Empirical Nash equilibrium and its applications

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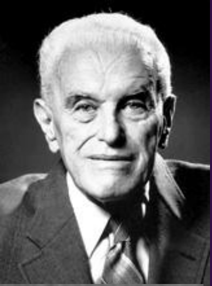
Today's Talk

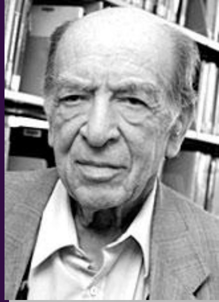
- **FORMULARING** game theory for the empirical Analysis.

- **PRESENTING** the typical example.

- **EXTENDING** this game theory to the dynamical framework.

1. INTRODUCTION





Nobel Prize in Economics and Game Theory

- 1994 - J.C. Harsanyi , J.F. Nash and R. Selten (Non-Cooperative Game Theory)
- 2005 - R.J. Aumann and T.C. Schelling (Conflict and Cooperation)
- 2007 - L.Hurwicz, E.S. Maskin and R.B. Myerson (Mechanism Design Theory)
- 2009 - E.Ostrom and O.E. Williamson (Governance : commons, boundaries of the firm)

The Kyoto Prize :2001 – J. Maynard Smith (Evolutionarily Stable Strategy)

WHAT IS THE “GAME” ?
(Non-cooperative Game)

There are two interacting players (Player 1, Player 2).

If player 1 chooses strategy 1 and player 2 chooses strategy 1, player 1‘s payoff is *a* , player 2’s payoff is *b*.

In this situation, which strategy does each player choose ?

(The game is played only once.)

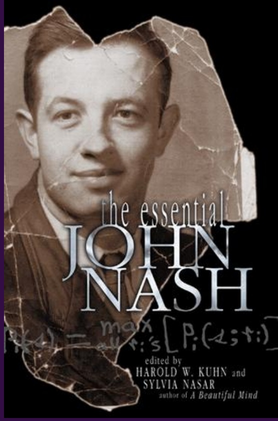
→This game’s solution is **Nash Equilibrium**.

player 2

player1		S1	S2
	S1	a,b	0,0
	S2	0,0	c,d

Nash equilibrium depends on the signs: *a,b,c,d*.

Interpretation of Nash Equilibrium (J.F.Nash's Ph D. Thesis)



- 1. "**Rationality**" (normative) ･･･ the players are perceived as rational and they have complete information about the structure of the game, including all of the players' preferences regarding possible outcomes, where this information about each other's strategic alternatives and preferences, they can also compute each other's optimal choice of strategy for each set of expectations. If all of the players expect the same Nash equilibrium, then there are no incentives for anyone to change his strategy.
- 2. "**Statistical Populations**" (descriptive) ･･･ is useful in so-called evolutionary games. This type of game has also been developed in biology in order to understand how the principles of natural selection operate in strategic interaction within among species.(→ **Mass Action**)

2. FORMULATION

DEF. Strategic Game

DEF. A strategic game is

$$G = \left(N, \{\pi_{i\alpha}\}_{i\in n,\alpha\in M}, \{p_{i\alpha}\}_{i\in N}\right).$$

where (i) $N<\infty$ is the set of **players** and a population in the sense of statistics of players for each position $i (i=1,\ldots,n)$ of the game in this set.

(ii) $\pi_{i\alpha}$ $(\alpha=1,\ldots,m)$ is the set of **strategies/actions** in the position i.

(iii) $p_{i\alpha}$ is a player's **utility function**.

IMAGE: Players’ Set : $N(<\infty)$

 : player: large population

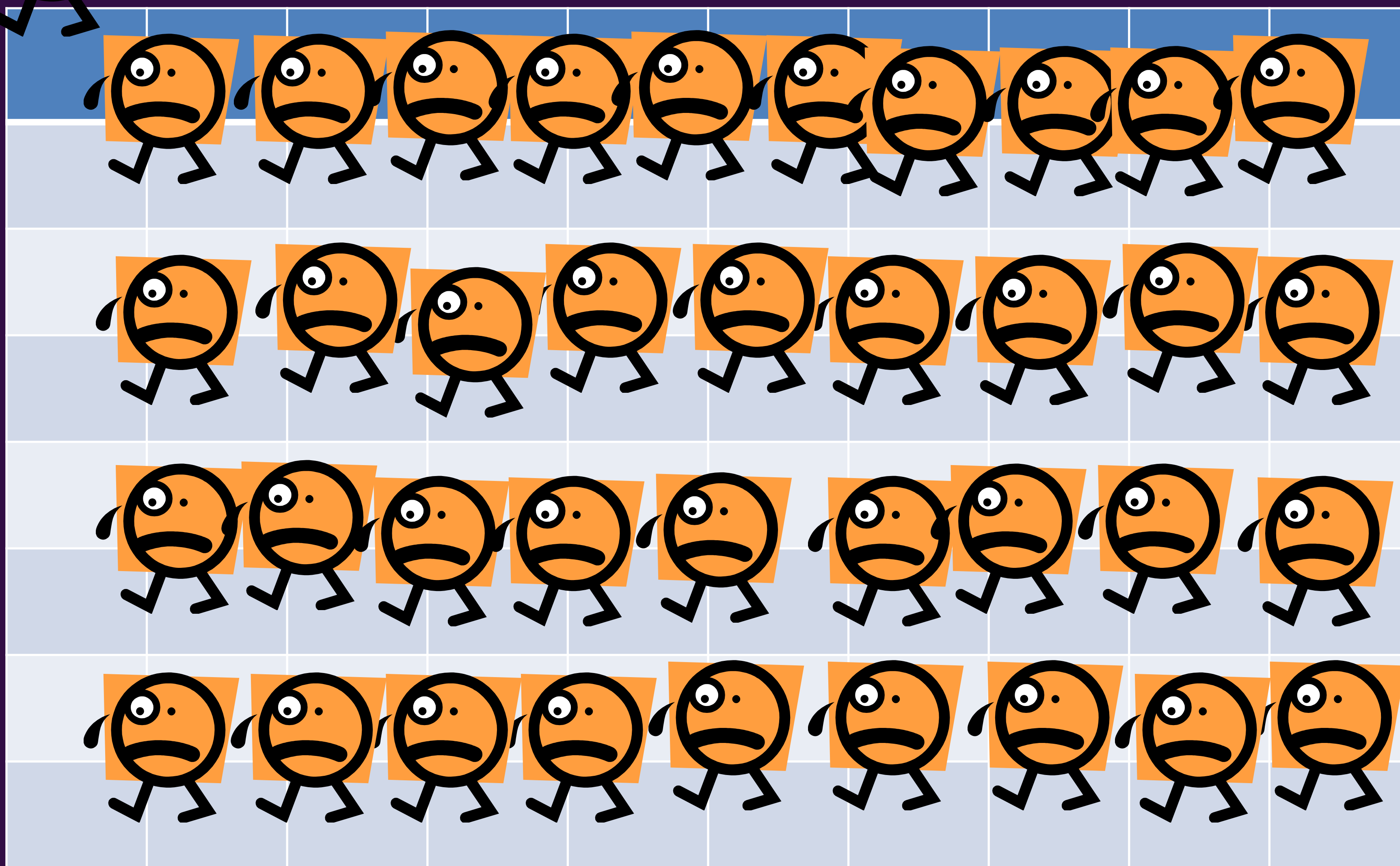
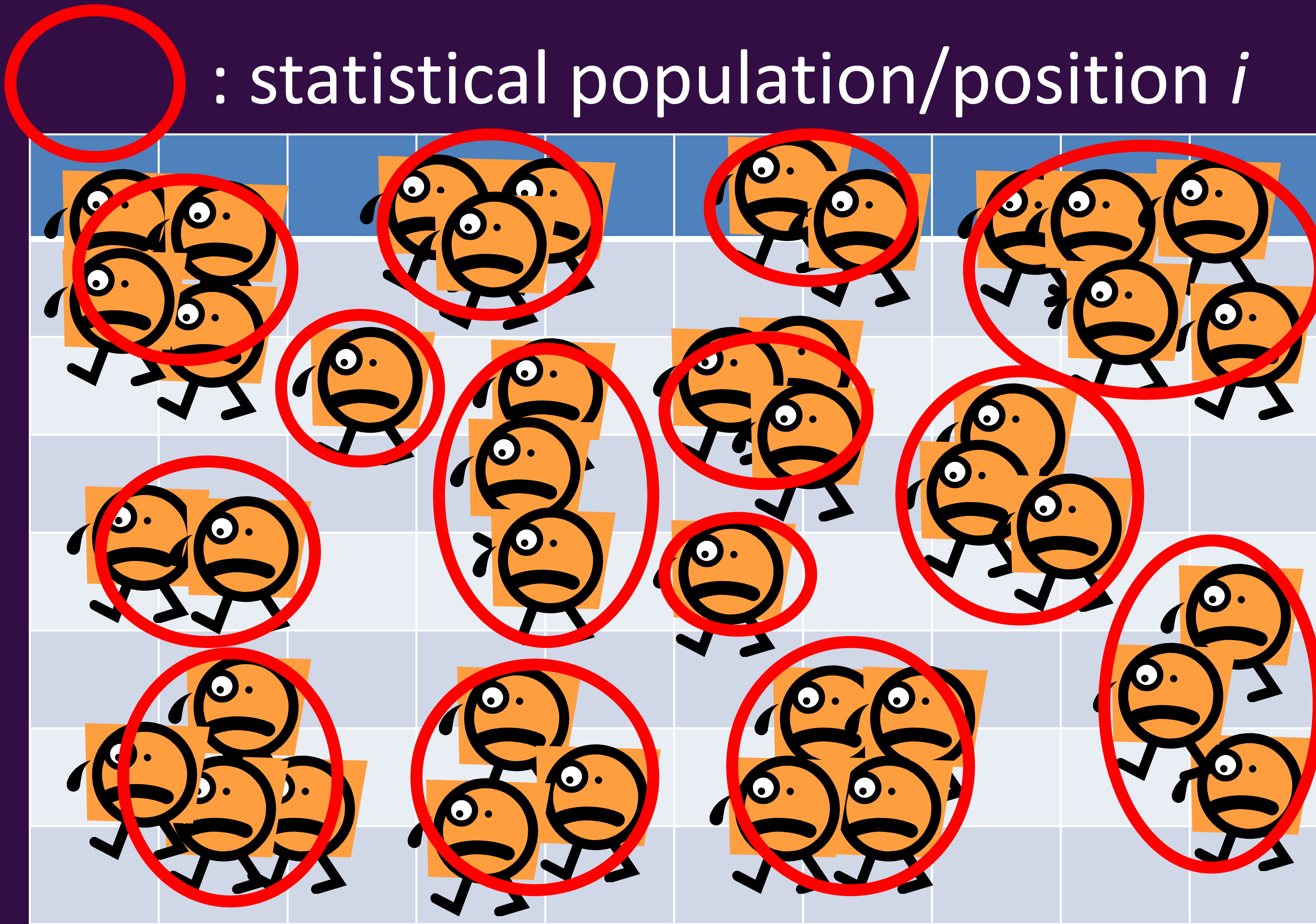
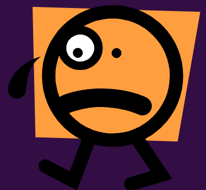


IMAGE: Statistical Population: $n(<\infty)$

: statistical population/position *i*



-  has m ($<\infty$) choices ($\pi_{i\alpha}$ ($\alpha=1,\ldots,m$), **pure strategy**).

- Introduce s_i (what strategy does each player play in *i*th position : **mixed strategy**)

$$s_i=\sum c_{i\alpha}\pi_{i\alpha},\ \sum c_{i\alpha}=1,\quad s=(s_1,\ldots,s_n).$$

<u>Harsanyi (1973)</u>

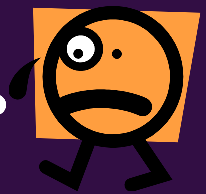
-  has a **pay-off function** : $p_i(s;\ \pi_{i\alpha})=p_{i\alpha}(s)$

IMAGE: *si (i=1,...n)*

DEF. Best Response, Nash eq.

DEF. : A **best response** of the position *i*'s strategy $\pi i\alpha$

For another *n-1* positions' strategy sets $s_{-i}=(s_1,\dots,s_{i-1},s_{i+1},\dots,s_n)$ is

$$p_{i\alpha}(s)=\max_{\beta} p_{i\beta}(s).$$

The whole best response for position *i* is $B_i(s_{-i})$ for strategy set s_{-i}.

DEF. A **Nash equilibrium** of a strategic game *n*-person game *G* is a profile $s^*=(s^*_1,\dots,s^*_n)$ with the property that for every position $i=(1,\dots,n)$ we have the best response for another position's strategy set s^*_{-1}.

Existence of Nash equilibria

TH. The mixed strategy $s^*=(s^*_1,\ldots,s^*_n)$ on G is a Nash equilibrium if and only if

$$s^* \in B(s^*)$$

TH. The strategic game G has a Nash equilibrium.

Proof. Kakutani's fixed point theory.

Evolutionarily Stable Strategy（ESS)

DEF.：Maynard Smith and Price (1973) : $x \in \Delta$ is an *evolutionarily stable strategy (ESS)* if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

INTERPRETATION：incumbent payoff is higher than that of the post-entry strategy

Proposition

PRO. : $x \in \Delta$ is an ESS if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$

$$p_{iy}(x) \leq p_{ix}(x), \qquad \forall y,$$

$$\left| s_i - x^* \right| < \varepsilon$$

where $x^* \in Z$ is the index of the equilibrium action.

This equilibrium concept is like a *perfect* (Selten, 1975)

Kikkawa (2009)

- Kikkawa (2009) formulates this situation with statistical mechanics.

Th1. We obtain the action, $\pi_{i\alpha}$ in the position i, and the player's payoff from the outcome is $p_{i\alpha}(s)$,

$$c_{i\alpha}=Z^{-1}\exp(\gamma\, p_{i\alpha}(s)),$$

γ: non-negative constant, Z: normalization parameter.

- Kikkawa (2009) is similar to **Quantal Response Equilibrium**. (Mckelvey and Palfrey (1995, 1996))

Multinominal Logit Model

- From Kikkawa (2009), we can know the probability of choosing the strategy for each player.

$$+$$

- Data (the probability of choosing the strategy for each player)

- Regression analysis

$$Y_i = \alpha + \gamma f + u, \qquad u : \text{logistic distribution.}$$

- We can estimate optimal parameters in this model with **Least Squares Method**.

3. EXAMPLE

EXAMPLE (Experimental Economics)

We consider the following game with the payoff.

	B_1	B_2	B_3
A_1	15,-15	0,0	-2,2
A_2	0,0	-15,15	-1,1
A_3	1,-1	2,-2	0,0

Payoff matrix 1 (Lieberman (1960))

Nash equilibrium: (A_3,B_3)

\+ Experimental Data.

We can obtain the following payoff matrix with our method.

	B_1	B_2	B_3
A_1	3.55,2.10	1.75,3.63	-1.14,-2.19
A_2	0.98,4.59	-2.31,1.18	-1.15,-1.47
A_3	-0.01,-0.624	-0.37,-0.257	-0.01,0.021

Payoff matrix 2

Nash equilibria:

(A_3,B_3), (A_1,B_2)

ESS : (A_3,B_3)

4. DYNAMICAL SYSTEM

The dynamical framework

Assu. 1 (evolutionary approach) If an expected utility is greater, then the probability of playing the strategy will be higher in the next step.

Th. 2 Under Assumption 1, the following relationship about between the payoff and the population size is realized empirically :

$$\gamma' = \frac{\hat{p}\Delta r - E\left[\overline{p}_i \Delta r_{i\alpha}\right]}{\hat{p}_{i\alpha}(s)},$$

where $\hat{p}$ is the average payoff of the total population, $\hat{p}_{i\alpha}(s)$ is the position *i* 's average payoff, *Δ r* is the whole population size variation, $E\left[\overline{p}_i \Delta r_{i\alpha}\right]$ is the expected utilities' variation by the population size changed.

Proof. Price's law + OLS

5. SUMMARY

Summary

FORMULATONG

- **FORMULARING** game theory for the empirical Analysis.

EXAMPLE

- **PRESENTING** the typical example.

DYNAMICAL SYSTEM

- **EXTENDING** this game theory to the dynamical framework.

REFERENCE

[1] Harsanyi, J. C.: ‘’ Games with Randomly Distributed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points, ‘’ International Journal of Game Theory, 2 (1973), 1-23. **[HP]**

[2] Kikkawa, M. : ''Statistical Mechanics of Games — Evolutionary Game Theory —,'' *Progress of Theoretical Physics Supplement*, **179** (2009), 216-226. **[HP]**

[3] Lieberman, B. : ‘’Human behavior in a strictly determined 3 X 3 matrix game,’’ *Behavioral Science*, **5** (1960), 317-322. **[HP]**

[4] McKelvey, R. D. and Palfrey, T. R. : ‘’Quantal Response Equilibria for Normal Form Games,'‘ *Games and Economic Behavior*, **10** (1995), 6-38. **[HP]**

[5] Mckelvey, R. D. and Palfrey, T. R. : ‘’A Statistical Theory of Equilibrium in Games,’’ *Japanese Economic Review*, **47** (1996), 186-209. **[HP]**

[6] Selten, R. “Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games,” International Journal of Game Theory, 4 (1975), 25-55. **[HP]**

Text Book

For Detail, See my Website(**Bookguide** **Readinglist)**

Classsic:

[1] Maynard Smith, John **Evolution and the Theory of Games** , Cambridge University Press, 1982/10. 日本語訳

[2] Axelrod, Robert **The Evolution of Cooperation** , Basic Books, 1984/03. 日本語訳

Text Book:

[1] Weibull , Jorgen W. **Evolutionary Game Theory** , MIT Press, 1995/08/14. 日本語訳

[2] Hofbauer, Josef and Sigmund, Karl **Evolutionary Games and Population Dynamics** , Cambridge University Press, 1998/07. 日本語訳

[3] Vega-Redondo , Fernando **Evolution, Games and Economic Behaviour** , Oxford University Press, 1997/01.

[4] Samuelson, Larry **Evolutionary Games and Equilibrium Selection (Mit Press Series on Economic Learning and Social Evolution, 1)** , MIT Press, 1997/04.

For Beginner :

[1] 石原英樹, 金井雅之 **進化的意思決定 (シリーズ意思決定の科学)** , 朝倉書店, 2002/04/05.

[2]大浦宏邦 **社会科学者のための進化ゲーム理論—基礎から応用まで**
書房, 2008/09/25.

Thank You For Your Attention

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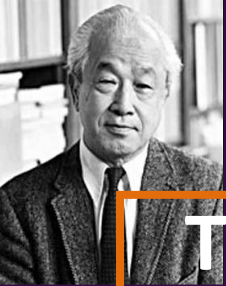
APPENDIX

Situation (Traditional Evolutionary Game Theory)



Kakutani's Fixed Point Theory

TH.: Let S be a nonempty, compact convex subset of the separable complete metric space and let $F(.) : S \rightarrow S$ be a set-valued mapping for which

(i) For all $x \in S$ the set $F(x)$ is a nonempty set and convex on S.

(ii) for all sequence $\{x_\nu\}_{\nu=1}^{\infty}$ and $\{y_\nu\}_{\nu=1}^{\infty}$ such that

$y_v \in F(x_v)$, $v=1,2,\ldots,x_v \rightarrow x_O$, $y_v \rightarrow y_O$ $(v \rightarrow \infty)$,

we have $y_0 \in F(x_O)$.

Then there exists $x^* \in F(.)$ such that $x^* \in F(x^*)$