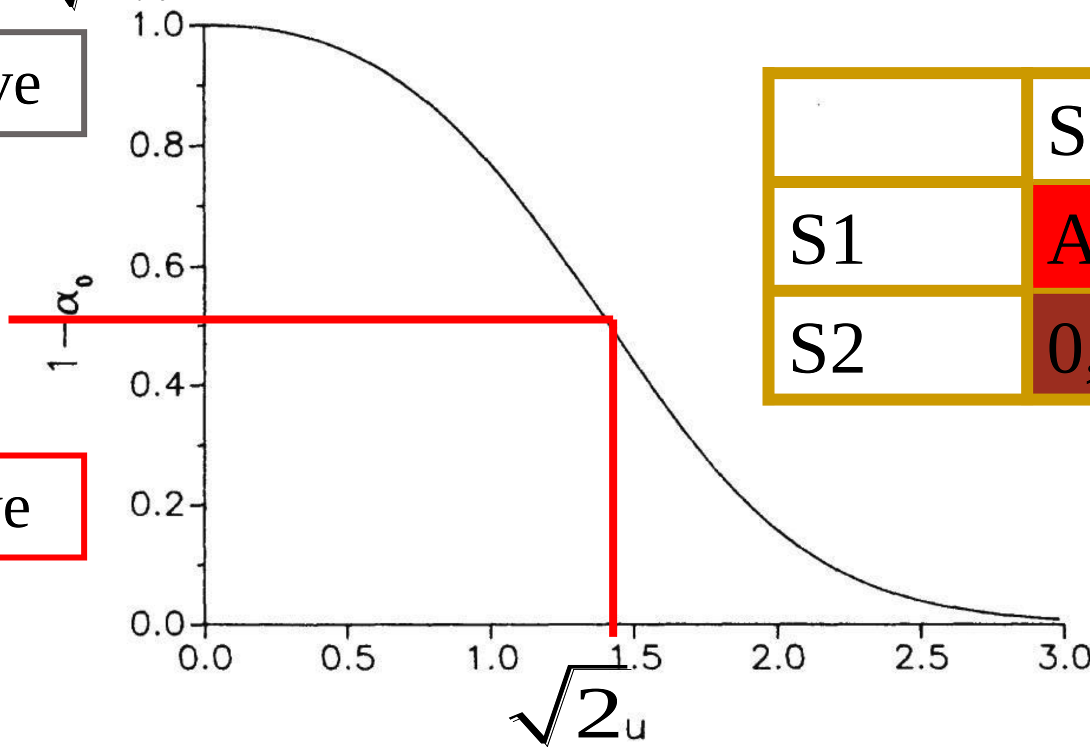
Competitive

↑↓、↓↑

0．5

Cooperative

↑↑、↓↓



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S2	0,0	B,B

Parameter u

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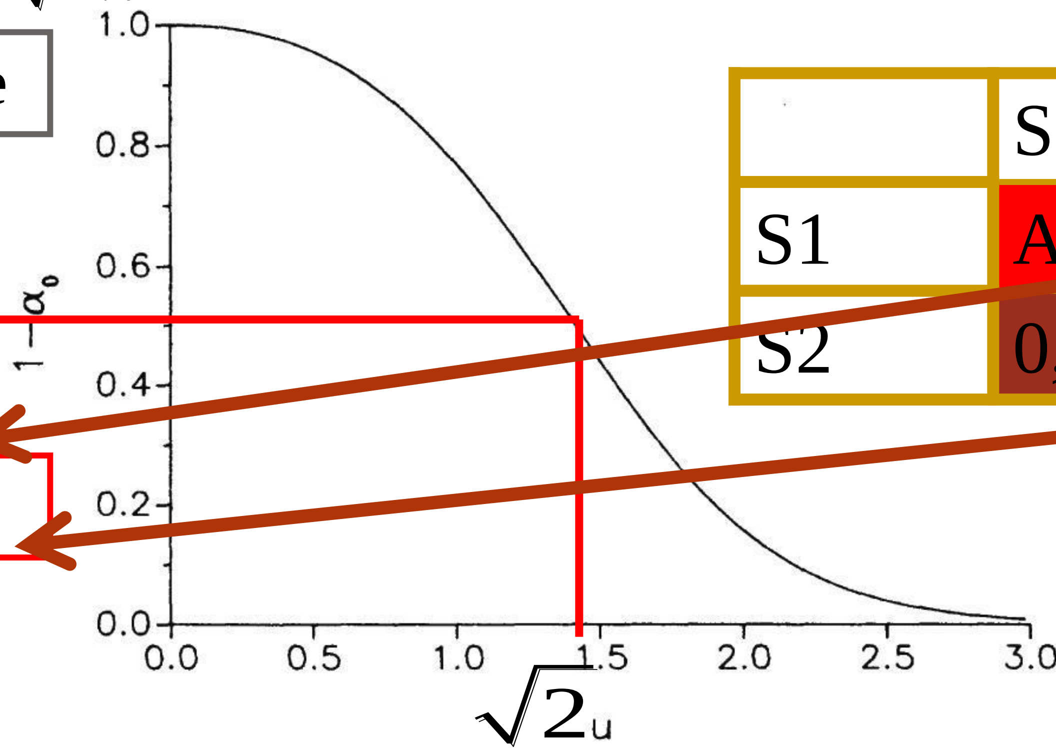

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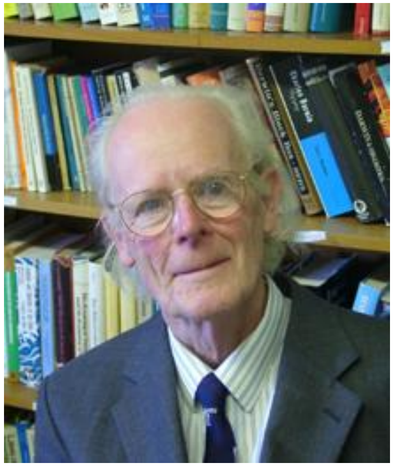
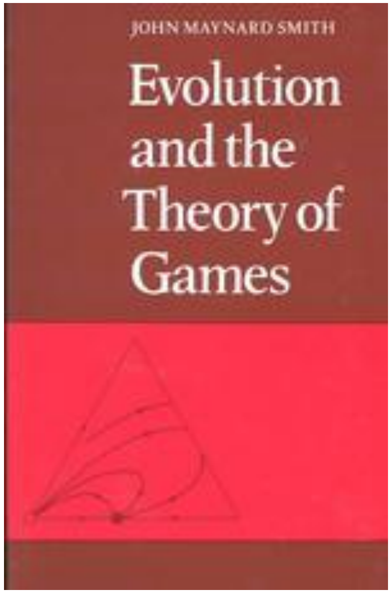
WHAT IS " EVOLUTIONARY GAME THEORY" ?



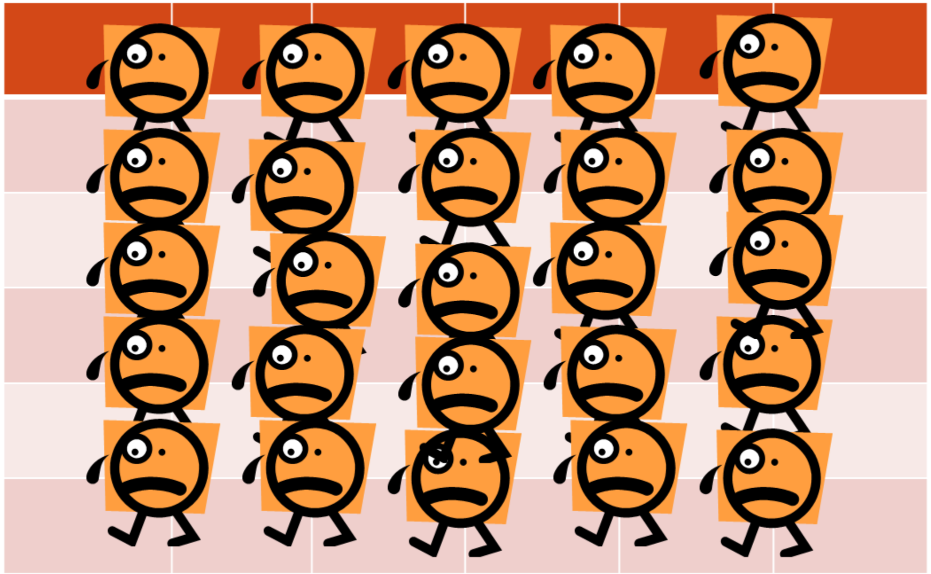


In 1973 Maynard Smith formalized a central concept in game theory called the evolutionary stable strategy (ESS), based on a verbal argument by G.R.Price. This area of research culminated in his 1982 book *Evolution and the Theory of Games.* The Hawk-Dove game is arguably his single most influential game theoretical model.

ASSUMPTION:

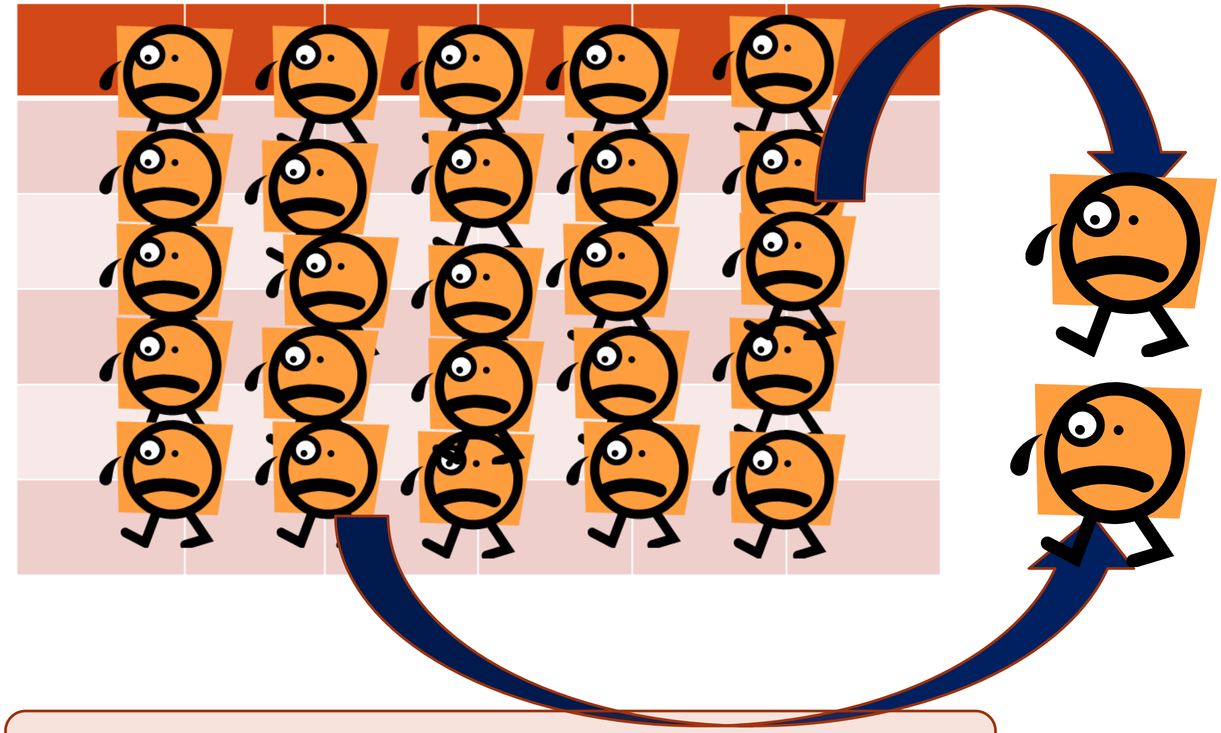
Large Number of Population (randomly matched) , **Monotone** (the strategy with higher payoff increases its shares)

Situation (Traditional Evolutionary Game Theory)



Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



Another players look at the game.

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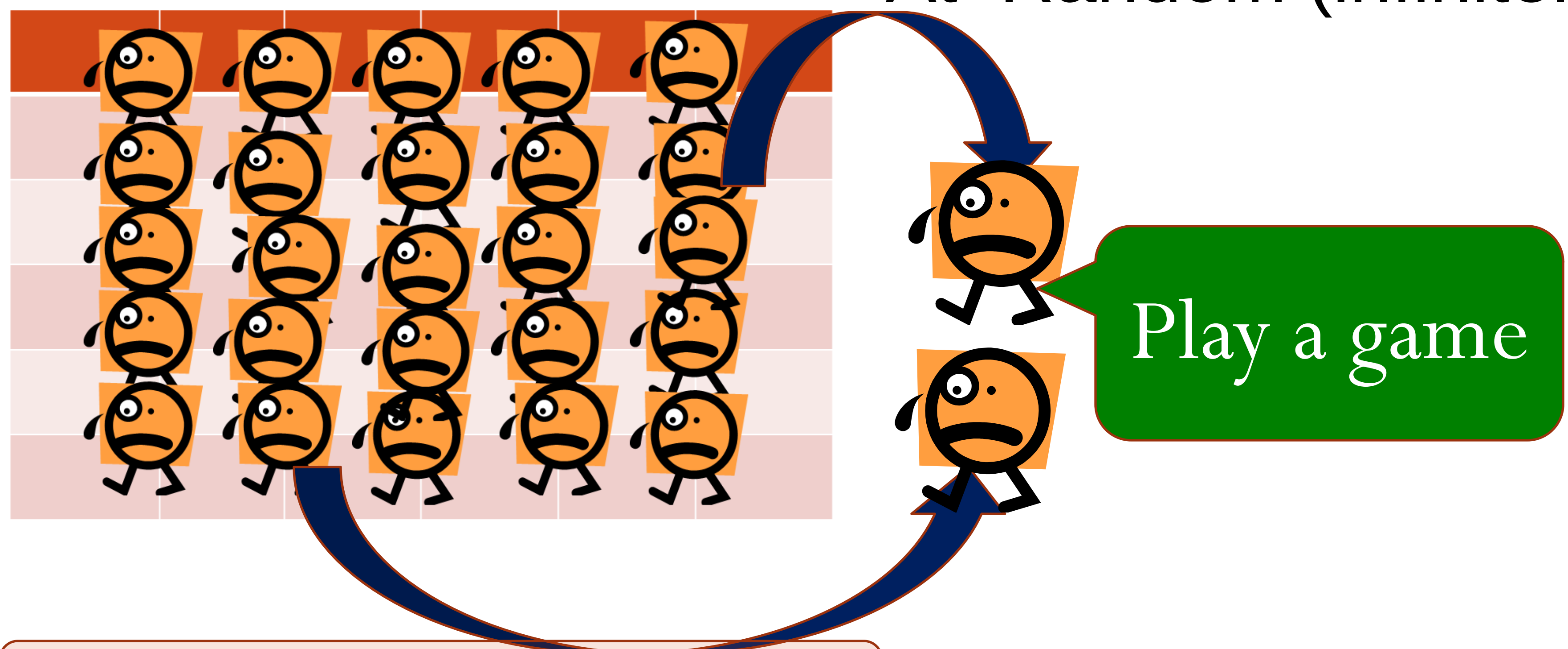

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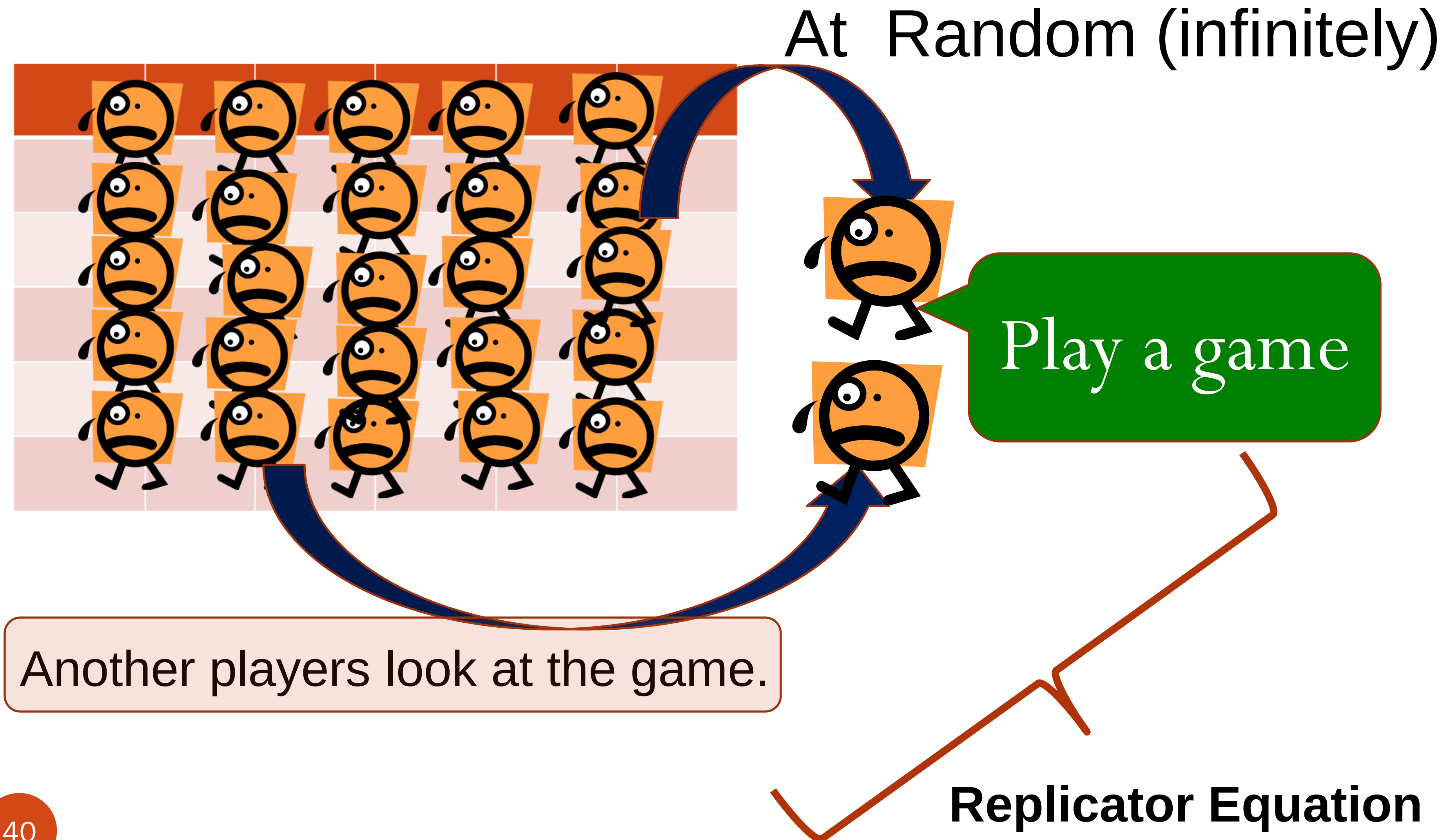

REVIEW: Replicator Equation

REPLICATOR EQ. $\dot{x}_i = x_i\left((Ax)_i - x \cdot Ax\right), i = 1, \cdots, n.$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before.

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Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \cdots (*)$$

Classification:

(I) Non-dilemma: a > 0. b < 0, ESS : one

(II) Prisoner's dilemma **:** a < 0. b > 0, ESS :one

(III) Coordination : a>0,b>0, ESS two

(IV) Hawk-Dove : a<0,b < 0, ESS one (mixed strategy)

2

1

	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

REVIEW: Symmetric and Asymmetic Games

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Type 2

Type 1

	S1	S2
S1	A,A	C,B
S2	B,C	D,D

Symmetric Two Person Game

Type 2

Type 1

	S1	S2
S1	A,E	C,G
S2	B,F	D,H

Asymmetric Two Person Game

Replicator Equation : one two

Situation :

Symmetric :

Asymmetric : seller and buyer etc.

REVIEW: Ising Model, Spin Glass

- **Ising model** • • •

- **Spin Glass** • • •

REVIEW: Ising Model, Spin Glass

- **Ising model・・・相転移（異なる相へ移る）を記述する最も簡単なモデル。**
- **金属に外場から磁化をかけ、ある臨界値（Curie温度）を超えると、磁石となる。**
- **格子上にある（スピンの）状態** S_j ： {-1, +1}, j=1,…,N
- 　N**個状態が**「+1 or -1」**にすべて揃ったら**「cooperative」、「-1, 1」**の組ならば**「competitive」、
- Hamiltonian (Energy) $H = -J \sum_{i,j} S_i S_j$
- **Spin Glass・・・**

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- Hamiltonian (Energy) $H = -J\sum_{i,j} S_i S_j$
- **Spin Glass ・・・相互作用の符合が場所に一定ではないというミクロ的な特徴を持っている。**

例） CuMn **・・・銅(強磁性体にならない)に微量のマンガン(磁性原子)を混ぜ合わせて合金を作ると、マンガンの原子は銅の結晶格子中でランダムな位置を占め、ガラスの性質に似たスピン秩序を示すので、**Spin Glass **と呼ばれる。**

REVIEW: PERCOLATION

[*d*-dimensional Percolation]
We examine each edge of Z^d, and consider it to be *open* with probability p and closed otherwise, independent of all other edges. The edges of Z^d represent the inner passageways of the stone, and the parameter p is the proportion of passages that are broad enough to allow water to pass along them. Suppose we immerse a large porous stone in a bucket of water. What is the probability that the center of the stone is wetted ?

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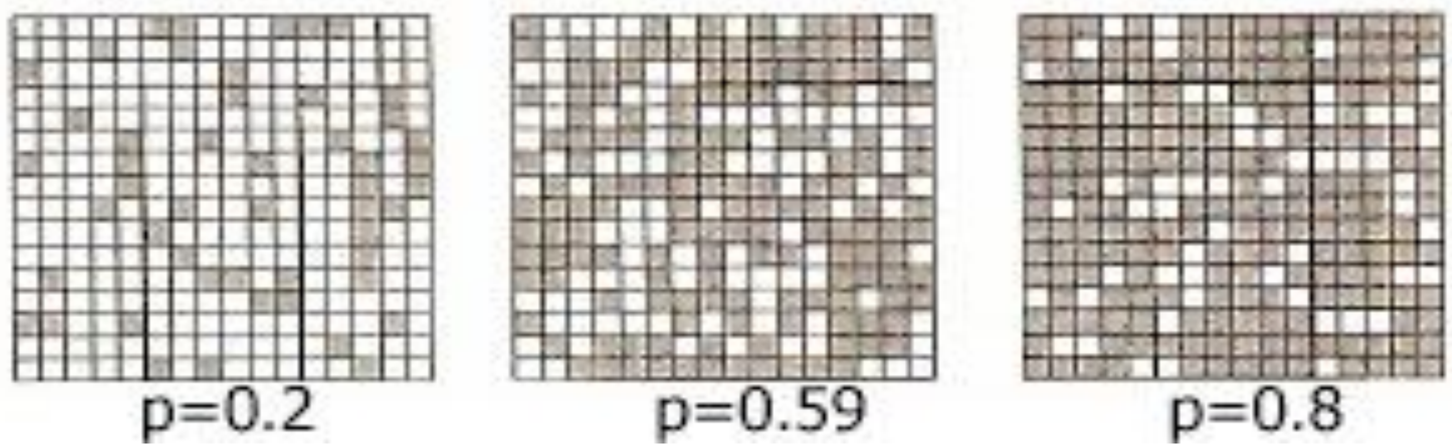

3． BASIC MODEL

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2. Related Literatures and Preliminaries
3. Our Model
   3.1 Nearest neighbor（Ising TYPE）
   3-2. Random Matching（SK MODEL）
   Annealed System, Quenched System
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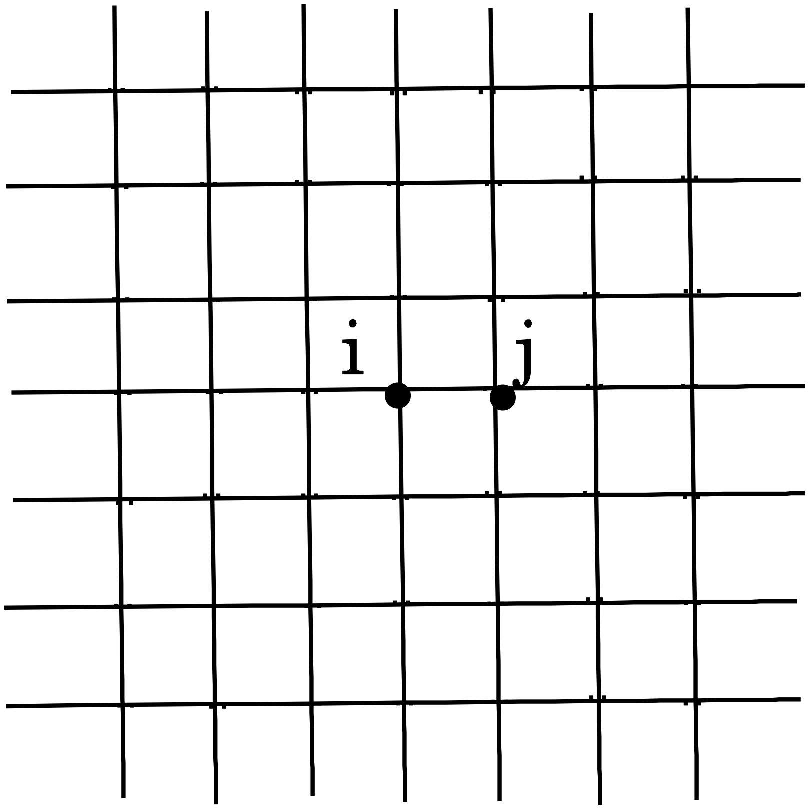
MODEL：

- Each site on the lattice is the address of one player.

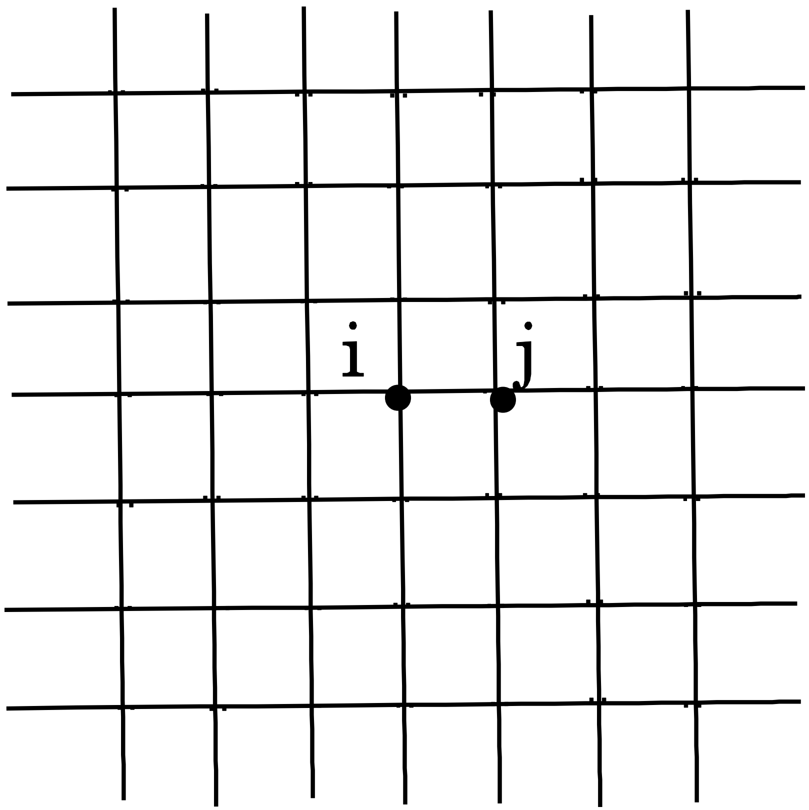


SQUARE LATTICE

MODEL：

- Each site on the lattice is the address of one player.

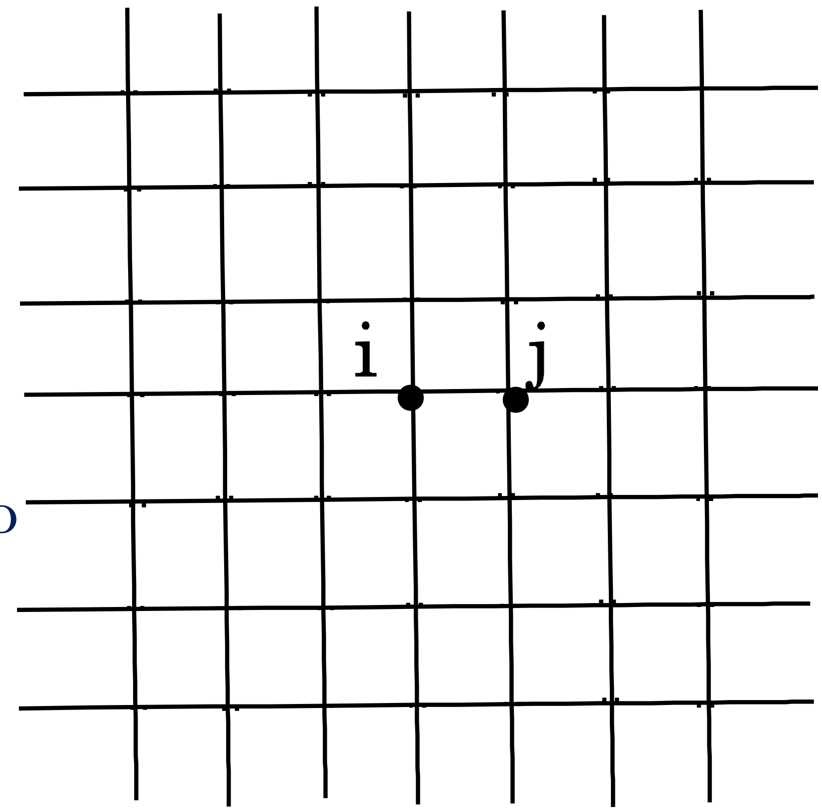
- In Sec.2, player *i* and *j* play a game with nearest neighbor interaction.



SQUARE LATTICE

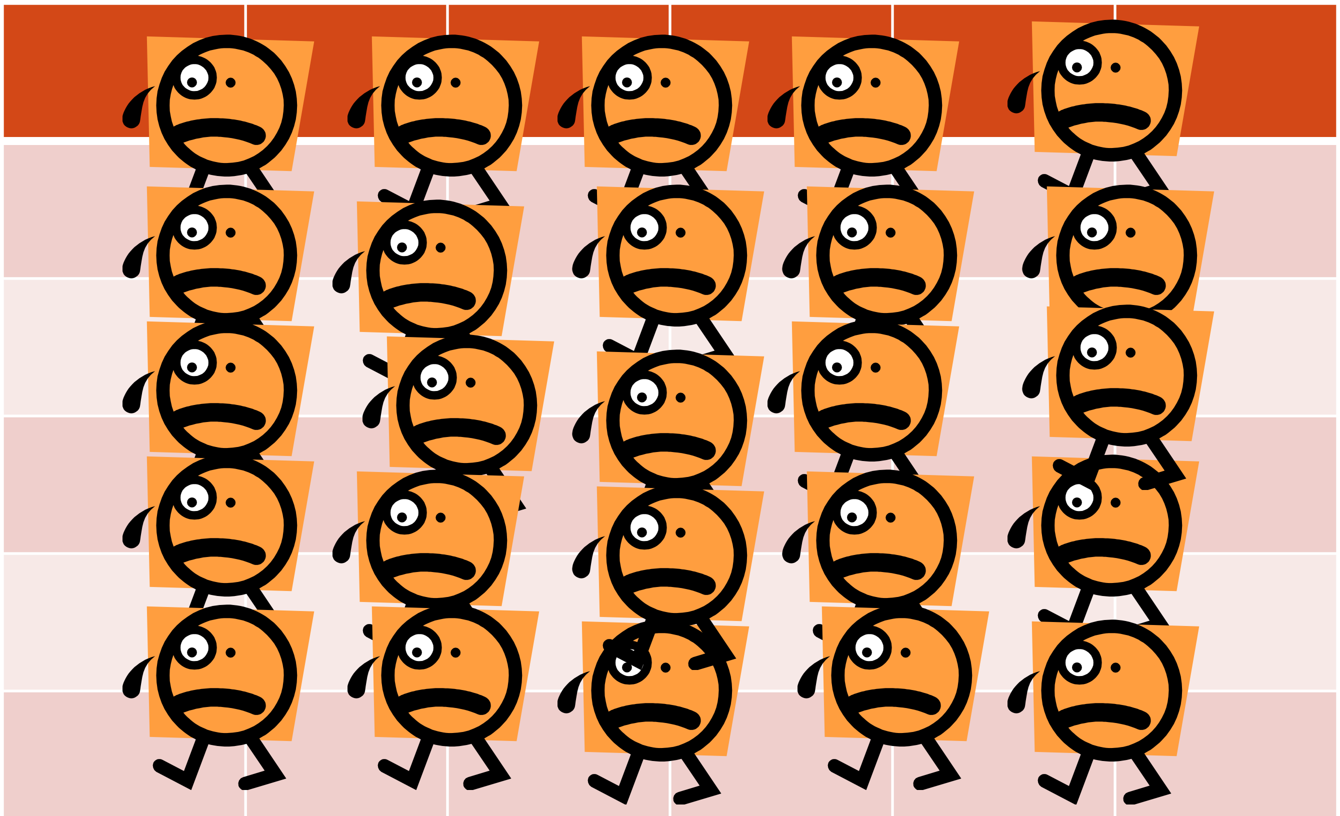
MODEL：

- Each site on the lattice is the address of one player.
- In Sec.2, player i and j play a game with nearest neighbor interaction.
- In Sec.3, the players are assumed to search at random for trading opportunities and when they meet the terms of game are started.



SQUARE LATTICE

Situation (nearest neighbor interaction)



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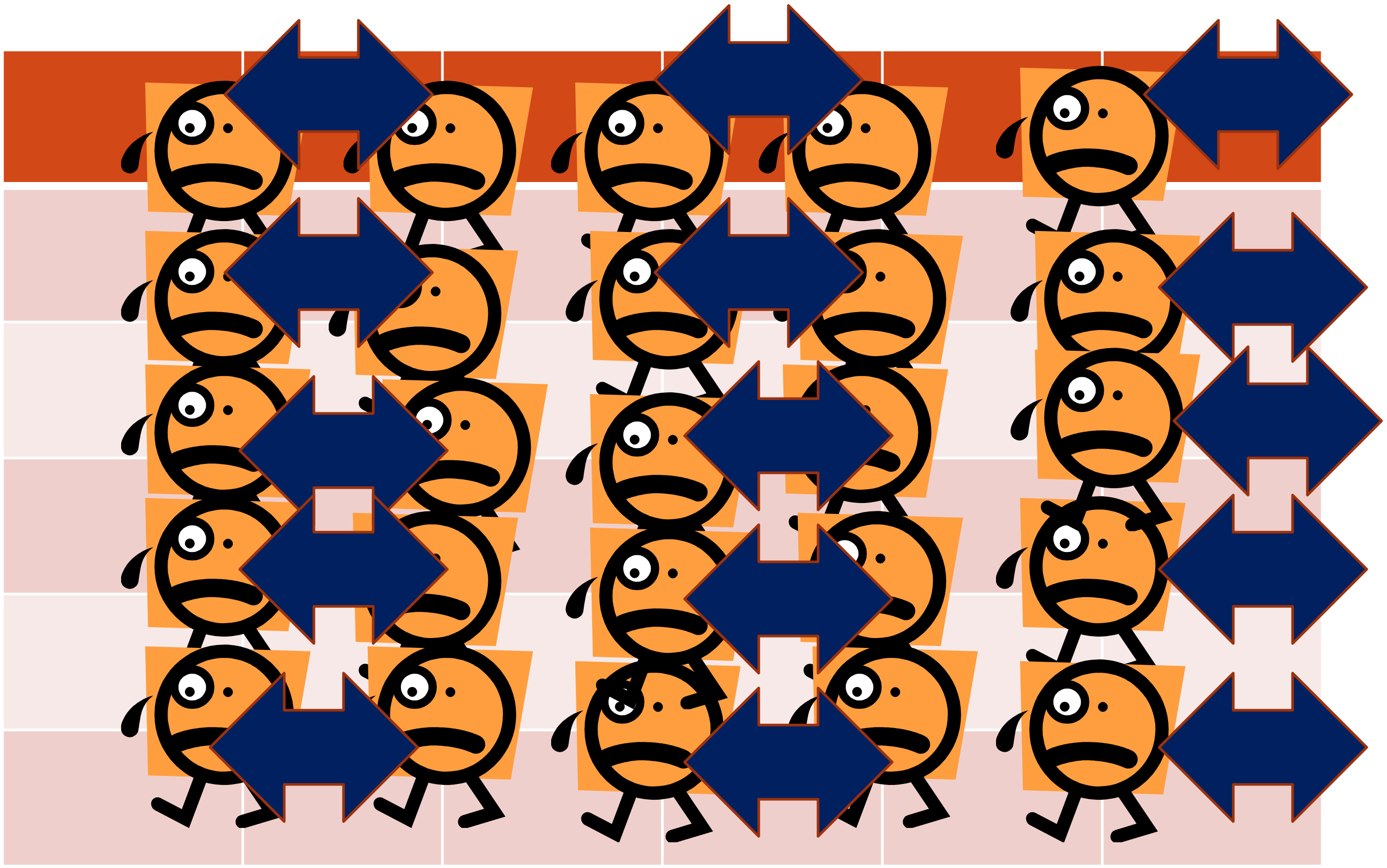

EXAMPLE

	S1(1)	S2(2)
S1(1)	A,A	0,0
S2(2)	0,0	B,B

	S1(-1)	S2(+1)
S1(-1)	A,A	0,0
S2(+1)	0,0	B,B

where A,B > 0

Ising Model

PROBABILITY SPACE

- Probability Space (Ω, F, P)

$$\Omega = \{-1, +1\}^{Z^2}$$

$$\mu \propto \exp[\gamma H(S)]dS \in F \qquad \text{(Prop.1)}$$

μはそれ上の確率測度で, dSはΩ上の一様分布とする。確率論的にはdS は密度1/2 のBernoulli 分布と呼ぶものである。

ASSUMPTION , PROPOSITON

ASSU. : All players are "rational".

ASSUMPTION , PROPOSITON

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PROP. : Under Assu., we obtain the probability distributions of actions, $\{S_i\}, i=1,\ldots,N$, and the palyer's payoff from the outcome is f

(2.1)

$$P(\{S_i\}) = Z^{-1}\exp(\gamma f)$$

where $\{S_i\}$ is player i's action, γ is non-negative constant; for instance, γ is the optimal choice behavior f is the player's payoff from the outcome $\{S_i\}$, and Z is the normalization parameter.

ASSUMPTION , PROPOSITON

ASSU.： All players are "rational".

PROP.： Under Assu., we obtain the probability distributions of actions, $\{S_i\}, i=1,\ldots,N$, and the palyer's payoff from the outcome is f

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- **INTERPRETATION：** If payoff f is greater, then the probability of choosing the action is higher.
- **Distinction：** STATICS, Non- Externality

Classical EVOLUTIONARY GAME

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INTERPRETATION: If the payoff f_i is greater then the expected utility, the player choose the action with probability 1.

Distinction: DYNAMICS, EXTERNALITY

REVIEW: Replicator Equation

REPLICATOR EQ. $\dot{x} = x_i\left((Ax)_i - x \cdot Ax\right), i = 1, \cdots, n.$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before. And this equation shows that the probability of the action *i* chosen by another players is also higher than before (**externality**). Furthermore, the equation is derived uniquely by the **monotonic** (that is if one type has increased its share in the population then all types with higher profit should also have increased their shares).

Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \cdots (*)$$

Classification:

(I) Non-dilemma: a > 0. b < 0, ESS : one

(II) Prisoner's dilemma **:** a < 0. b > 0, ESS :one

(III) Coordination : a>0,b>0, ESS two

(IV) Hawk-Dove : a<0,b < 0, ESS one (mixed strategy)

1 \ 2	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

DEFINITION

DEF. : We define an *order parameter*, as how often a player has chosen an action in this game.

(2.2)

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$$m = \sum_{i=1}^{N} S_i P(\{S_i\})$$

where N is the number of the actions.

EXAMPLE

- The actions' index $\{S_i\} = \{1,2\}, N=2$, and the order parameter for each case is computed as follows.

	S1(1)	S2(2)
S1(1)	A,A	0,0
S2(2)	0,0	B,B

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(i) If all the players' actions are $\{\text{Action } 1\}$, then we obtain $m=1$.

(ii) If all the players' actions are $\{\text{Action } 2\}$, then we obtain $m=2$.

(iii) If half of all the players' actions are $\{\text{Action } 1\}$, then we obtain $m=3/2$.

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→ If the order parameter m is near 1, then we know that there are many more players choosing {Action 1} than {Action 2}.

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- $Si=\{-1,1\} \rightarrow m= -1, 0(\text{random}), 1$

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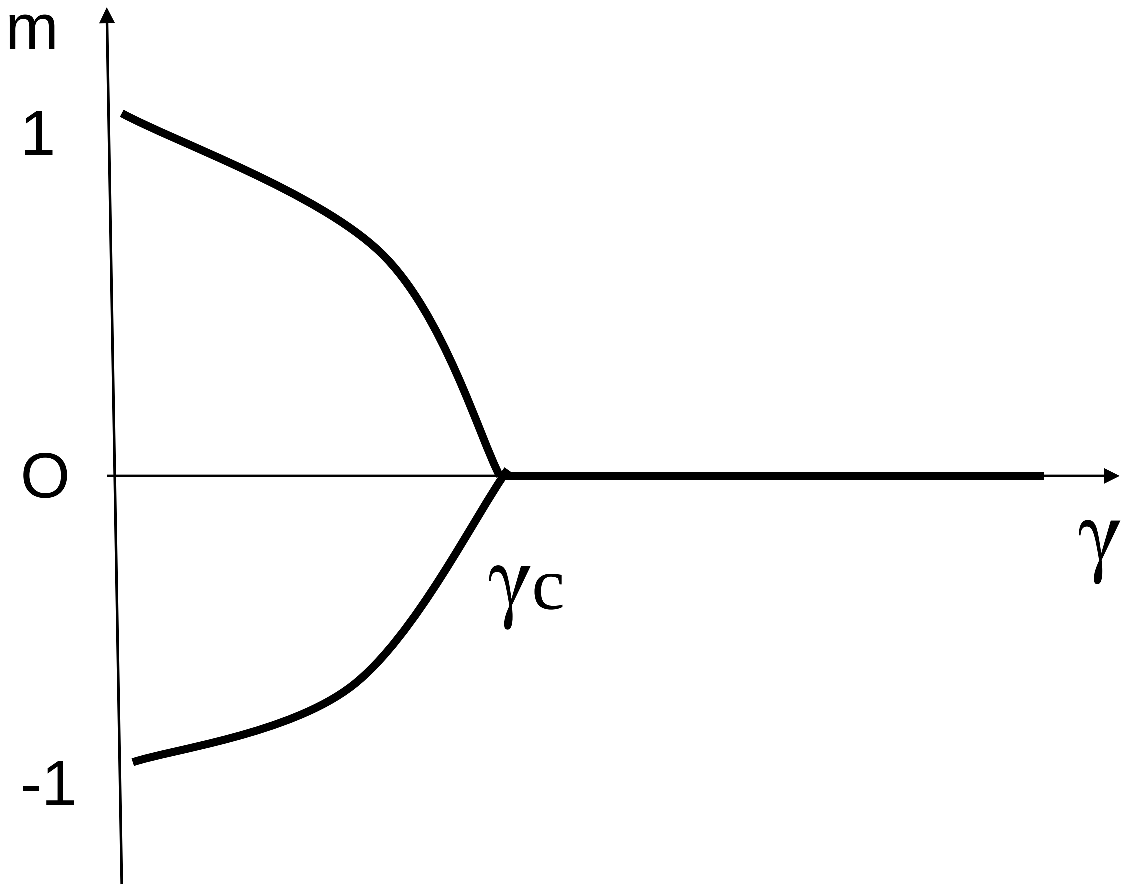

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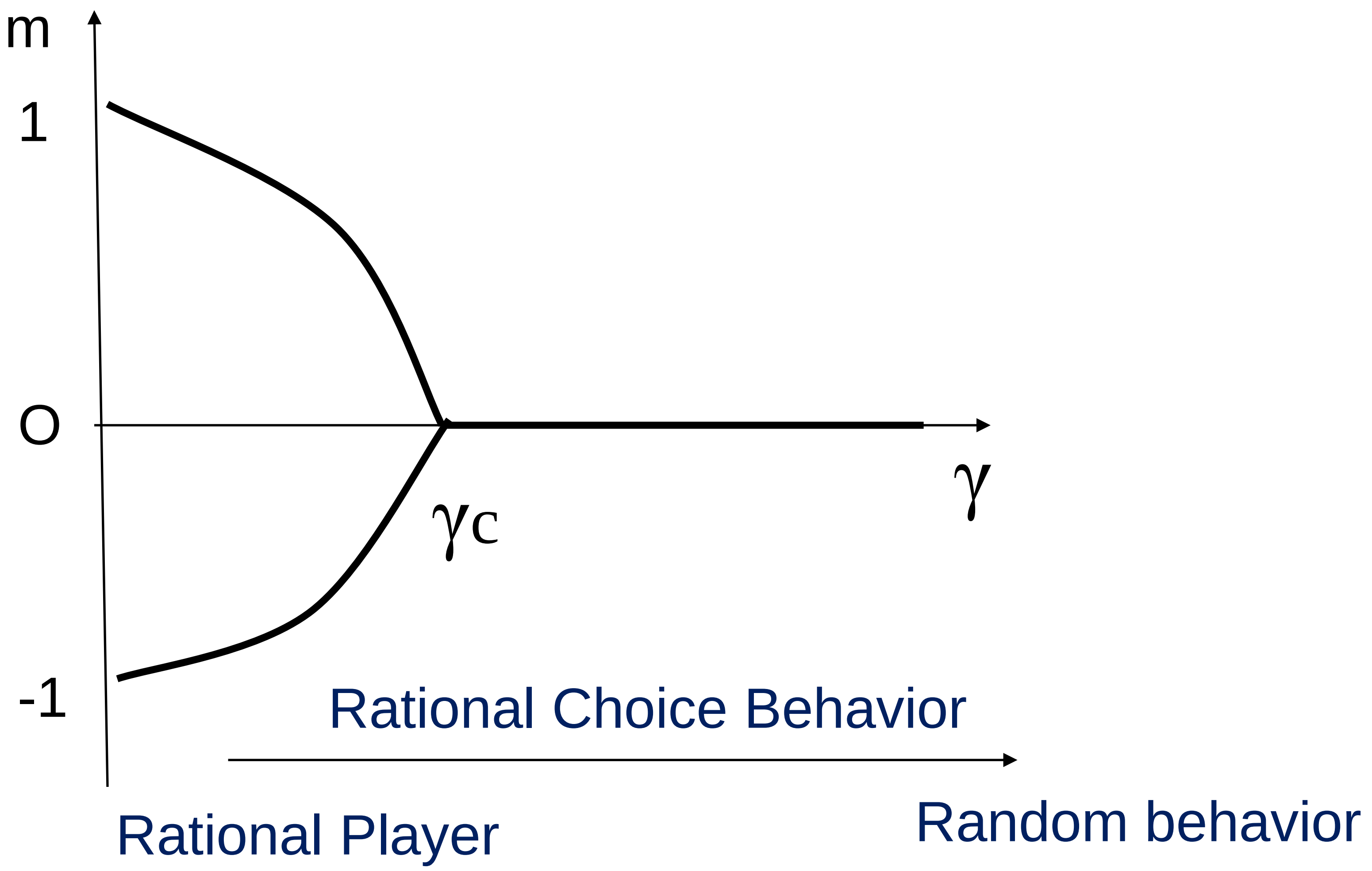
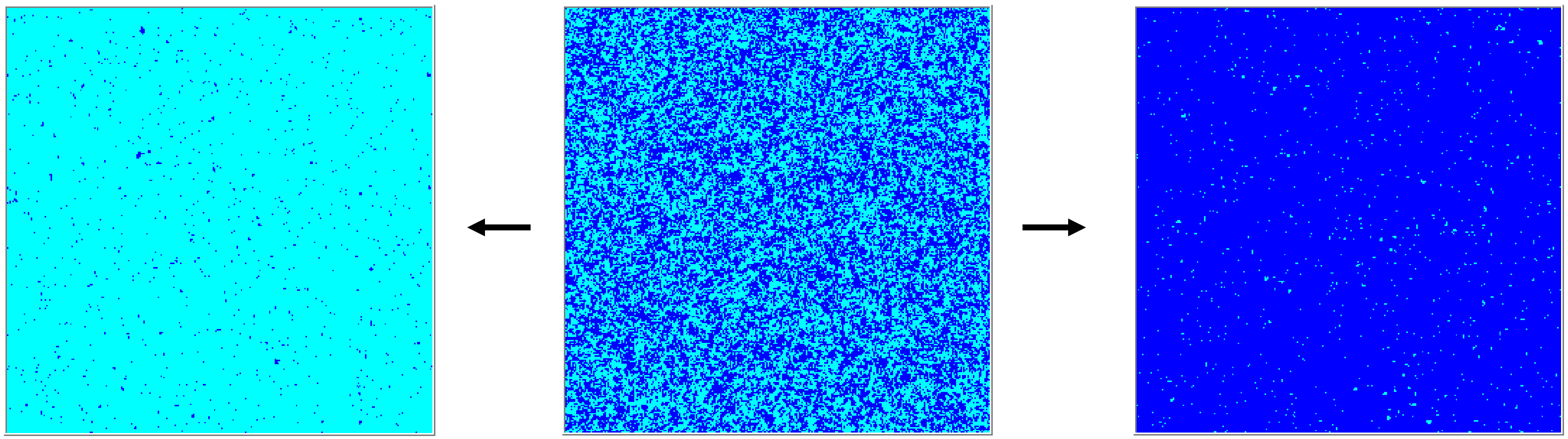


SIMULATION

SKY BULUE＝Strategy 1, BLUE＝Strategy 2

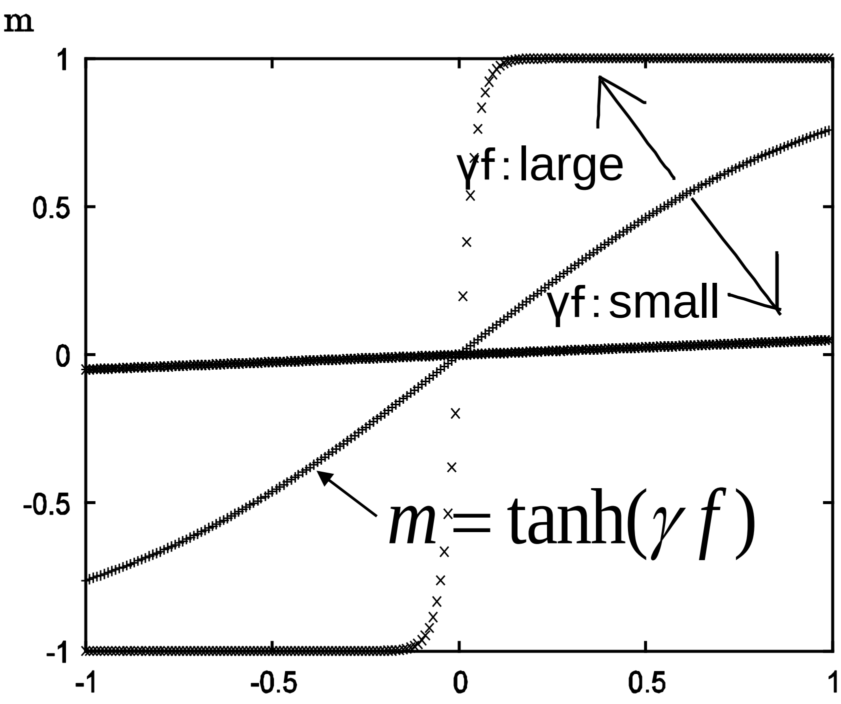


ORDERED TYPE 1 | NO ORDERED | ORDERED TYPE 2

$m^* > 0$ | $m^* = 0$ | $m^* < 0$

(s1, s1) | Random | (s2,s2)

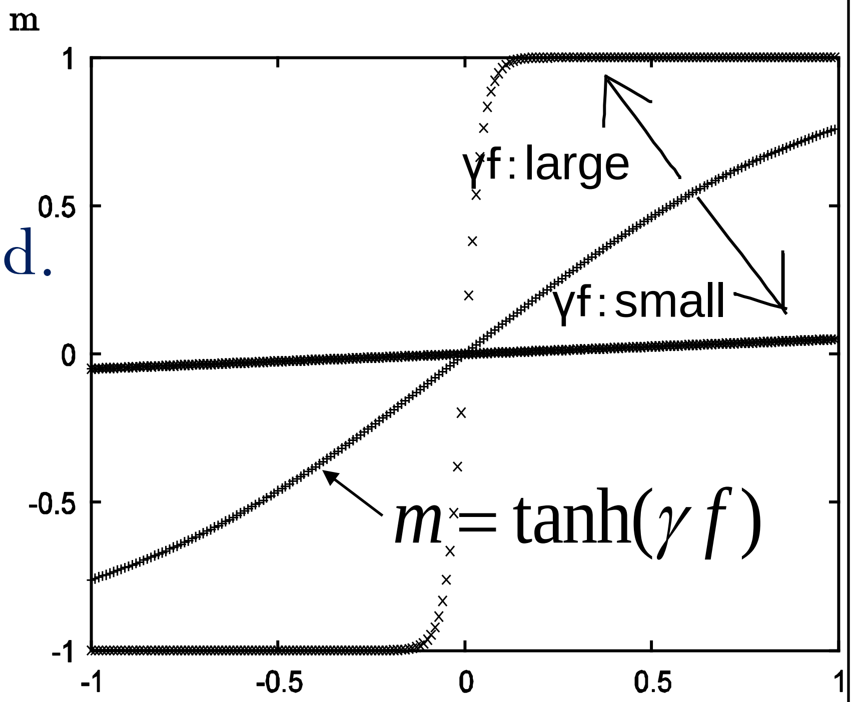
Relation between order parameter and product of profit f and parameter γ



Relation between order parameter and product of profit f and parameter γ

- If the γf is large, order parameter approaches to 1.

→ We can find which action is occupied.



Relation between order parameter and product of profit f and parameter γ

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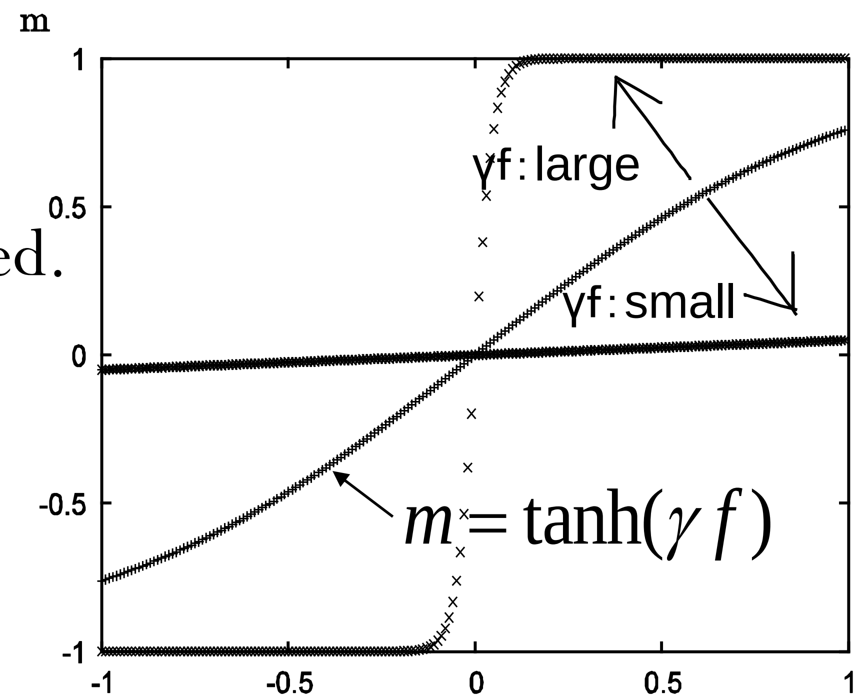
$\rightarrow$ We can find which action is occupied.

- If the γf is small, order parameter approaches to 0.



ORDERED PARAMETER IN REPLICATOR SYSTEM

REPLICATOR Equation (symmetric two person game, the number of the strategy is two.)

$$\dot{x} = x(1-x)\{b-(a+b)x\}$$

Stationary point (Nash equilibrium)

$$x^* = 0, 1, 0 < \frac{b}{a+b} < 1$$

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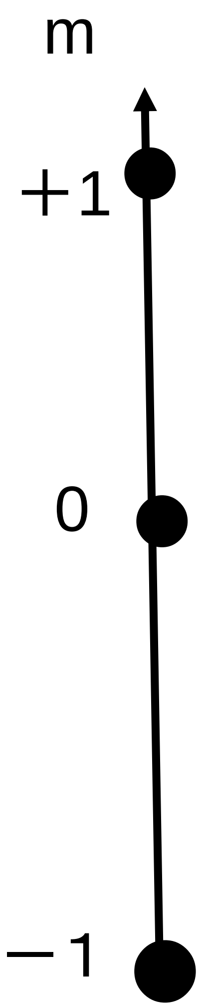

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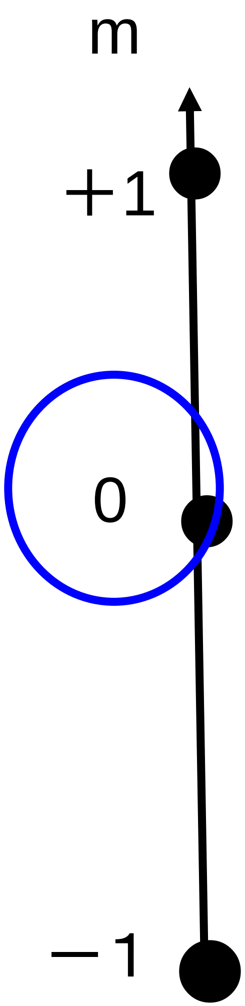

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ORDERED PARAMETER has **three points** (corner point(-1,+1), interior point) in RE. SYS.

EVOLUTIONARY STABLE STRATEGY (ESS)

DEF. : Weibull(1995): $x \in \Delta$ is an ***evolutionary stable strategy (ESS)*** if for every strategy $y \neq x$ there exists some $\bar{\varepsilon}_y \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \bar{\varepsilon}_y)$.

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

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INTERPRETATION: incumbent payoff (fitness) is higher than that of the post-entry strategy

（ESS : ①the solution of the Replicator equation + ② asymptotic stable.）

PROPOSITION

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Nash Eq.

$$(2.5) \quad \begin{array}{l} u(y,x) = u(x,x) \\ \Rightarrow u(y,y) < u(x,y), \end{array} \quad \forall y \neq x,$$

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Asymptotic Stable Conditon

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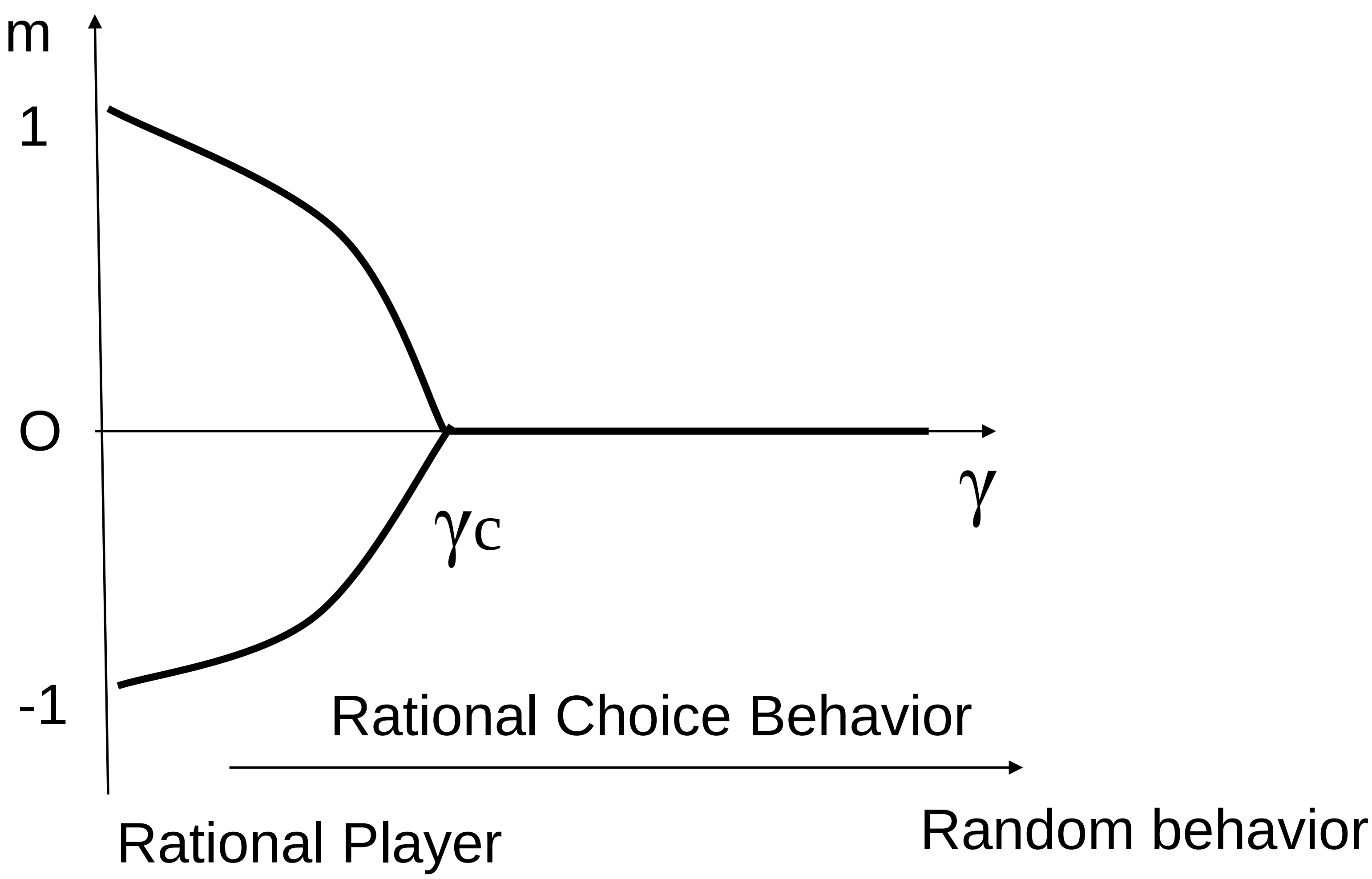
$$(2.4) \quad u(y,x) \le u(x,x), \quad \forall y,$$

$$(2.6) \quad \left| m - m^* \right| < \varepsilon \quad ,$$

Lyapunov Stable Condition

where, m^* is the index of the equilibrium action.

- EXAMPLE : Ising model
- $S_i=\{-1,1\} \rightarrow m= -1,0(\text{random}),1$



ASYMMETRIC TWO PERSON GAME

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- Equilibrium Condition:

$$\left|m'_1 - m^*_1\right| < \varepsilon_1 \quad , \left|m'_2 - m^*_2\right| < \varepsilon_2$$

PERCOLATION

- The fundamental relationship between percolation and phase transition

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THE. (Coniglio, *et al.*(1976)) In the two-dimensional Ising model, we obtain

(i) If $\gamma > \gamma_c$, $\mu^{+}_{\gamma,0}\left(\left\{\left|C_0^+\right| = \infty\right\}\right) > 0, \mu^{-}_{\gamma,0}\left(\left\{\left|C_0^-\right| = \infty\right\}\right) > 0.$

where $\mu^s, s = \{+,-\}$ is Gibbs measures.

(ii) if μ is external to the set of all Gibbs states $G(\gamma, h)$

$$\mu\left(\left|C_0^+\right| = \infty\right)\mu\left(\left|C_0^-\right| = \infty\right) = 0$$

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(i) → there exists a.e. an infinite cluster of the corresponding sign and no infinite clusters of the opposite sign.

(ii) → there exists an infinite cluster for neither actions,

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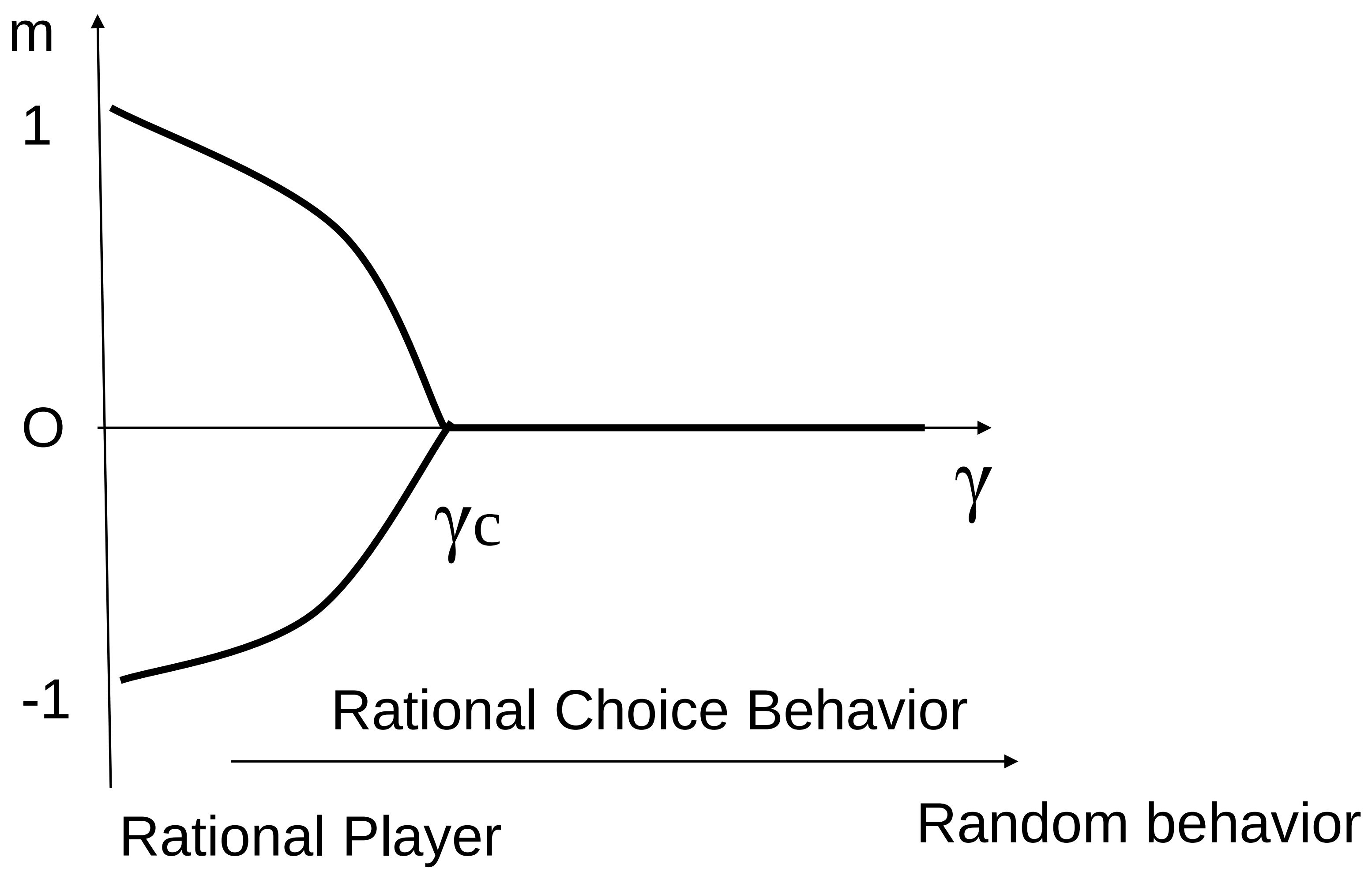
- Si={-1,1} → m= -1,0(random),1



DEFINITION(CONNECTED)

DEF. A subset $A \subset B^2$ is called *connected* if and only if for every $x, y \in \overline{A}$, there exists a sequence $\{b_1, b_2 \cdots, b_n\} \in A$ such that

(a) $x \in b_1$ and $y \in b_n$

(b) For every , $1 \leq i \leq n-1$

there exists a point $x_i \in Z^2$ such that $b_i \cap b_{i+1} = x_i$.

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DEF 2.11 A subset $A \subset Z^2$ is called (✱) *connected* if and only if for every $x, y \in A$, there exists a sequence of points $\{x_1, x_2, \cdots, x_n\} \subset A$ such that $x_0 = x, x_{n+1} = y$ and for every ,$1 \le i \le n+1$,

$$\|x_i - x_{i+1}\| = 1.$$

where , $x = (x^1, x^2) \in Z^2$, $\|x\| = \max\{|x^1|, |x^2|\}.$

Concentric Circle Pattern and Chess Pattern

- What kind of pattern do the actions' distribution on the lattice make ?

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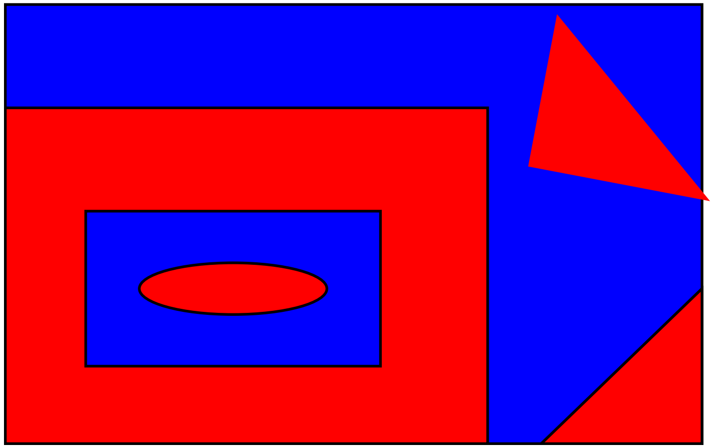
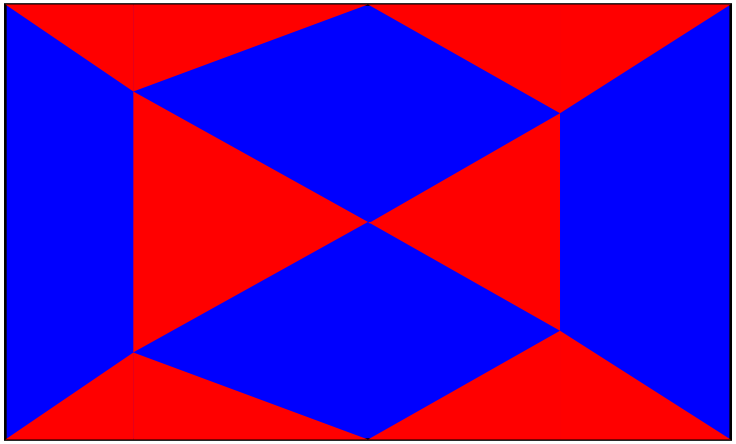


Concentric Circle Pattern

→ red surrounded by a bigger blue, which is surrounded by a bigger red , ….

coexistence of finite (*) connected



Chess Pattern

→ red and blue placed alternately

coexistence of infinite (*) connected

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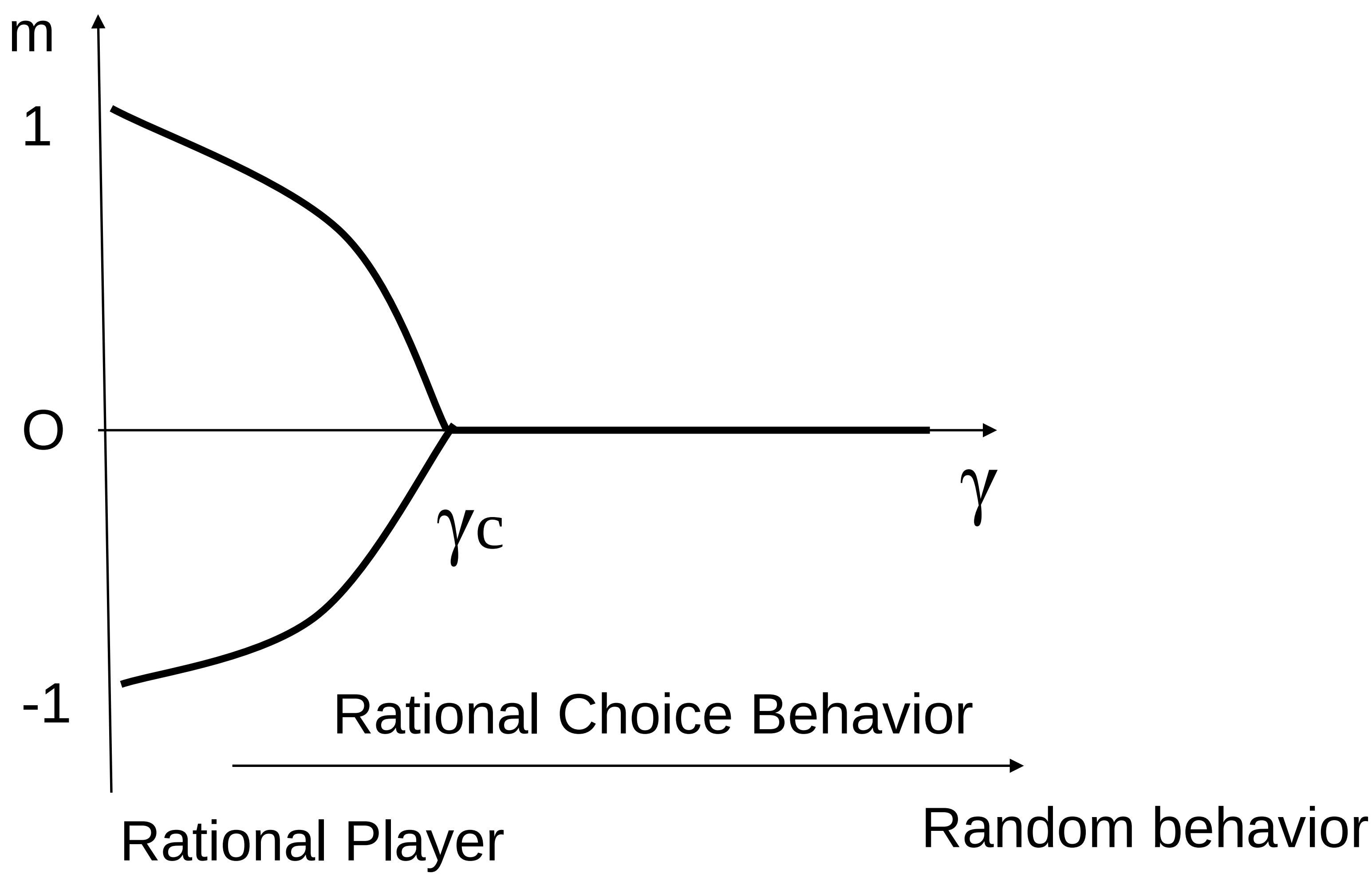

Coexistence of infinite (*)-clusters

TH. (Higuchi(1995)) For every $\gamma > 0$ is sufficiently small, there exists h such that $\gamma' h' < \frac{1}{2}\log\frac{p_c}{1-p_c} - 4\gamma'$, $\gamma h < \frac{1}{2}\log\frac{1-p_c}{p_c} - 4\gamma$, implies the coexistence of infinite (*)-clusters with respect to the Gibbs state for $\mu_{\gamma,h}$.

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OUTLINE OF THE PROOF.

Step 1. Lemma A.1 → **大小関係を表すための条件を得る**.

Step 2. **＋戦略が**Percolation**する確率**(p_c)**と－戦略が**Percolation **する確率**($1-p_c$)**を求める**. $1-p_c < p < p_c$ **であり，それらを同時に成り立つ条件を求めと，定理２の条件を導出することができる**. (QED)

- **→無限＊クラスターの共存が存在**

→このとき戦略の分布はチェス盤のパターン

Ωに大小関係を入れる.

- 任意の$x \in Z^2$ に対して $\varpi(x) \le \eta(x)$ となるときに, $\varpi \le \eta$ とかくことにする. この大小関係に対してΩ上の関数fが単調増加(減少)とは, $\varpi \le \eta$ なる$\varpi, \eta \in \Omega$ に対して常に
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DEF. Ω上の確率測度μとνに対して, μ≦ νとは, 任意のΩ上の連続かつ単調増加関数 f に対して

$$\int_\Omega f(\varpi)\mu(d\varpi) \leq \int_\Omega f(\varpi)\nu(d\varpi)$$

となるときに言う.

定理A.1. (FKG-Holley Inequalities)　$\Lambda \subset Z^2$を有限集合として, Ω_Λ 上の2つの確率測度μ, νが, 任意の $\sigma_1, \sigma_2 \in \Omega_\Lambda$ に対して

(A.1)
$$\mu(\sigma_1 \wedge \sigma_2)\nu(\sigma_1 \vee \sigma_2) \geq \mu(\sigma_1)\nu(\sigma_2)$$

を満たすならば, (Ω_Λ上の確率測度として) μ≦ ν である. ただし　$(\sigma_1 \wedge \sigma_2)(x) = \min\{\sigma_1(x), \sigma_2(x)\}$, $(\sigma_1 \vee \sigma_2)(x) = \max\{\sigma_1(x), \sigma_2(x)\}$ とする.

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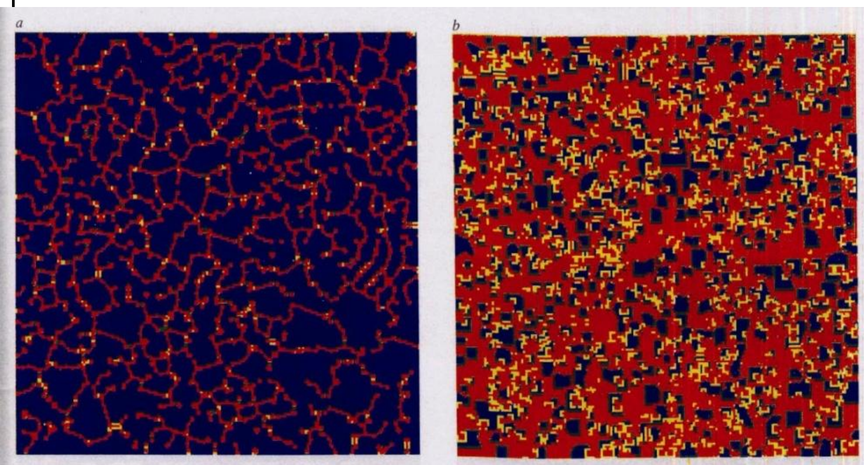
系A1. Λを Z^2の有限部分集合とする．このとき以下のことが成立する．

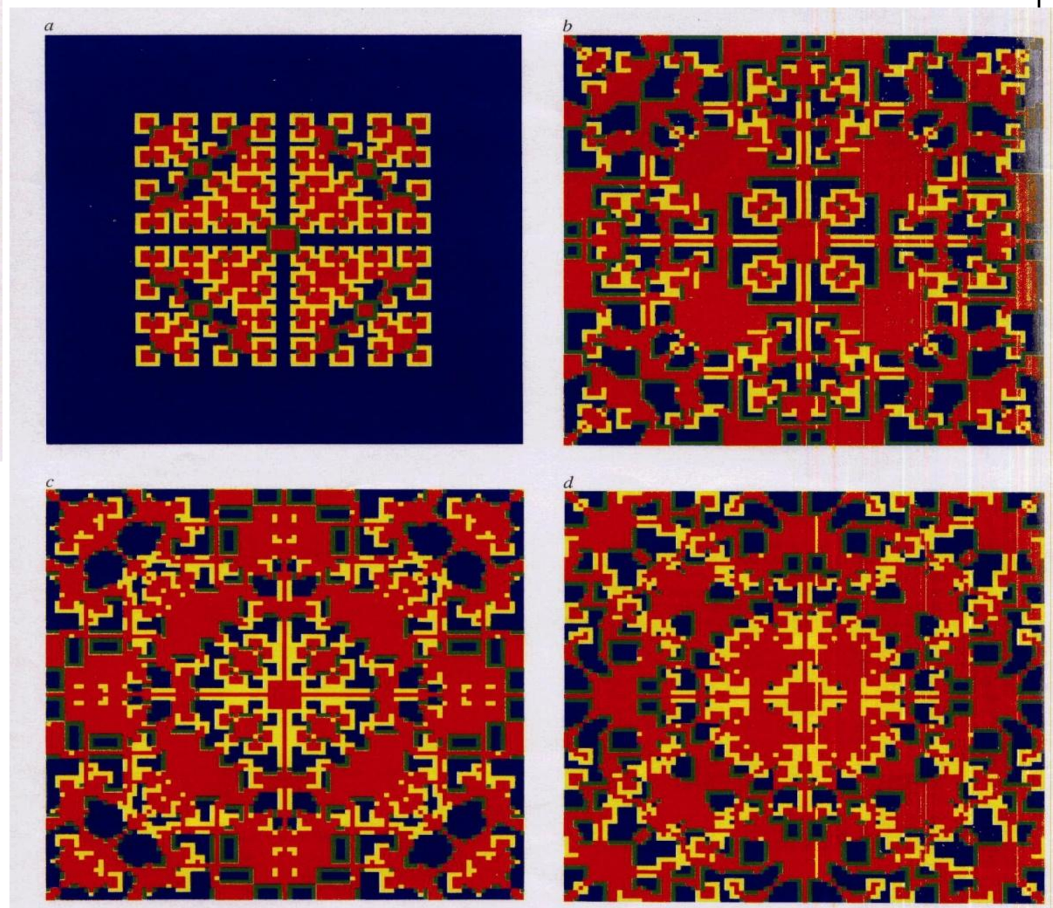
(i) $\varpi, \eta \in \Omega$ が Ω≦ η を満たすならば，$q_\Lambda^\varpi \leq q_\Lambda^\eta$

(ii) f,g を F_Λ可測な単調増加関数とすると任意の$\varpi \in \Omega$に対して $\int_{\Omega_\Lambda} fgdq_\Lambda^\varpi \geq \int_{\Omega_\Lambda} fdq_\Lambda^\varpi \bullet \int_{\Omega_\Lambda} gdq_\Lambda^\varpi$.

(iii) $\gamma h - \gamma' h' - 4|\gamma - \gamma'| \geq 0$ ならば，任意の$\varpi \in \Omega$に対して，$q_\Lambda^\varpi(\bullet|\gamma, h) \geq q_\Lambda^\varpi(\bullet|\gamma', h')$.

EX. :SPATIAL PRISONER'S DILEMMA GAME, Nowak and May(Nature, 1992)





Blue:C(cooperate),

Red: D (defect),

Yellow: D following a C,

Green : C following a D

Coexistence of infinite (*)-clusters

EXTENSION :

Random Matching

1. Introduction (Motivation, Purpose)
2. Related Literatures and Preliminaries
3. Our Model
   3.1 Nearest neighbor（Ising TYPE）
   3-2. Random Matching（SK MODEL）
   Annealed System, Quenched System
4. Implication : Cont- Bouchaud's Model
5. Summary and Future Works

SK MODEL

- Random Matching

SK MODEL

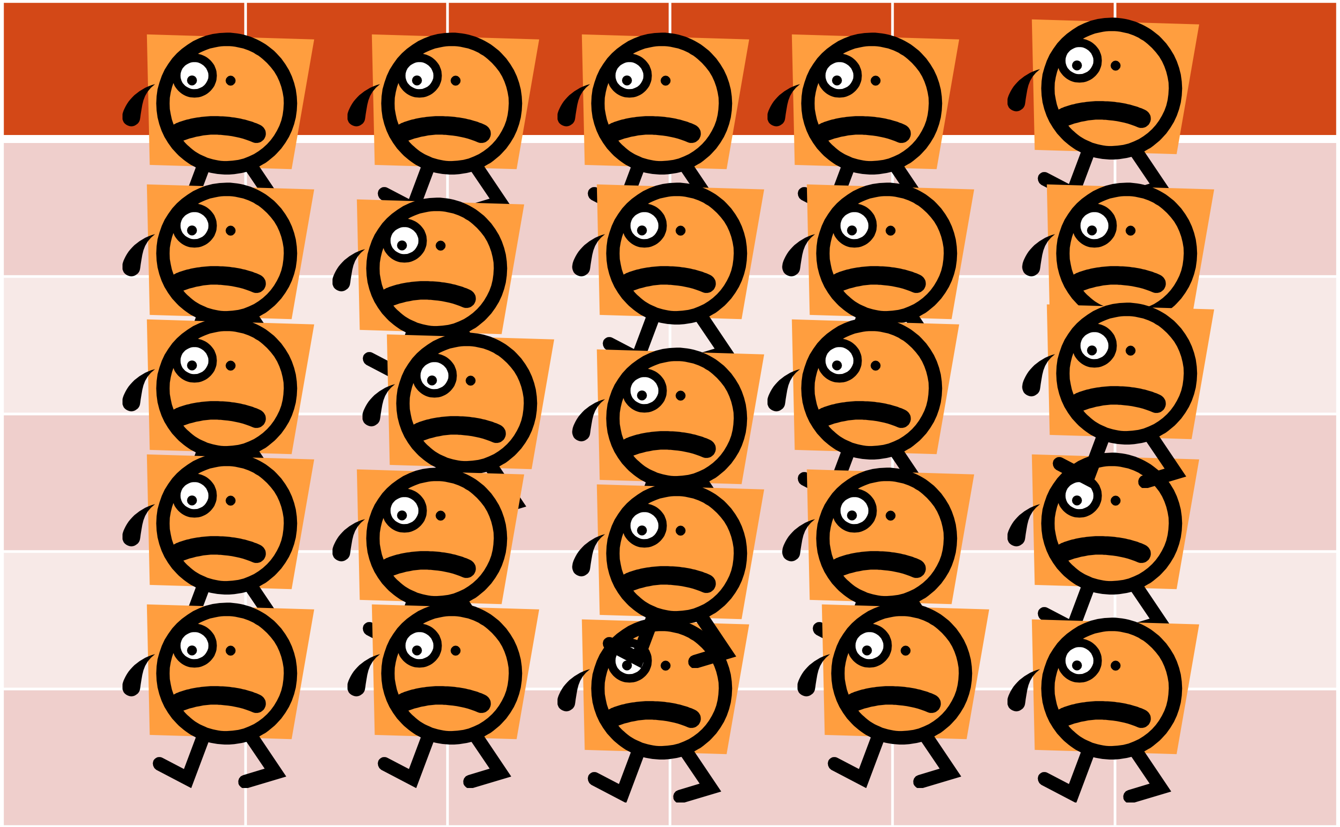
- Random Matching
- Payoff, Fitness

$$H\left(\left\{J_{ij}\right\}\right)=\sum_{i\neq j}J_{ij}S_iS_j$$

$$where \quad P\left(J_{ij}\right)=\frac{1}{\sqrt{2\pi J^2}}\exp\left\{-\frac{(J_{ij}-J_0)^2}{2J^2}\right\}$$

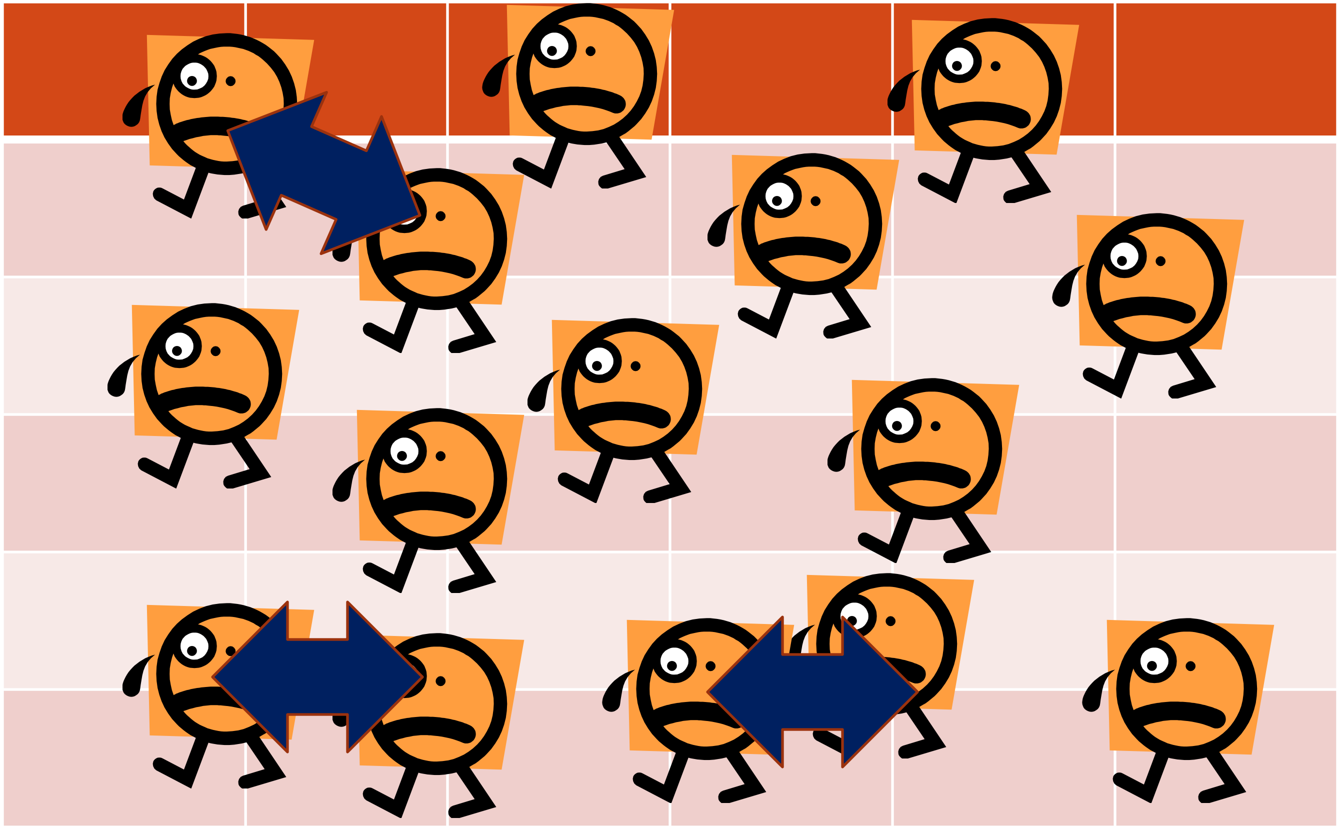
J_0 : Average , J^2 : Variance

Situation



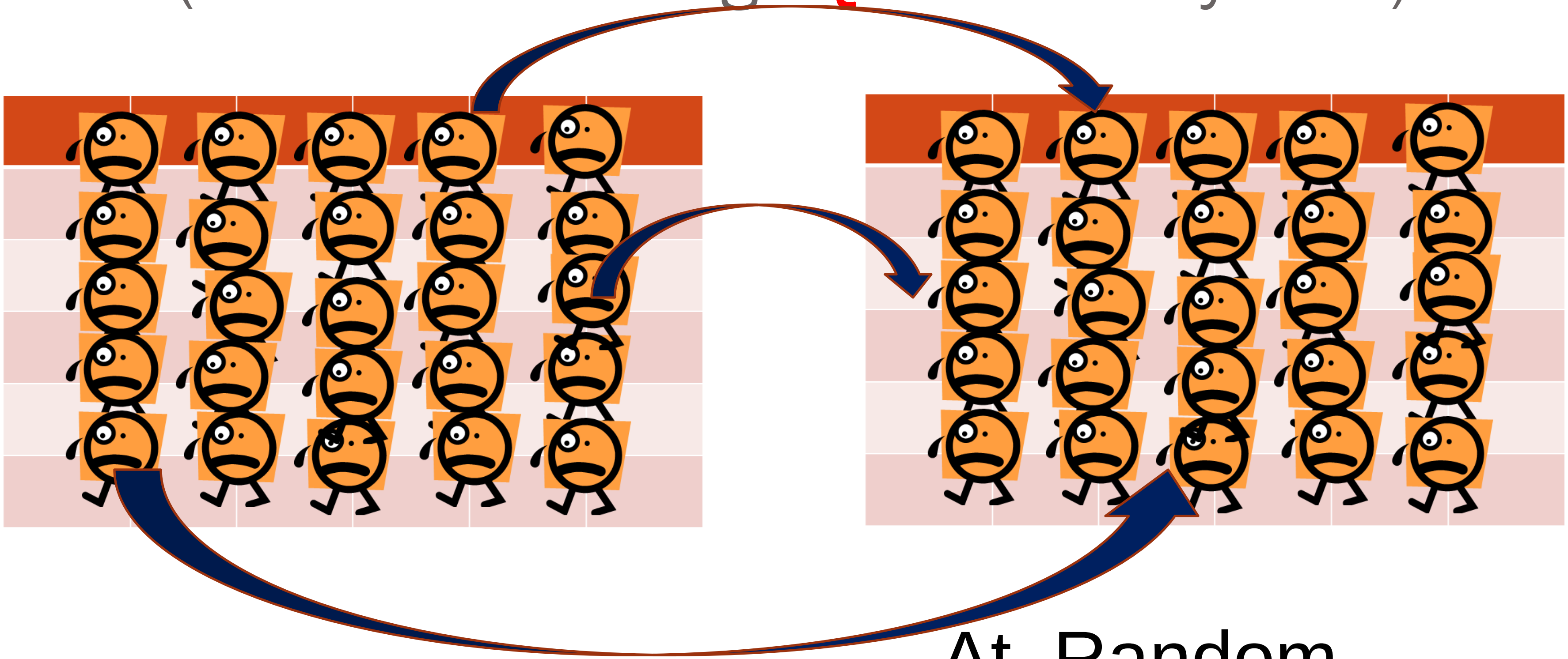
Situation

(random matching : annealed system)



Situation

(random matching : Quenched System)



At Random

ANEALED SYSTEM

- Social Welfare Function, **分布関数の配位平均**.

$$F = \gamma \log \langle Z \rangle ,$$

Probability of Matching

$$\langle Z \rangle = \sum_{\{S_i\}} \int_{-\infty}^{\infty} \prod_{(ij)} dJ_{ij} P\{J_{ij}\} \exp(\gamma H\{J_{ij}\}),$$

Fitness

$$= \sum_{\{S_i\}} \exp\left[\sum_{(ij)} \left\{ \gamma J_0 S_i S_j + \frac{(\gamma J)^2}{2} (S_i S_j)^2 \right\} \right]$$

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$$\underset{m}{Max} \quad F$$

Solved

$$F = \gamma \left[\sum_{[S_i]} \left\{ \gamma J_0 (\sum_i S_i)^2 + \frac{1}{2} (\gamma J)^2 (\sum_i S_i)^4 - \gamma J_0 N \sum_i S_i^2 - \frac{1}{2} (\gamma J)^2 N \sum_i S_i^4 \right\} \right]$$

- m=<Si>,

$$\frac{\partial F}{\partial m} = 2\gamma^2 J_0 N^2 m + 2\gamma^3 J^2 N^4 m^3 = 0$$

$$m = 0 \quad or \quad \pm\sqrt{\frac{-J_0}{\gamma J^2 N^2}}$$

As N $\rightarrow \infty$, m = 0 .

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1 . In Ising type, the order parameter is a tanh function; however, the order parameter is a point, like a replicator system.

2. If there are infinite players on this lattice, then the order parameter is 0.

QUENCHIED SYSTEM

- Quenched system :

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Hubbard-Stranovich Trans. $\exp\left[\frac{a^2}{2}\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[ax - \frac{x^2}{2} \right] dx$

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Solved →

$$m = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{z^2}{2}\right) \tanh\left(\gamma \tilde{J} \sqrt{qz} + \gamma \tilde{J}_0 n\right) dz$$

$$q = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{z^2}{2}\right) \tanh^2\left(\gamma \tilde{J} \sqrt{qz} + \gamma \tilde{J}_0 n\right) dz$$

→ Like a Ising Type

TAP EQUATION

- Let a model add another parameter h_j (an effect of externality).

$$H\left(\left\{J_{ij}\right\}\right)=\sum_{i\neq j}J_{ij}S_iS_j+\sum_{i\neq j}h_jS_j$$

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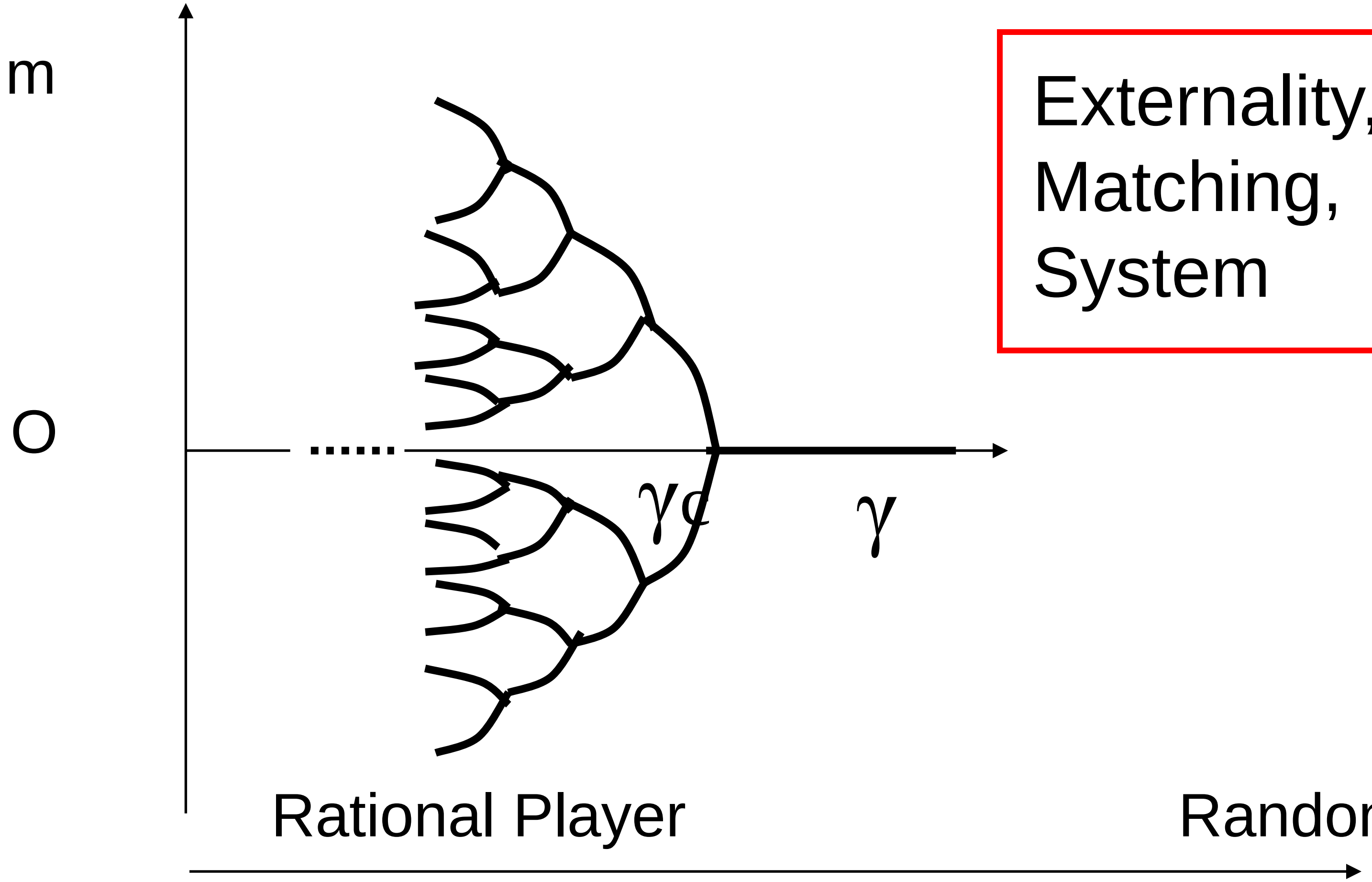
→If the maximal eigenvalue of J_λ is 2J, the order parameter is discontinuous.

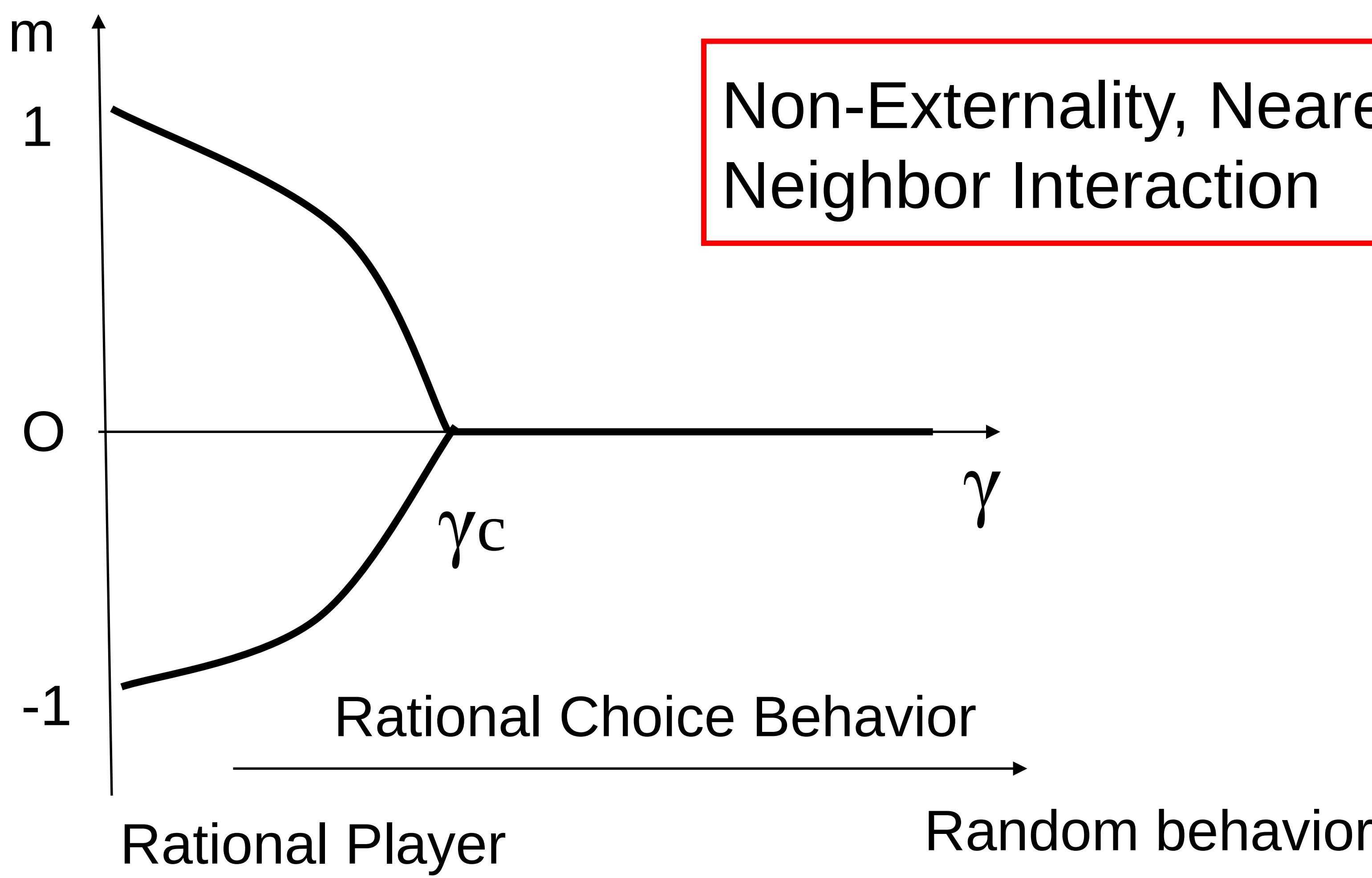
→Multiple Equilibria

MULTIPLE EQUIRIBRIA



Externality, Random Matching, Quenched System

- EXAMPLE : Ising model
- Si={-1,1} → m= -1,0(random),1



Non-Externality, Nearest Neighbor Interaction

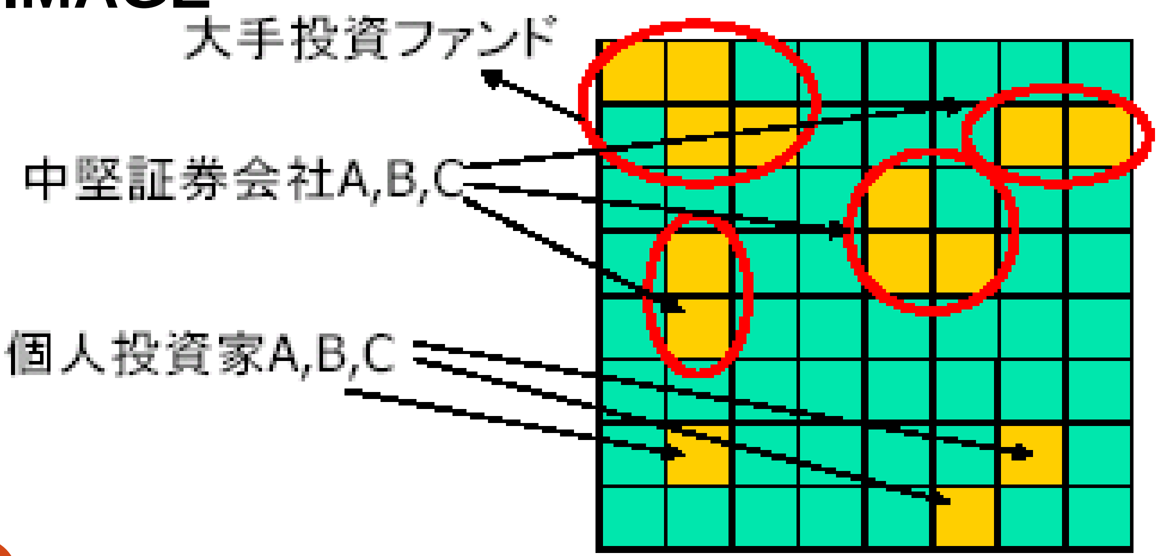
4. IMPLICATION

Cont-Bouchaud's Model

Cont-Bouchaud 's model ≒ § 2 's model.

- This study discusses the simplified Cont and Bouchaud model through our models.

IMAGE

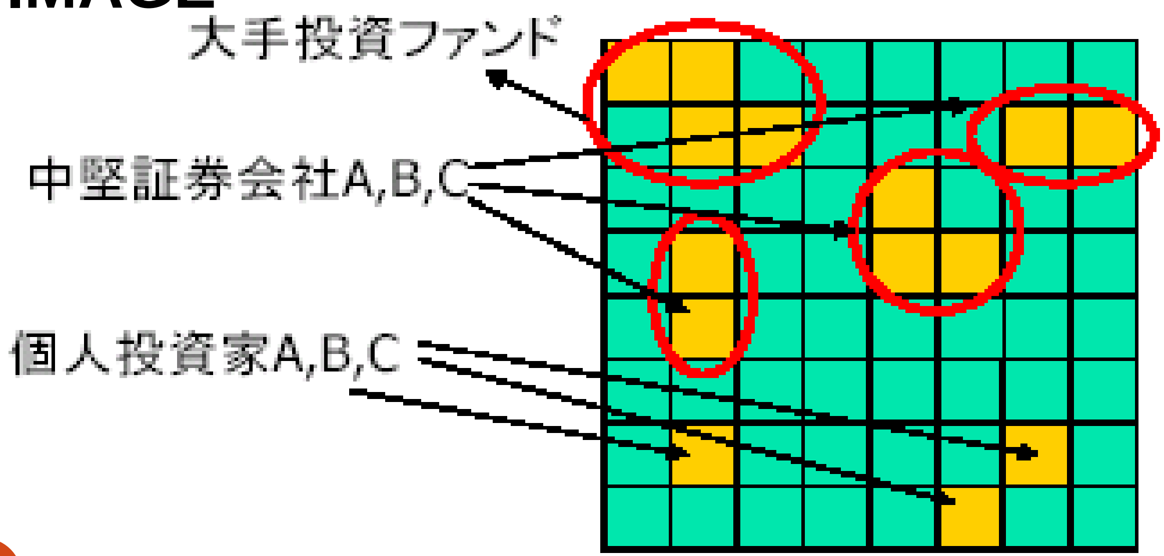


Cont-Bouchaud 's model ≒ § 2 's model.

- This study discusses the simplified Cont and Bouchaud model through our models.

→ We can understand the player's behavior in Cont and Bouchaud model.

IMAGE



POINT! :
Percolation Cluster
⇔ trading groups

- A stock market with *N* AGENTS
- Trading a SIGNLE asset

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- The demand for stock of agent *i* is represented by a random variable Φi(t)(∈ {-1,0,1})

Φi(t)>0 : BULL , <0 : BEAR, 0: not trade

$$P(\phi_i = +1) = P(\phi_i = -1) = a, P(\phi_i = 0) = 1 - 2a.$$

- A stock market with N AGENTS
- Trading a SIGNLE asset
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Φi(t)>0 : BULL , <0 : BEAR, 0: not trade

$$P(\phi_i = +1) = P(\phi_i = -1) = a, P(\phi_i = 0) = 1 - 2a.$$

- Between price changes and excess demand:

$$x(t) = x(t+1) - x(t) = \frac{1}{\lambda}\sum_{i=1}^{N}\phi_i(t)$$

λ- Market Depth

It measure the sensitivity of price to fluctuations in excess demand

CLUSTER

The number of clusters (coalitions)

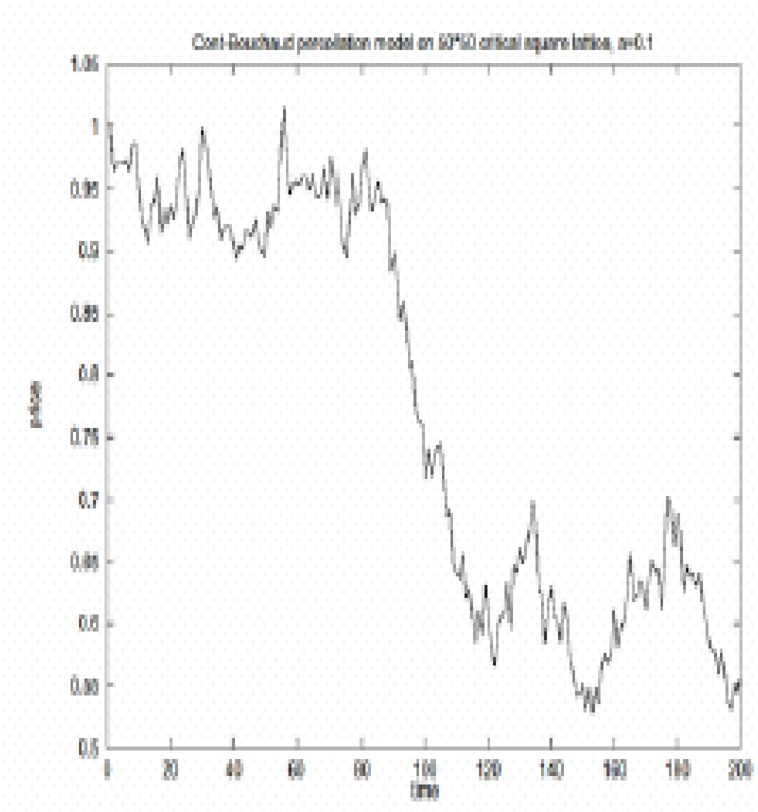
The size of cluster

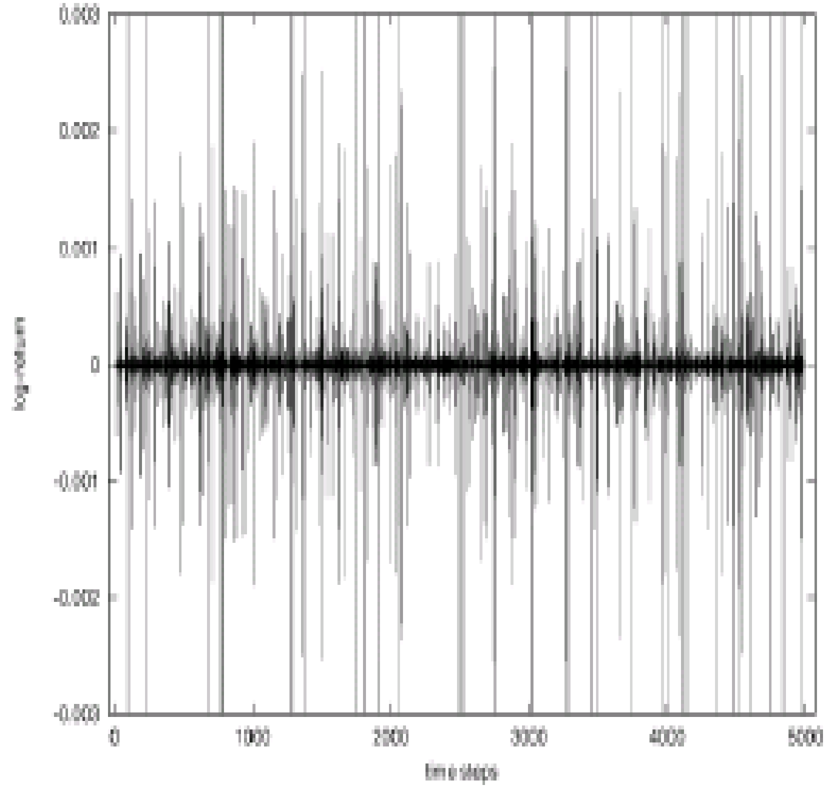
$$x(t) = \frac{1}{\lambda}\sum_{\alpha=1}^{k}W_\alpha\phi_\alpha(t)$$

Aggregate Excess Demand

Price Valuation

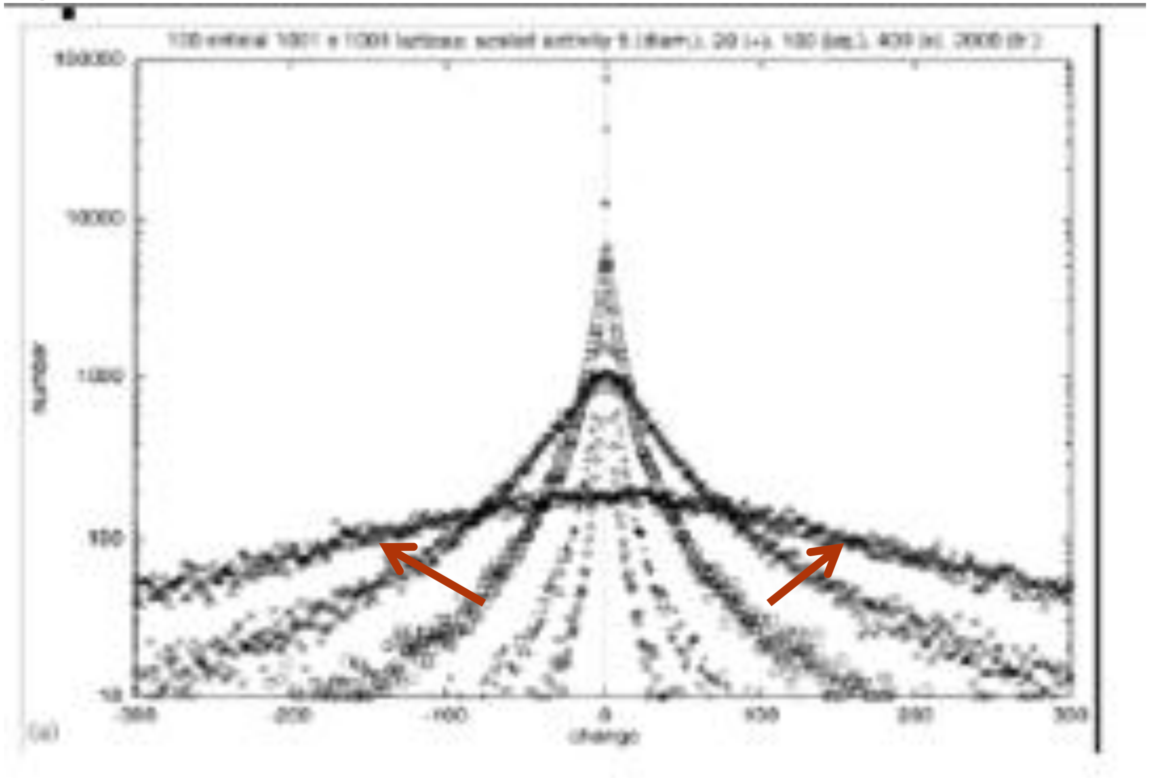
$$p = p_c$$





陳昱 パーコレーションと金融市場の価格変動 より転載

Heavily tails



陳昱 パーコレーションと金融市場の価格変動 より転載

- For decrease in the activity parameter a showing its similarity with real stock market phenomena: the *heavily tails* observed in the distribution of stock market.

Random Matching (Cont-Bouchaud)

- Annealed Sys. (+ externalitiy)

$\longrightarrow$

- Quenched Sys

$\longrightarrow$

- Quenched Sys. + externality

$\longrightarrow$

Random Matching (Cont-Bouchaud)

- Annealed Sys. (+ externalitiy)

→One action occupied.

The price is higher or lower than before.

- Quenched Sys

→

- Quenched Sys. + externality

→

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→multiple equilibria (The rate of price change is dependent on the size of γ).

5. Summary and Future Works

SUMMARY

Add the parameter (optimal choice behavior).

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- **Sec. 2**: We construct the nearest neighborhood model (Ising Type)
- **Sec. 3**: We construct the randomly matched model (Annealed Sys., Quenched Sys.)
- **Sec.4**: Quenched System + Externality

$\longrightarrow$ multiple equilibria

- **Sec.5**: Apply to econo-physics' model

SUMMARY

Add the parameter (optimal choice behavior).

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→ multiple equilibria

- **Sec.5**: Apply to econo-physics' model

FUTURE WORKS： relation between this model and DMBG(Dynamic Matching and Bargaining Game), Simulation (Monte Calro Simulation)

Fin

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FAQ.

Q１）なぜ、1対1のゲームなのですか？

数理生物学の分野でも格子モデルを使って、空間構造のあるゲームを分析している研究があると思います(例えば, Nowak and May (1992), Nowak (2006) **など)が、これらの研究はノイマン近傍の相手とのゲームなどあると思いますが、なぜ隣の相手とのゲームなのでしょうか？**

A1) **まず理論としてきちんと定式化したかったので、**Ising**モデルや**SK**モデルのアイデアを借り、定式化するために、最近接の相手とのゲームとしました。数理生物学の分野では多くはシミュレーションによるアプローチだと思います。そこが我々のものとは異なります。相手が複数ある場合のゲームなどは今後の拡張となると思います。**

- Q2) このモデルを拡張するとしたら、どのようなことが考えられますか？
- A2) まず考えられるのが、戦略が2以上の場合。例えば、Celluer Automata からのアプローチ（Domany and Kinzel (1984), PRL, vol. 53, number 4. pp. 311-314）などが考えられます。つまりIsingモデルでは状態が2つでしたが、0から1までの実数とすれば、無限個の戦略がある場合に拡張することが出来るわけです。

  次にはゲームの相手が1対1ではなく、グループでゲームをする場合。これは主に数理生物学で研究されています。

  さらには、このモデルで重要なパラメーターを内生化する研究（超統計(super statistics)）。

などいろいろ考えることができます。

- Q3) 統計力学という言葉には聞き馴染みがないのですが、経済学ではよく使われている概念なのでしょうか？
- A3) はい。ミクロ, マクロ経済学教科書レベルのものでは取り扱われていませんが、経済学の研究にも使われています。
- 取り上げた研究以外にも、Follmer (JME, 1974)ではIsingモデルを。Grandmont(JET, 1992), Foley (JET, 1994) など多数あります。
- 元来統計力学は、古典力学では分析できない高次元系を分析するために開発されたものです。もちろん市場などの経済システムは大多数の人間が売買を繰り返す複雑なシステムであるので、有効であると思います。シミュレーションまで行うと、より現実に迫れるのではないかと思います。

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2008年11月　　　吉川　満。

http://kikkawa.cyber-ninja.jp/index.htm