“Theory of Biomathematics and Its Applications V”
in RIMS, Kyoto Univ. (420: 2009/01/16 10:20-)

Coevolution and Diversity in Evolutionary Game Theory : Random Environment

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OUTLINE

1． INTRODUCTION（Motivation, Purpose）
2． RELATED LITERATURES and PRELIMINARIES
3． OUR MODEL
   3-1. HARSANYI TYPE
   3-2. SELTEN TYPE
4． EXTENSION (Global Game)
5． SUMMARY and FUTURE WORKS

1． INTRODUCTION

OUR PROBLEM

- Q How does each player choose the action in stochastic environment ?

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- A.1：Each player randomly chooses the action. (mixed strategy) (Harsanyi , 1973)

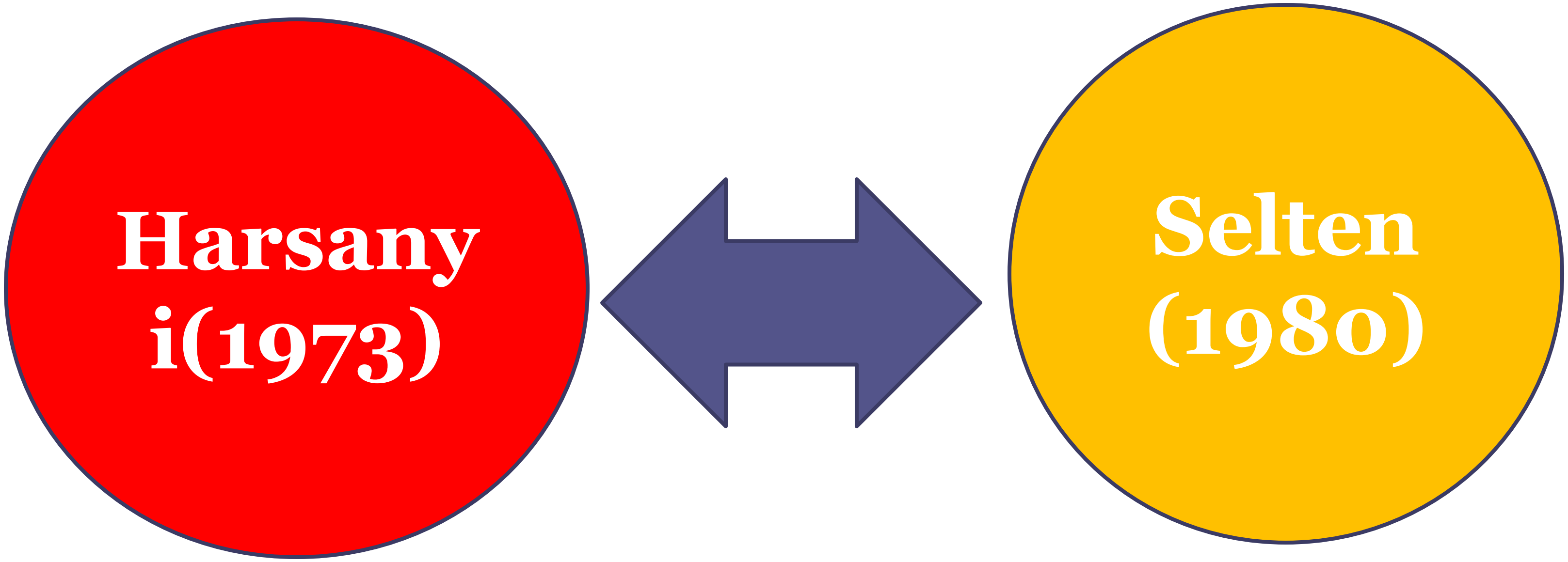
- A.2：Each player chooses the better action. (pure strategy) (Selten, 1980)

Research Fields (this study)

Harsanyi(1973)

Selten (1980)

Research Fields (this study)



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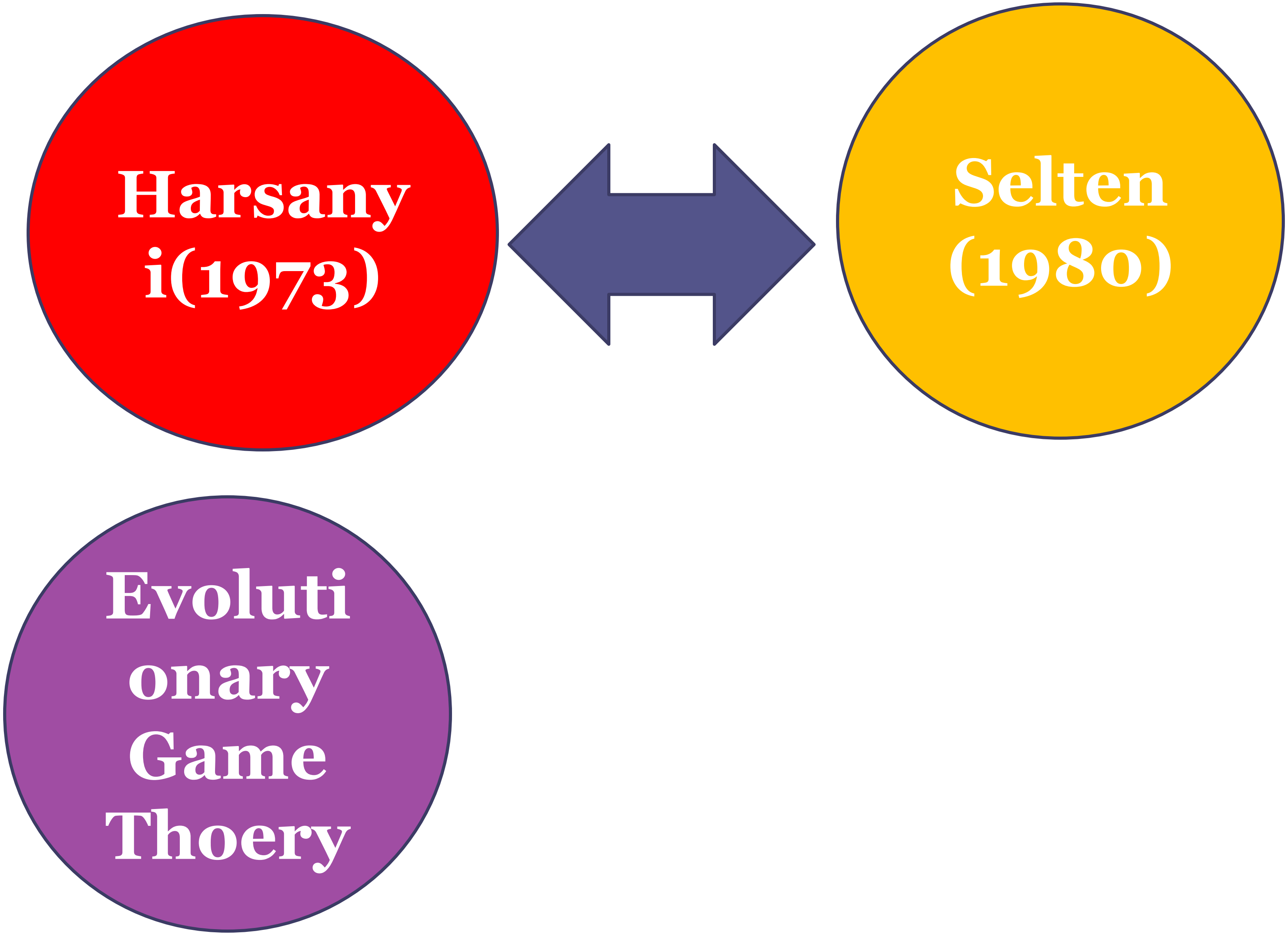

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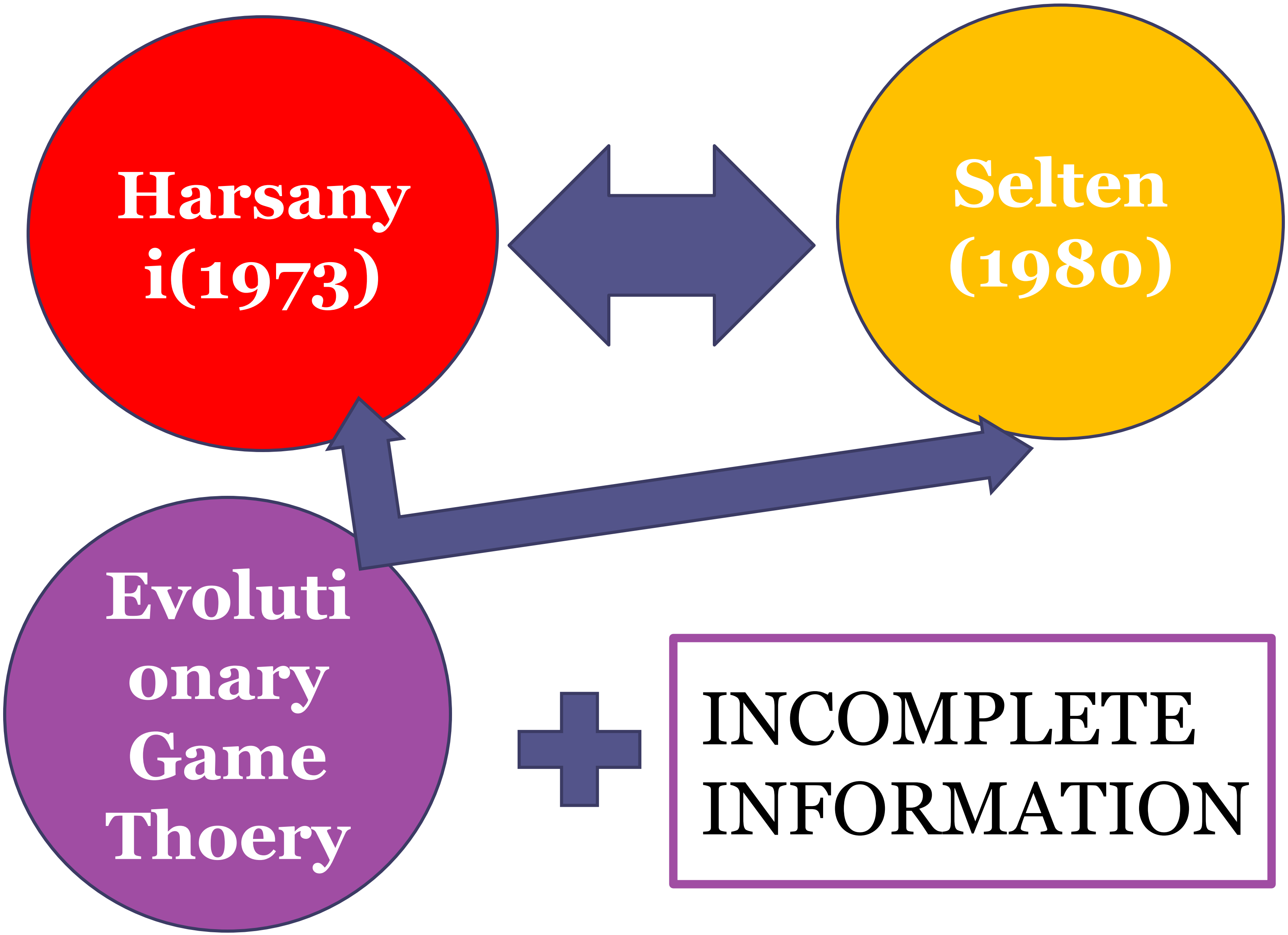
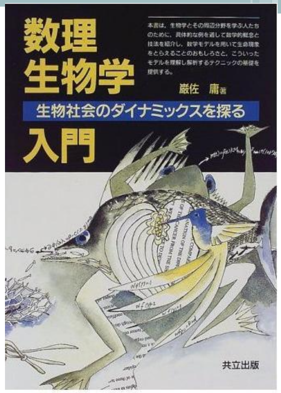


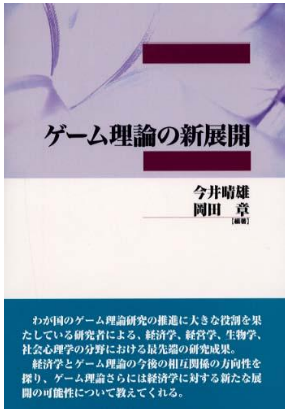
2. RELATED LITERATURES and PRELIMINARIES

MATHEMATICAL BIOLOGY

- Stochastic Environment
- Bet-Hedging Strategy (=Mixed Strategy)



IWASA, Y. (1998)



IMAI and OKADA (2002)

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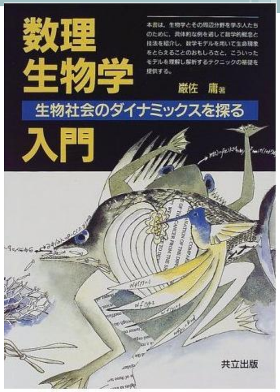
- Stochastic Environment
- Bet-Hedging Strategy (=Mixed Strategy)

- Fitness Function is
- (i) Geometric mean
- (ii) Arithmetic average



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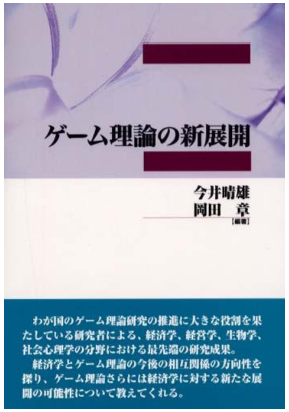

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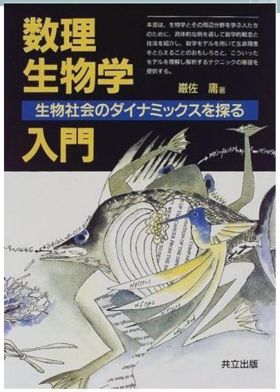

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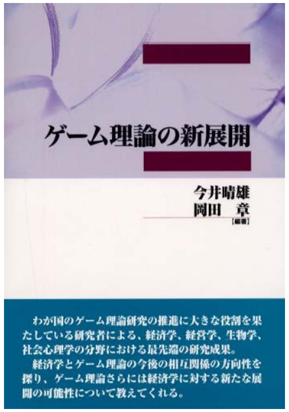

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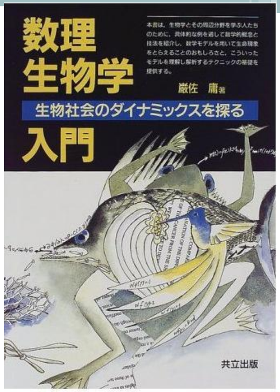

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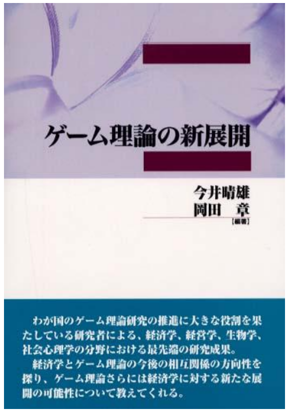

IMAI and OKADA (2002)

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Game Theory

- The fitness(utility) function is a von-Neumann-Morgenstern utility function .
- → No Bet-Hedging Strategy ?

Harsanyi (1973)

- Harsanyi, J. C. (1973): ”Games with Randomly Distributed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points,” *International Journal of Game Theory*, Vol.**2**, pp.1-23.

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- **Th.** Fix a set of I players and strategy spaces S_i. For a set of payoffs $\{u_i(s)\}_{i\in F, s\in S}$ of Lebesgue measure 1, for all independent, twice-differentiable distributions p_i on $\Theta_i=[-1,1]^{\#S}$, any equilibrium of the payoffs u_i is the limit as $\varepsilon\rightarrow 0$ of a sequence of pure-strategy equilibria of the perturbed payoffs $\tilde{u}_i$.

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→The probability distributions over strategies induced by the pure-strategy equilibria of the perturbed game converge to the distribution of the equilibrium of the unperturbed game.

EX. Battle of Sexes (BoS)

- Consider two-player games in which each player i has two pure strategies, a_i and b_i. Let δ_i for i=1,2 be independent random variables, each uniformly distributed on [-1, 1], and let the random variables $\varepsilon_i(a)$ for i=1,2 and $a \in A$ have the property that $\varepsilon_1(a_1,x)-\varepsilon_1(b_1,x)= \delta_1$ for $x=a_2,b_2$ and $\varepsilon_2(x,a_2)-\varepsilon_2(x,b_2)= \delta_2$ for $x=a_1,b_1$.
- All the equilibrium of BoS are approachable under ε.

	a2	b2
a1	$2+\gamma\delta_1$, $1+\gamma\delta_2$	$\gamma\delta_1$,0
b1	0, $\gamma\delta_2$	1,2

Proof Outline

(1) The pure equilibria are trivially approachable.

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(2) We consider the strictly mixed equilibrium.

For $i = 1, 2$ let pi be the probability that player i's type is one for which he chooses a_i in some Nash equilibrium of $G(\gamma\varepsilon)$.

(i) it is optimal for player 1 to choose a_1 if $(2 + \gamma\delta_1)p_2 \geqq (1 - \gamma\delta_1)(1 - p_2)$.

(ii) $-1 \leqq \delta_1 \leqq 1$

(i) + (ii) : $p_1 = 1/2(1 - (1 - 3p_2)/\gamma)$.

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Solving for p_1 and p_2 we find that $p_1 = (2 + \gamma)/(3 + 2\gamma)$ and $p_2 = (1 + \gamma)/(3 + 2\gamma)$ satisfies these conditions. Since $(p_1, p_2) \rightarrow (2/3, 1/3)$ as $\gamma \rightarrow 0$ the mixed strategy equilibrium is approachable.

Selten (1980)

- Selten, R. (1980): ”A Note on Evolutionary Stable Strategies in Asymmetric Animal Conflicts,” *Journal of Theoretical Biology*, Vol.**84**, pp.93-101.

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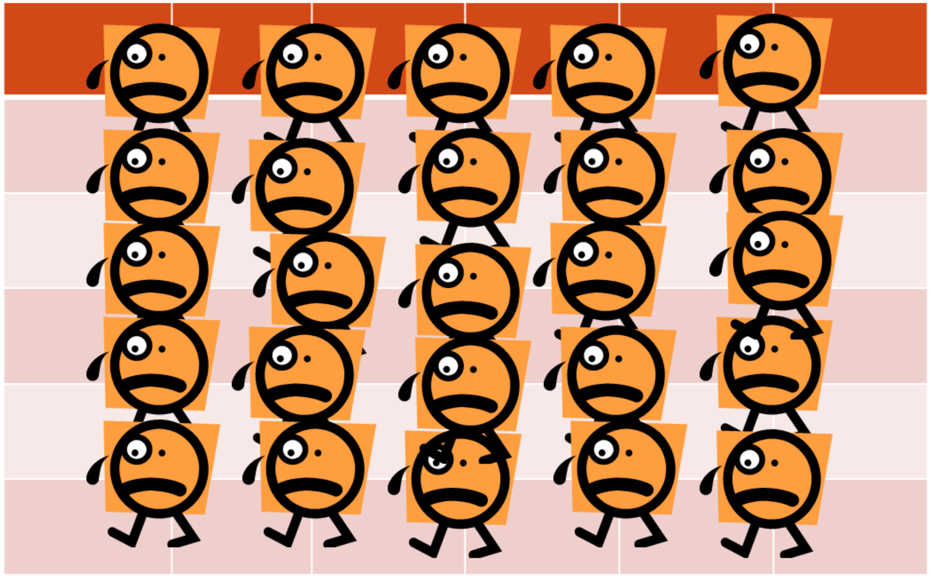
- $\boxed{\text{\textbf{Th.} A behavior strategy } x^* \text{ in } \Gamma \text{ is evolutionary stable if and only if } x^* \text{ is a strict Nash equilibrium of } G.}$

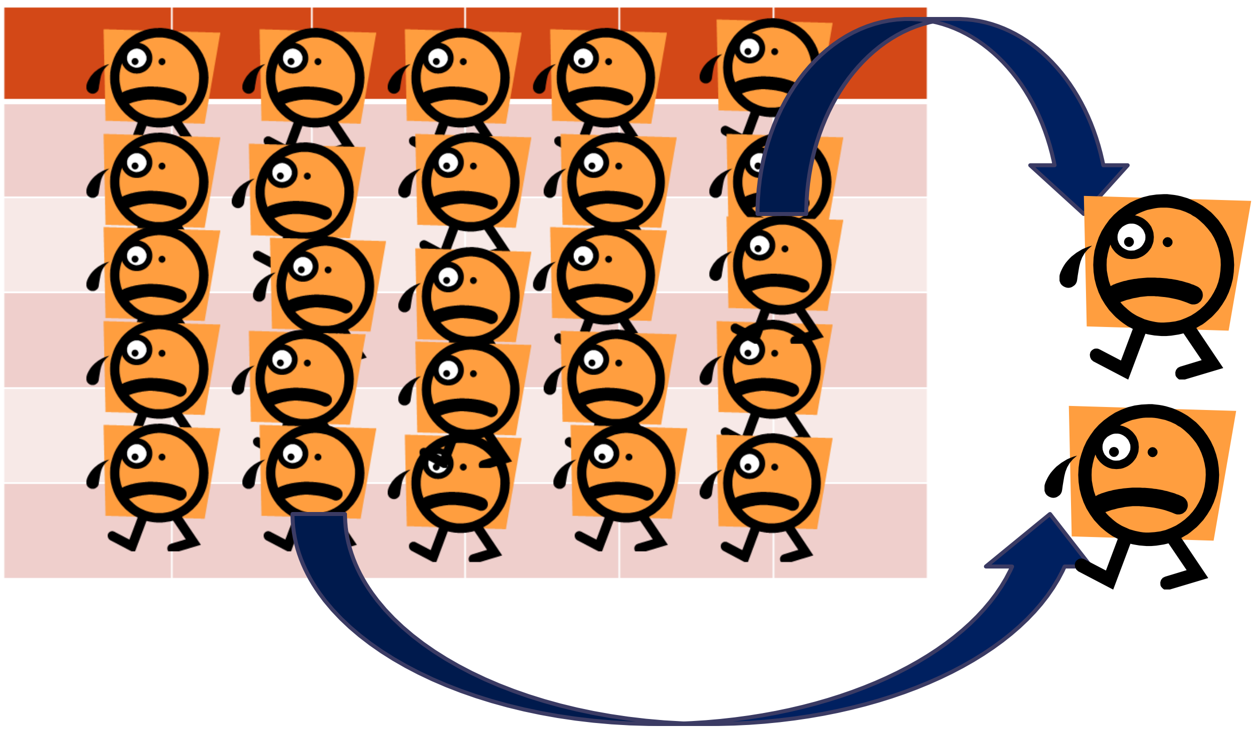
→No mixed equilibria are evolutionary stable when players can condition their strategies on their roles in a game.

Situation (Role Completed Game)



Situation (Role Completed Game)

At Random



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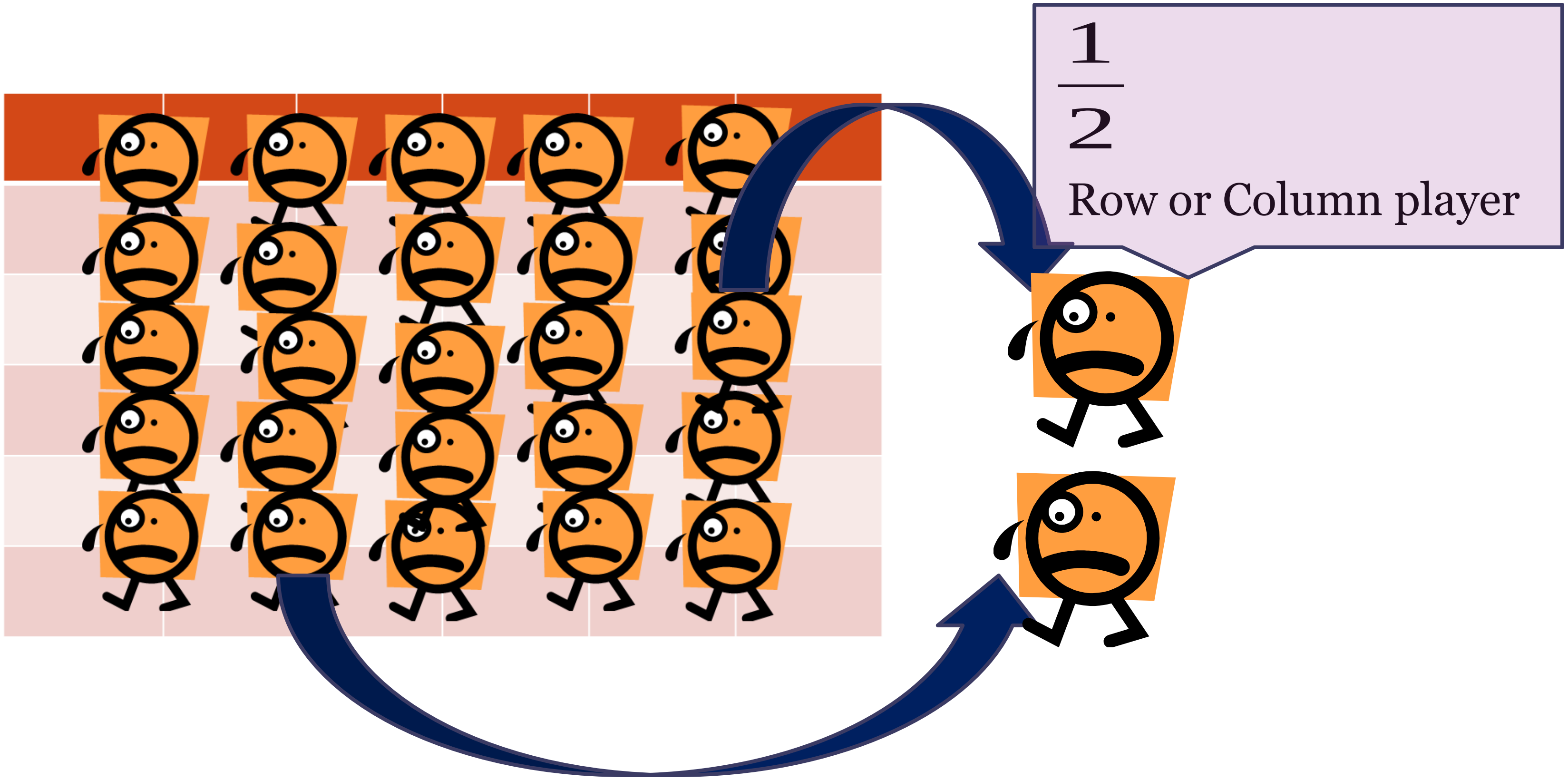

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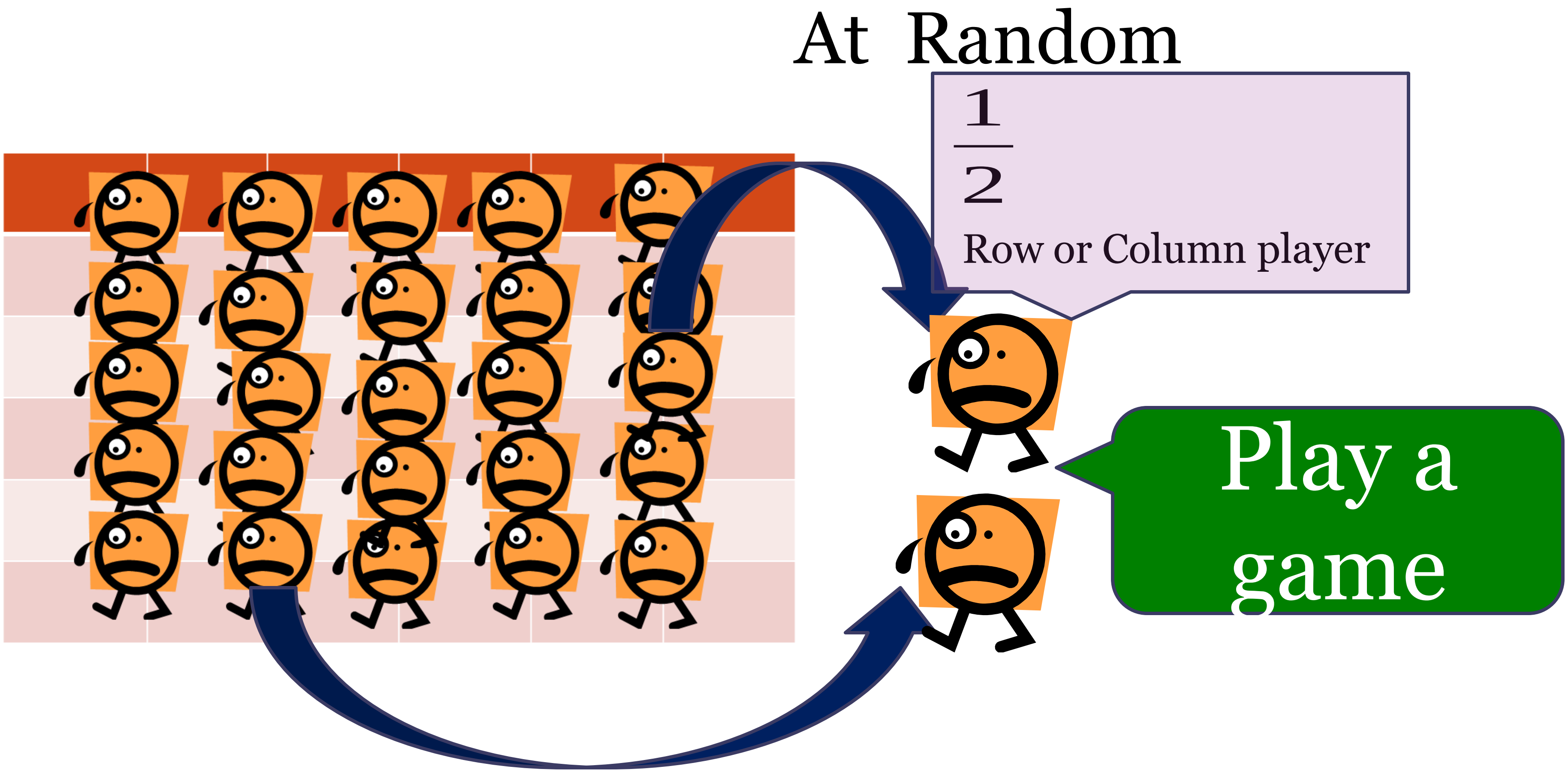
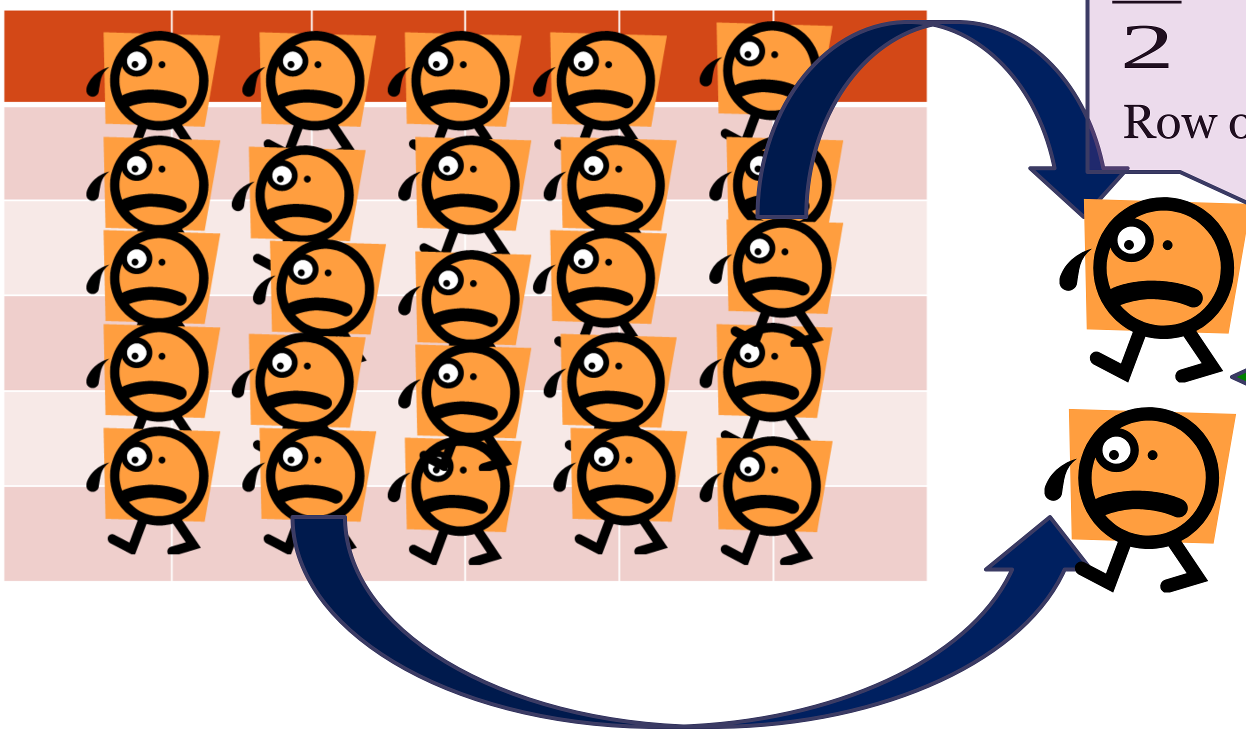


Situation (Role Completed Game)

At Random (infinitely)

$\frac{1}{2}$

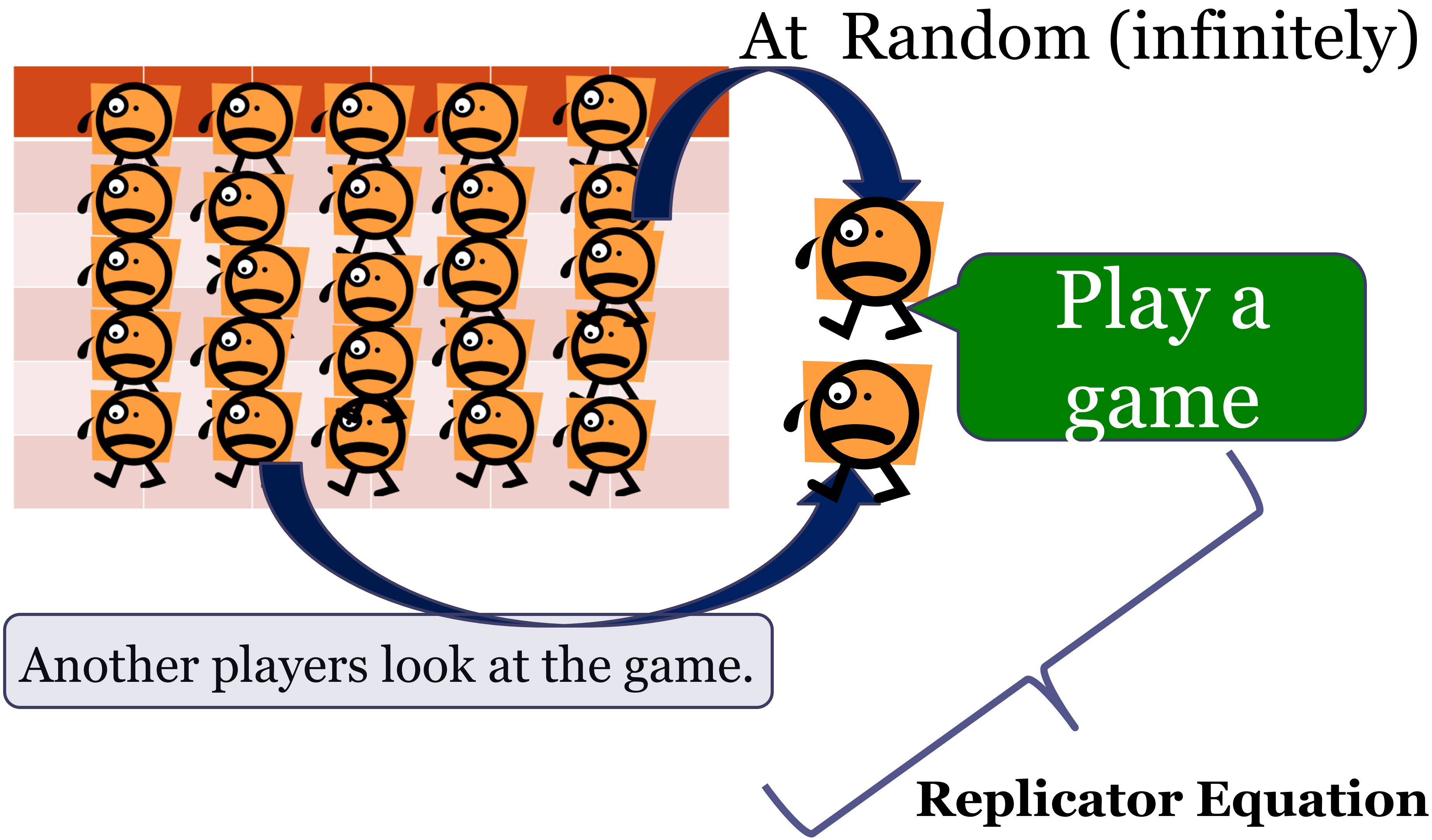
Row or Column player



Play a game

Another players look at the game.

Situation (Traditional Evolutionary Game Theory)



Ex. Hawk-Dove Game

	D	H
D	V/2, V/2	0,V
H	V, 0	V/2-C,V/2-C

H-D Game

- **Classical H-D Game** :
- **Strategy** : {Dove, Hawk}
- **Payoff** : $V>0$, $V<C$
- **Nash Eq.** : Pure {(H.D), (H,D)} + Mixed
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- **ESS :** Mixed (× strict Nash)
- **Stability** : Limit Cycle, Structurally Unstable.
- **Replicator Eq.**

$$\dot{x} = x(1-x)\{V/2 - C + Cx\}$$

- **Role Completed H-D Game**
- **Pure Strategy : {DD}, {DH}, {HD}, {HH}**
- **{DH} means play Dove if chosen to be a row player in the surface game and Hawk if chosen to be a column player.**
-

	DD	DH	HD	HH
DD	V/2,V/2	V/4,3V/4	3V/4,V/4	0,V
DH	3V/4,V/4	V/2,V/2	(V-C)/2, (V-C)/2	V/4-C/2, 3V/4-C
HD	V/4,3V/4	(V-C)/2, (V-C)/2	V/2,V/2	V/4-C/2, 3V/4-C
HH	V,0	3V/4-C,V/4-C/2	3V/4-C,V/4-C/2	V/2-C,V/2-C

Role Completed H-D Game

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- **ESS :** (DH,DH) , (HD, HD)

	DD	DH	HD	HH
DD	V/2,V/2	V/4,3V/4	3V/4,V/4	0,V
DH	3V/4,V/4	**V/2,V/2**	(V-C)/2, (V-C)/2	V/4-C/2, 3V/4-C
HD	V/4,3V/4	(V-C)/2, (V-C)/2	**V/2,V/2**	V/4-C/2, 3V/4-C
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Role Completed H-D Game

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- **ESS : (**DH,DH) , (HD, HD) (○ strict Nash)

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HH	V,0	3V/4-C,V/4-C/2	3V/4-C,V/4-C/2	V/2-C,V/2-C

Role Completed H-D Game

3. OUR MODEL

3-1. HARSANYI TYPE

Stochastic Environment (Harsanyi (1973) + Dynamics)

- Stochastic Environment = payoff variation

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- **Pro.** Let x be a strategy distribution. It satisfies :

$$P(x,t)dx = (2\pi\sigma^2 t)^{-1/2} \exp\left[-\frac{\left(\log x - \log x^*(t)\right)^2}{2\sigma^2 t}\right]\frac{dx}{x}.$$

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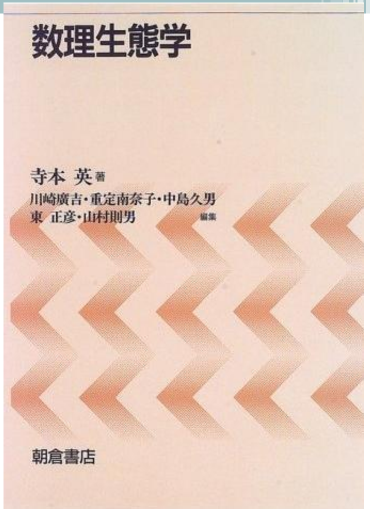
→Approachable under variance (σ^2)

PROOF OUTLINE

- Teramoto (1997)



Teramoto(1997)

PROOF OUTLINE

- Teramoto (1997)
- (i) transformation

$$\dot{x}_i(t) = x_i(t)\{g_i(t) - g(t)\},\ g_i(t) = g_i + \zeta(t)$$

$$\log\frac{x_i(t)}{x(0)} - g_i t + \int_0^t g(t)dt = \sum_{k=1}^{n} \xi_k$$

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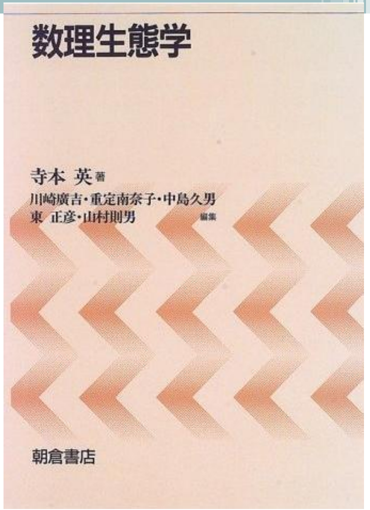

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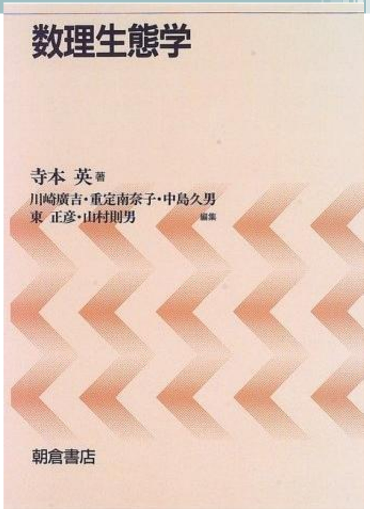

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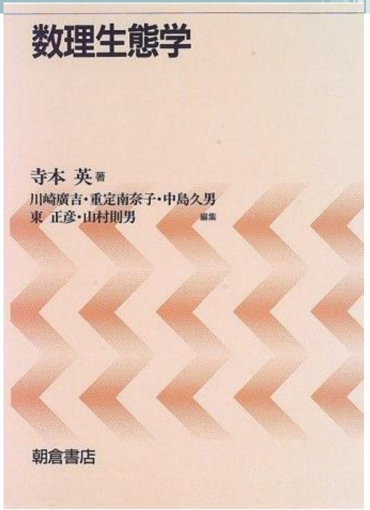
- (ii) apply central limit theorem



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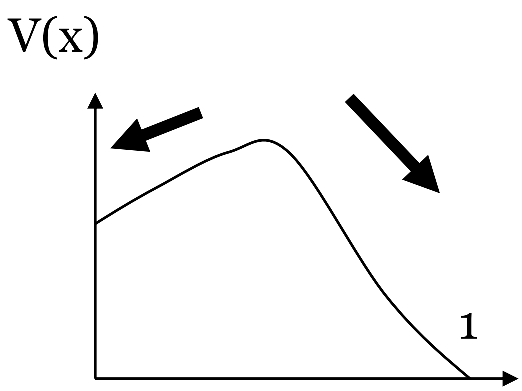
- (ii) apply central limit theorem
- (iii) transformation

EX-1.

	a_2	b_2
a_1	$2+\gamma\delta_1$, $2+\gamma\delta_1$	$\gamma\delta_1$,0
b_1	0, $\gamma\delta_1$	1, 1

- **Coordination Game**
- Replicator Eq. : $\dot{x} = x(1-x)\{3x - 1 + \gamma\delta_1\}$
- Equilibrium : 0, 1, $\dfrac{1-\gamma\delta_1}{3}$
- Potential Func. :

$$V(x) = \frac{3}{4}x^4 - \frac{4-\gamma\delta_1}{3}x^3 + \frac{1-\gamma\delta_1}{2}x^2 + C$$



- The equilibrium of the mixed strategy is **unstable**.

EX-2.

	a_2	b_2
a_1	$2+\gamma\delta_1$, $1+\gamma\delta_1$	$\gamma\delta_1$,0
b_1	0, $\gamma\delta_2$	1, 2

- Battle of Sexes (BoS)
- Replicator Eq. :

$$\dot{x} = x(1-x)\{2-\gamma\delta_2 - 3y\}, \qquad \dot{y} = y(1-y)\{2+\gamma\delta_1 - 3x\}$$

- Equilibrium point : (y^*,x^*)=(0,0), (1,0), (0,1), (1,1),

$$\left(\frac{2-\gamma\delta_2}{3}, \frac{2+\gamma\delta_1}{3}\right)$$

- The stability of the Mixed Strategy is saddle point.

3. OUR MODEL

3-2. SELTEN TYPE

Stochastic Environment (Selten (1980) + Dynamics)

- “Role” = “Group”

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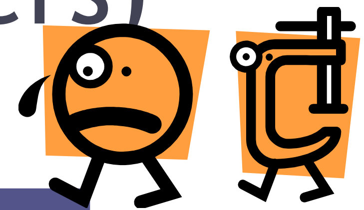
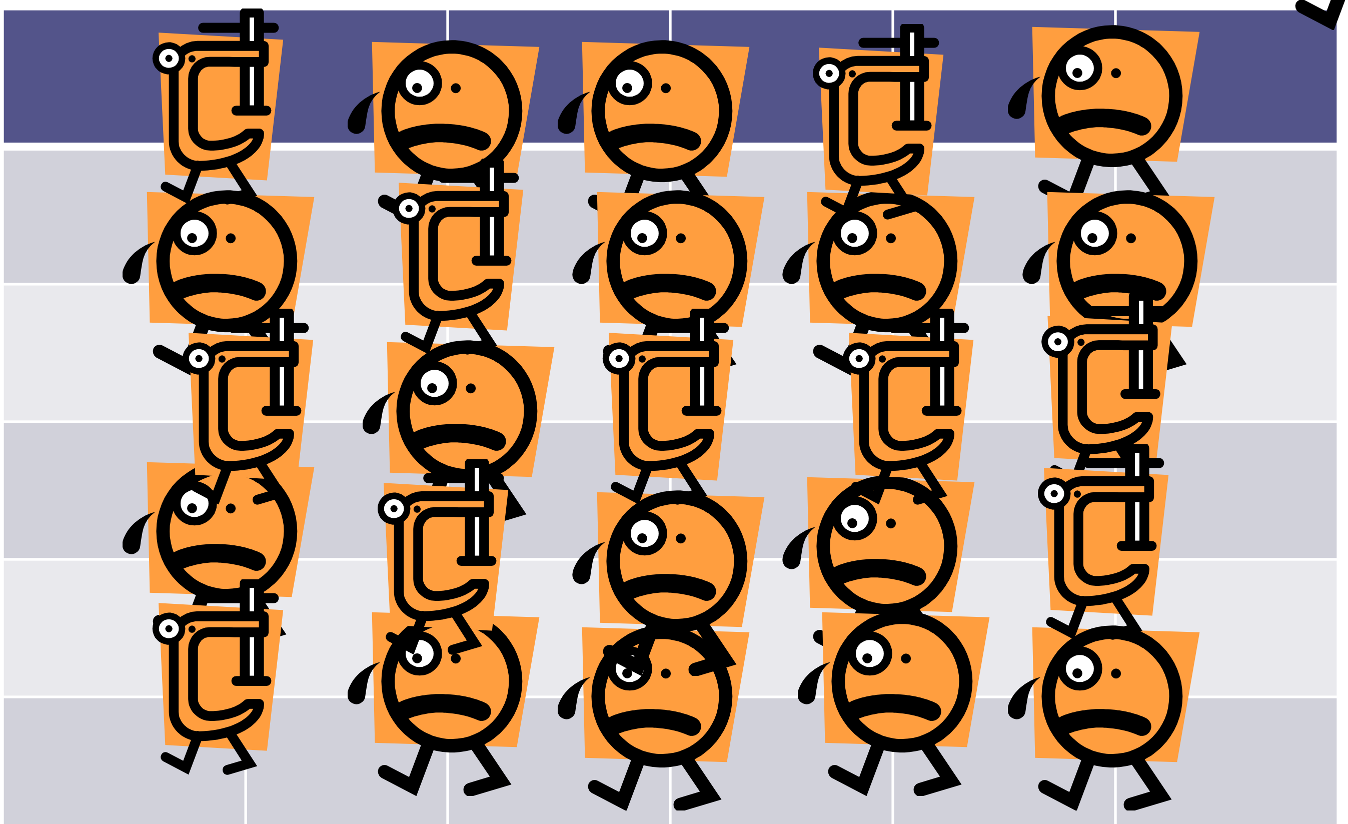
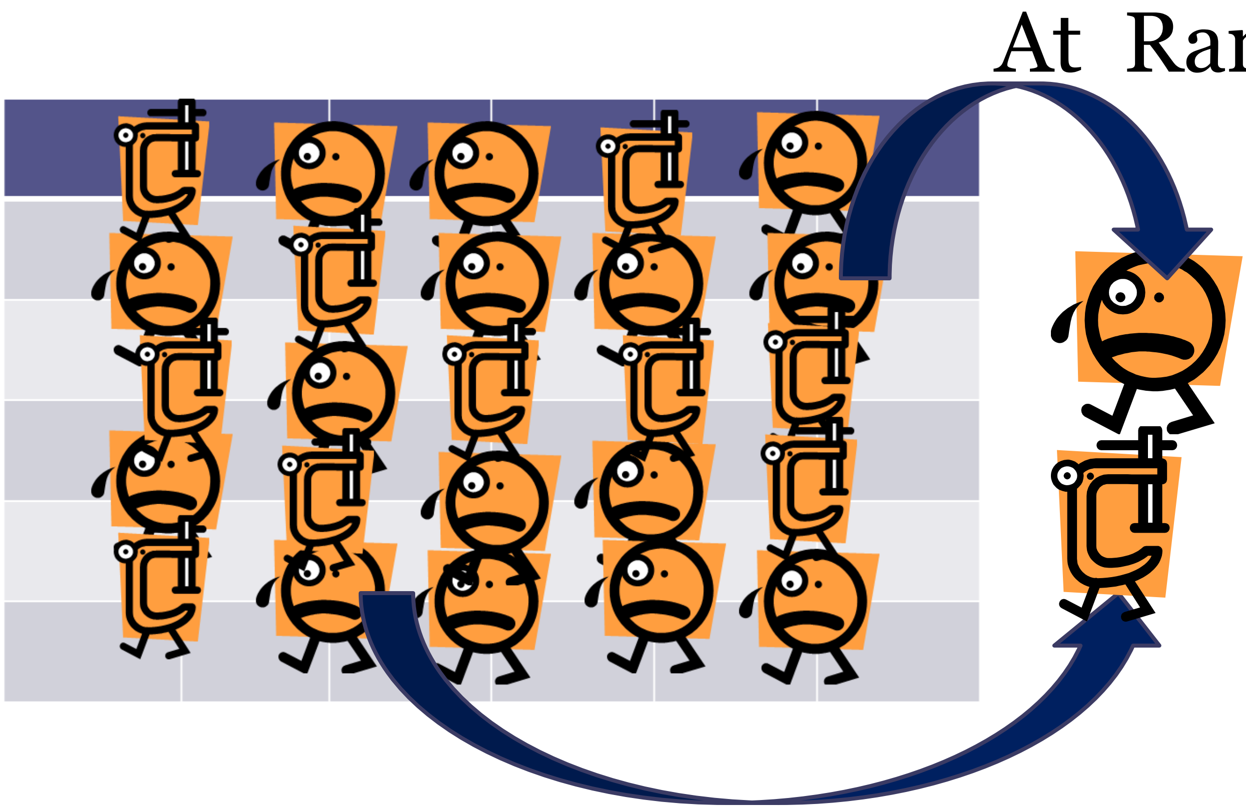
- **Situation** : see next slide.

Situation (two types players)





Situation (Evolutionary Game Theory with Group Structure)



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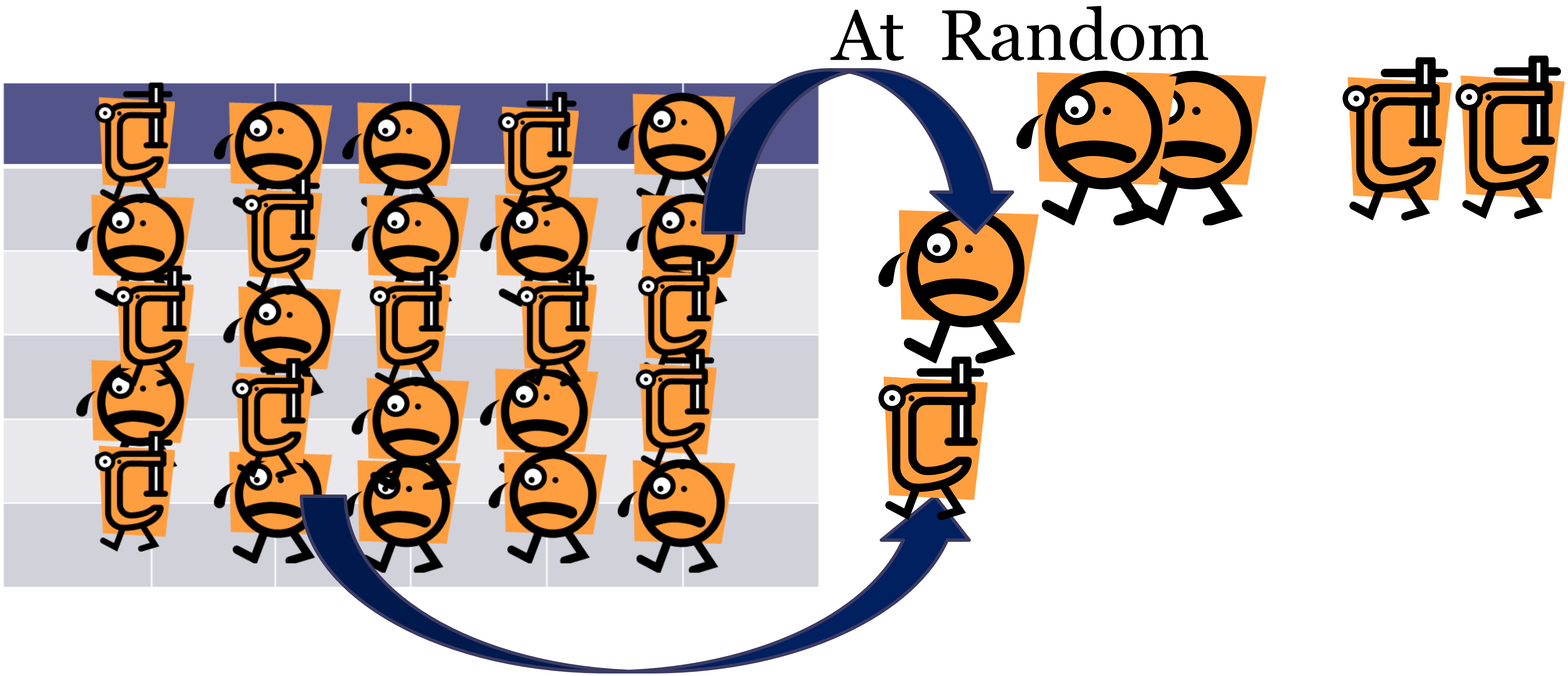

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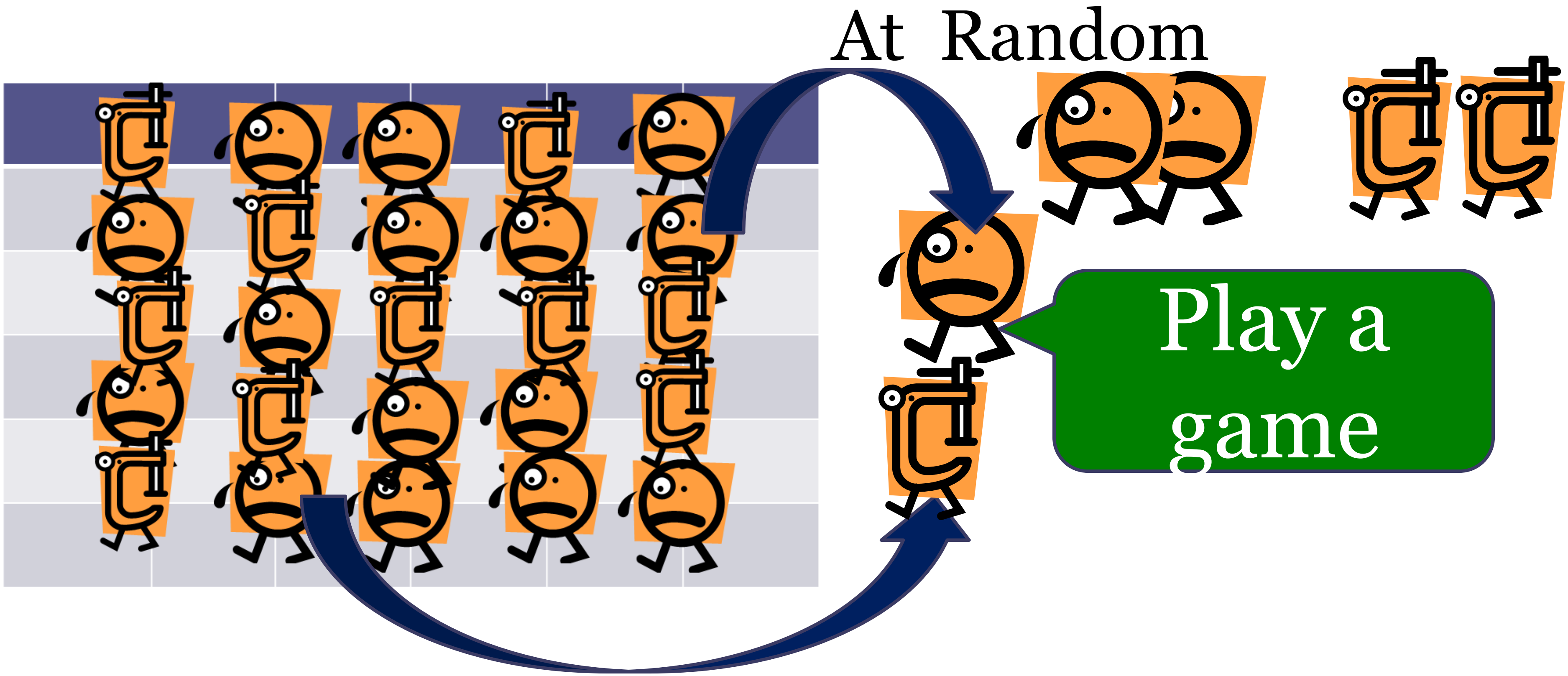
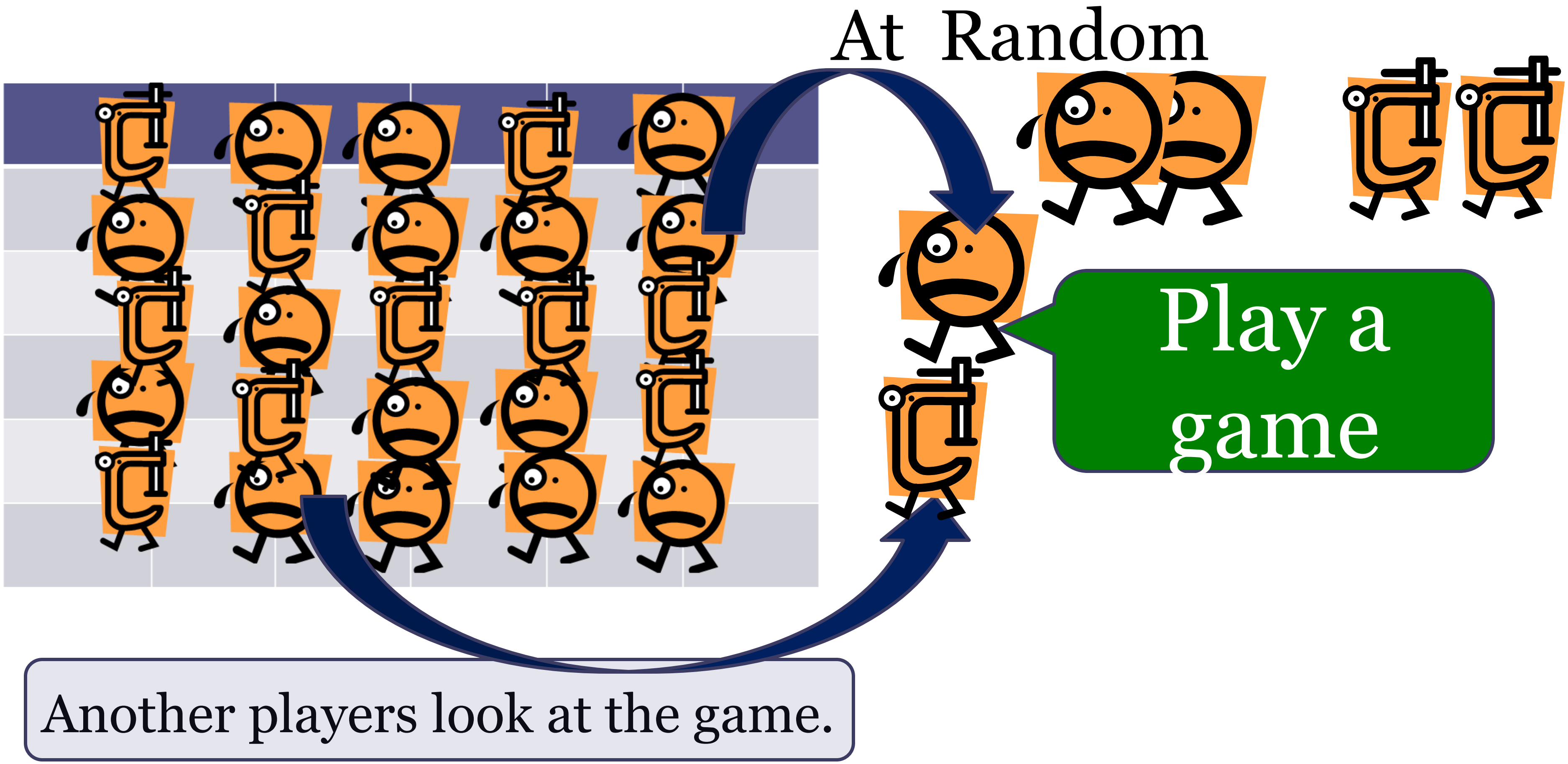


Situation (Evolutionary Game Theory with Group Structure)



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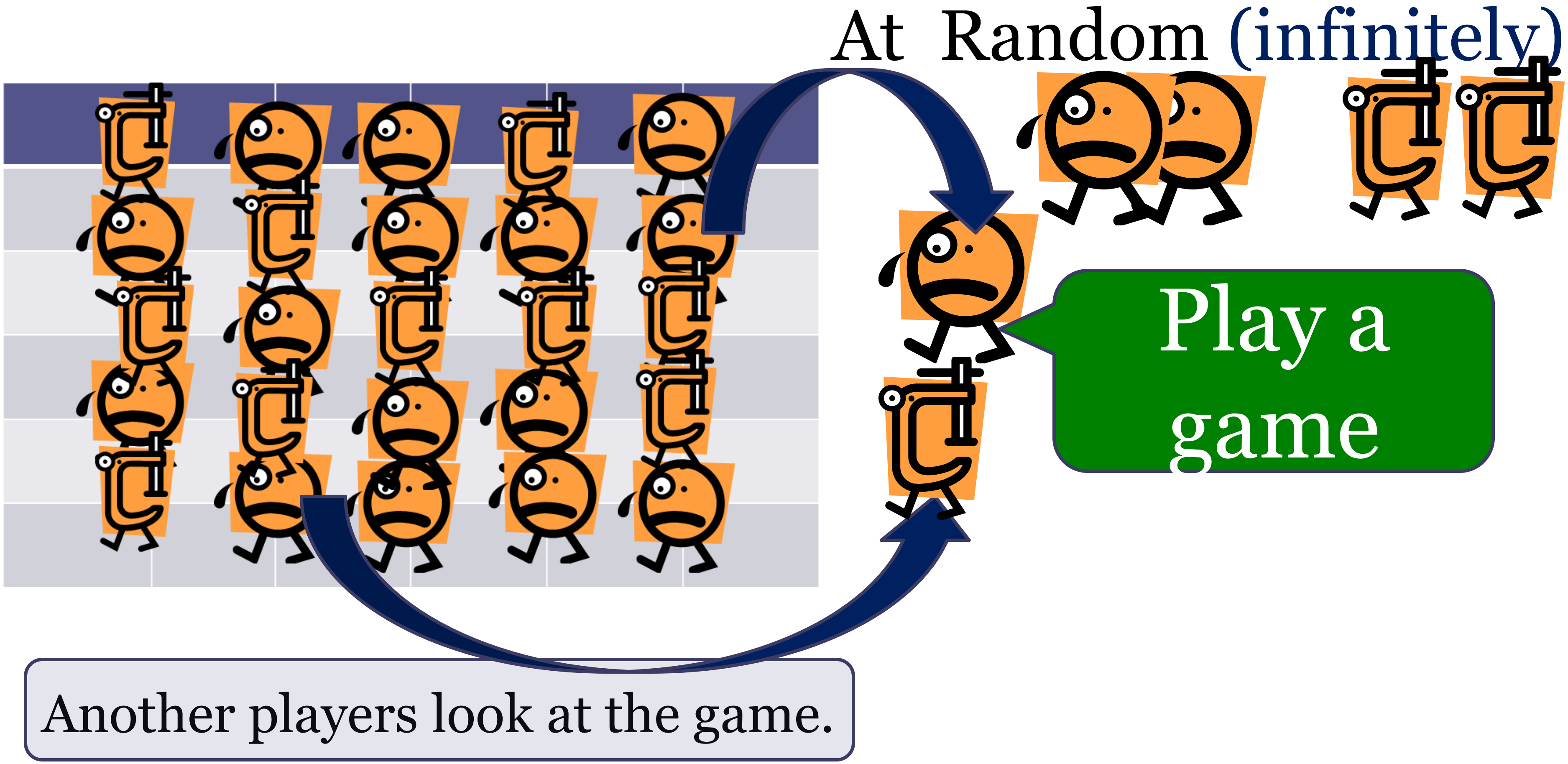

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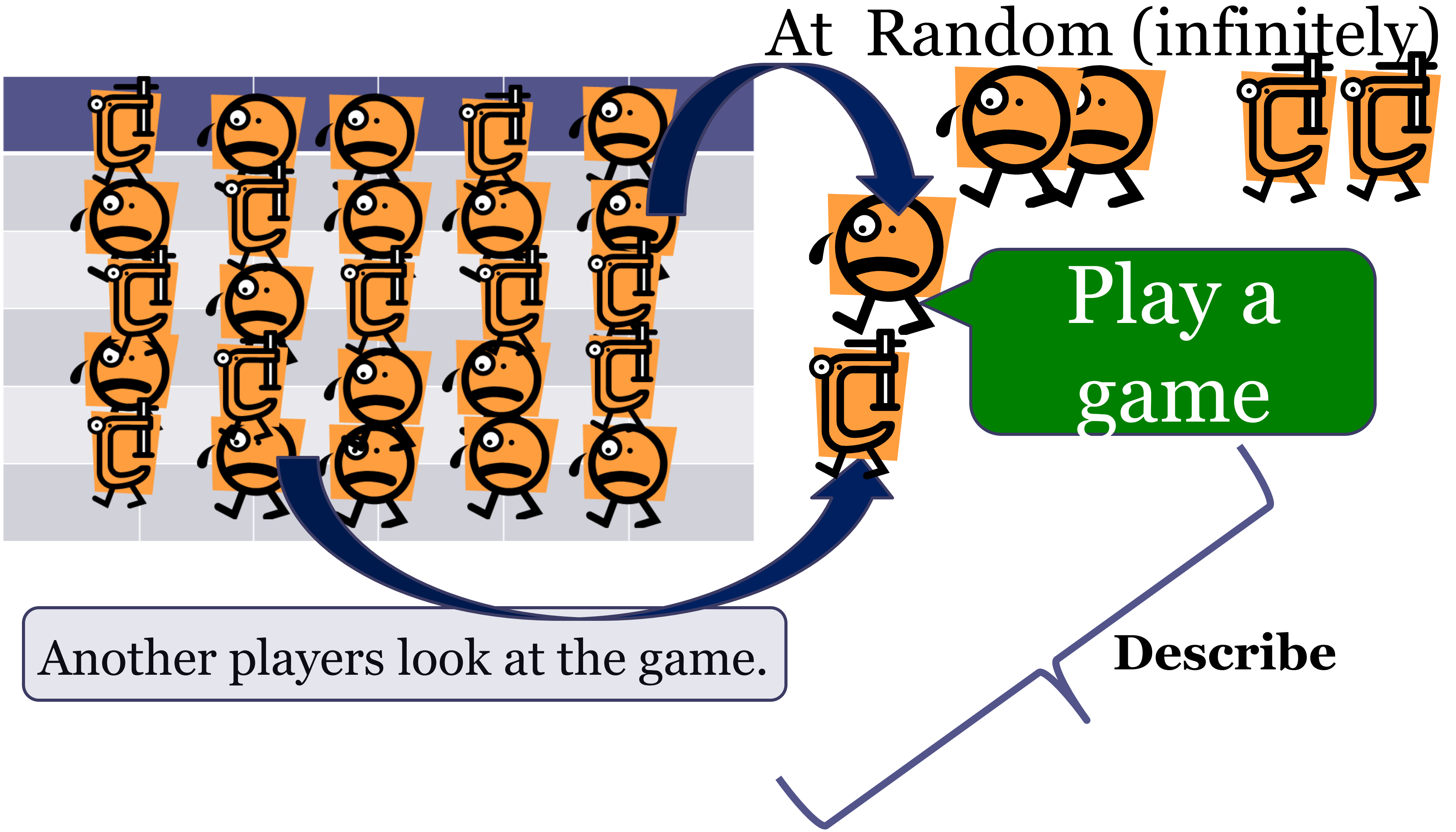

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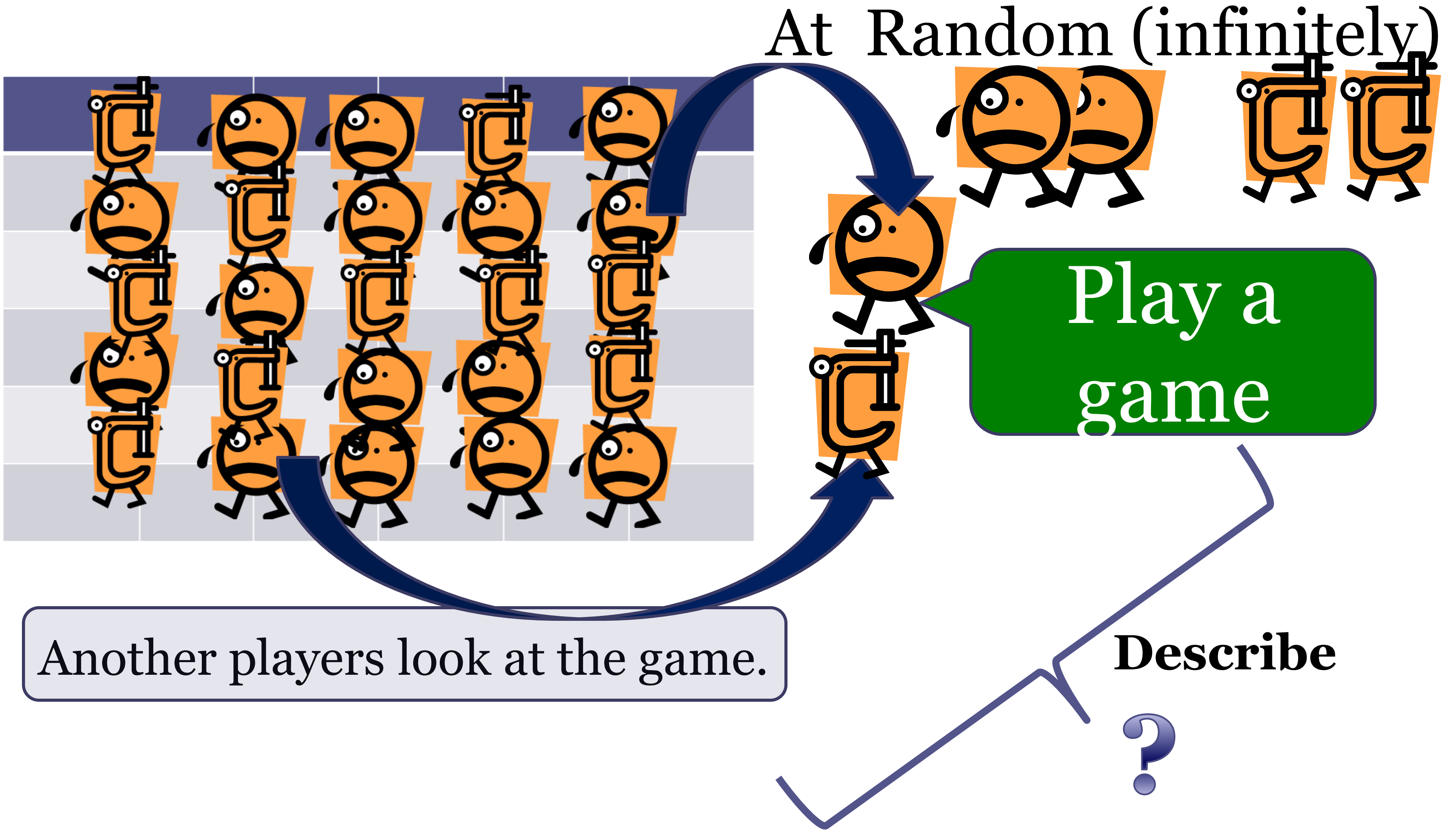
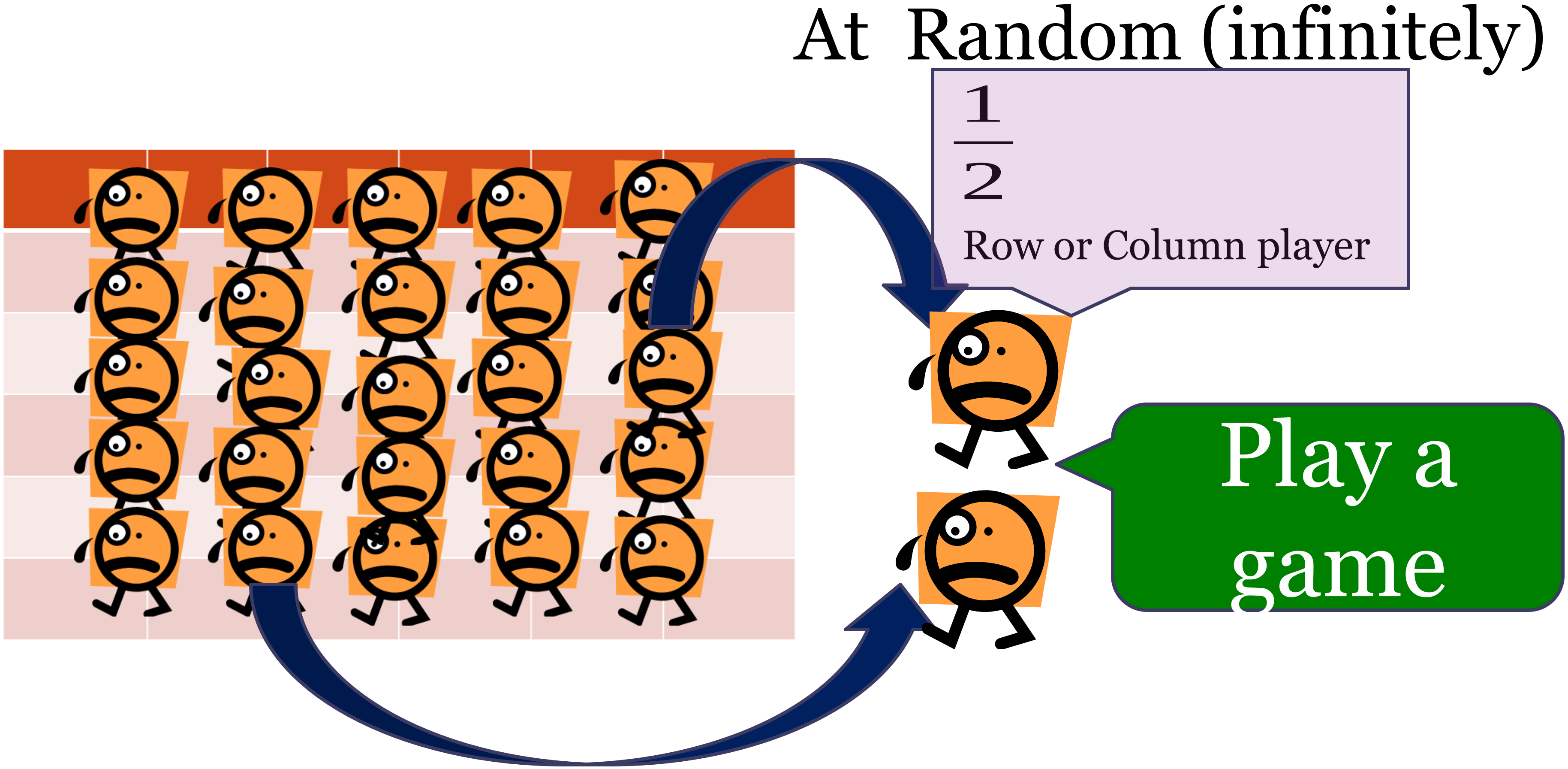
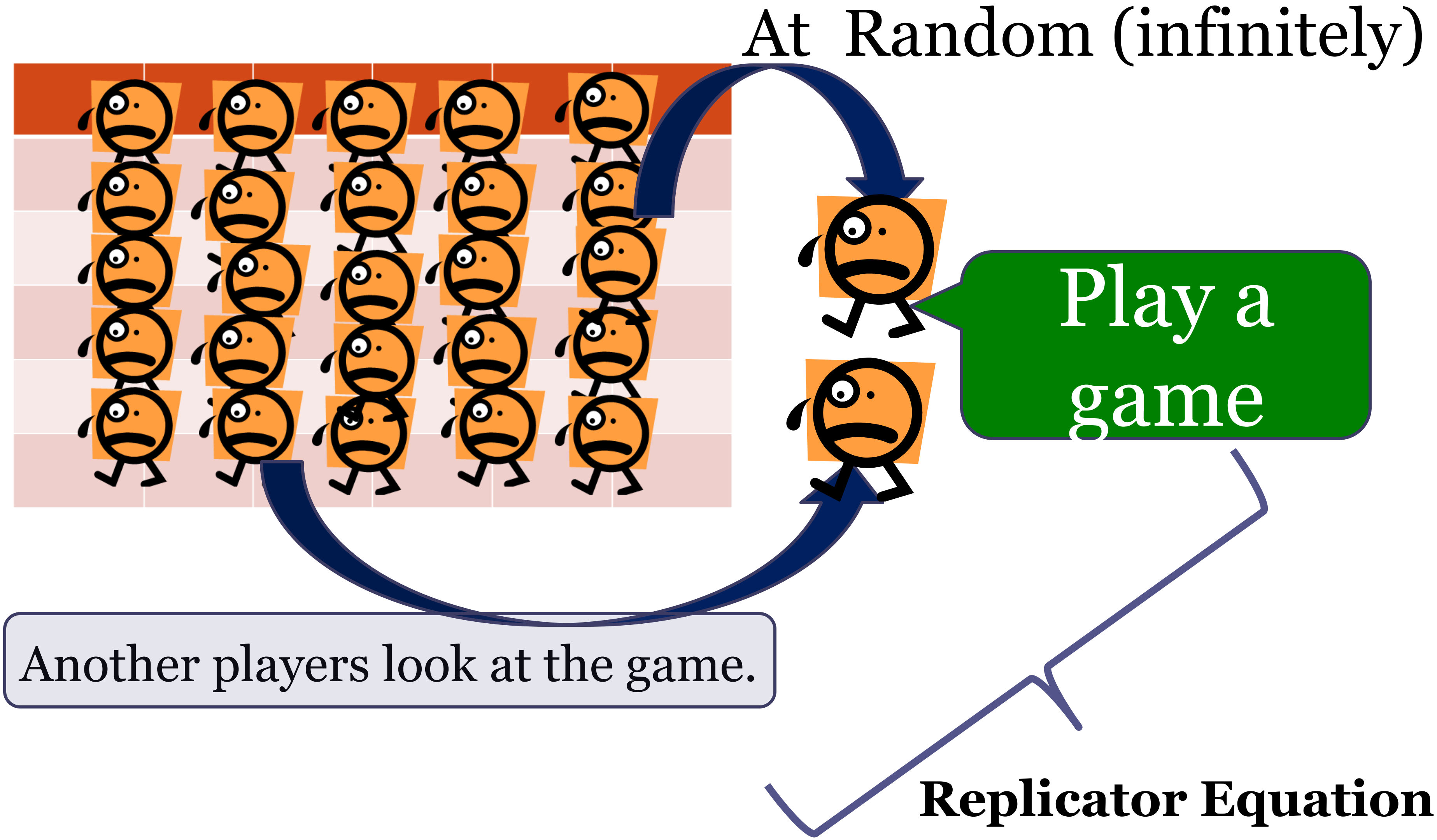


Situation (Role Completed Game)



Another players look at the game.

Situation (Traditional Evolutionary Game Theory)



Stochastic Environment (Selten (1980) + Dynamics)

- “Role” = “Group”

- **Pro.** Group size and it’s fitness in a game with group structure are as follows :

  Price equation $\dot{E}(p) = Cov(f, p) + E\left(\dot{p}\right).$

PROOF OUTLINE

1) transformation

$$\bar{x}' - \bar{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\bar{\pi}}\right) x_i - \sum_i f_i \frac{\pi_i}{\bar{\pi}} \Delta x_i$$

where $\Delta x_i = x'_i - x_i$

PROOF OUTLINE

1) transformation

$$\bar{x}' - \bar{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\bar{\pi}} \right) x_i - \sum_i f_i \frac{\pi_i}{\bar{\pi}} \Delta x_i$$

where $\Delta x_i = x'_i - x_i$

2) $\bar{\pi} \Delta \bar{x} = \sum_i f_i (\pi - \bar{\pi}) x_i + \sum_i f_j \pi_i \Delta x_i$ where $\Delta \bar{x} = \bar{x}' - \bar{x}$

PROOF OUTLINE

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3) Definition

$$Cov[\pi, x] = \sum_i f_i (\pi_i - \overline{\pi})(x_i - \overline{x}), \sum_i f_i (\pi_i - \overline{\pi}) \overline{x} = 0$$

$$\Rightarrow \quad \overline{\pi} \Delta \overline{x} = Cov[\pi, x] + E[\pi \Delta x].$$

PROOF OUTLINE

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- **Remark** : Price equation is equivalent to Replicator equation.

	H	D
H	a, a	0,0
D	0, 0	b, b

Payoff matrix

EX.

- Two type agent : {S,A}
- Random Matching : {SS}, {SA}, {AA}

$$Cov[\pi, x] = \sum_{i \in \{AA, AS, SS\}} f_i \left(\pi_i - \overline{\pi}\right)\left(x_i - \overline{x}\right) = f(1 - f)\left\{f(a+b) - b\right\}.$$

- Price Eq. = Replicator Eq.

- H-D game (a, b < 0)
- $Cov[\pi,x]=0 \Leftrightarrow f=0,1, b/(a+b).$

4. EXTENSION

GLOBAL GAME

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Global Game

(1) Complete information about x

(i) unique Nash eq.

$x<0$: strategy "D", $x > a$: strategy "C"

(ii) Multiple eq. $x \in [0,a]$: strategy "C" and "D"

•

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(2) Incomplete information about x

- Player i observes a private signal $s=x+\varepsilon_i$.

Pro. (Carlsson and van Damme, 1993) Let $\gamma \in \{\alpha, \beta\}$. If x lies on a continuous curve C such that $C \subseteq \Theta$, $g(C) \subseteq R^{\gamma}$, and $g(C) \cap D^{\gamma} \neq \emptyset$, then γ is iteratively dominant at x in Γ^{ε} if ε is sufficiently small.

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→ unique equilibrium : $x \in [0,a]$

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D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leq r \leq 1$, $r=0$: random matching)

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)
(2) Group Structure : {S,A}

	C	D
C	x, x	x,0
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(3) Price eq.

$$Cov[\pi, x] = f(1-f)\{af-(a-x)+r(x-af)\}$$

$$Cov[\pi,x] = 0 \Leftrightarrow f=0,\ 1\ ,\ \frac{a\{f(r-1)+1\}}{1+r}.$$

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$r \to 1$: $x>a/2$, $Cov[\pi,x]>0$, $x<a/2$, $Cov[\pi,x]<0$
$\to$ ESS Unique.

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Global Game

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→ unique equilibrium : $x \in [0,a]$

5. SUMMARY and FUTURE WORKS

OUR PROBLEM

- Q How does each player choose the action in stochastic environment ?

- A.1：Each player randomly chooses the action. (mixed strategy) (Harsanyi , 1973)

- A.2：Each player chooses the better action. (pure strategy) (Selten, 1980)

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Summary

1. Harsanyi(1973)+ Dynamics :

2. Selten (1980)+ Dynamics :

3. Global Game + Dynamics :

Summary

1. Harsanyi(1973)+ Dynamics :

$\rightarrow$ log-normal distribution (central limit theorem)

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→Approachable under variance (σ^2)

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$\rightarrow$ Bayesian Game = Game with Group Structure

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Stochastic Environment (Harsanyi (1973) + Dynamics)

- Stochastic Environment = payoff variation
- Replicator Eq.

$$\dot{x}_i(t) = x_i(t)\{g_i(t) - g(t)\},\ g_i(t) = g_i + \zeta(t)$$

- **Pro.** Let x be a strategy distribution. It satisfies :

$$P(x,t)dx = (2\pi\sigma^2 t)^{-1/2} \exp\left[-\frac{(\log x - \log x^*(t))^2}{2\sigma^2 t}\right]\frac{dx}{x}.$$

WANT YOUR COMMENTS

Stochastic Environment (Harsanyi (1973) + Dynamics)

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Thank you for your attention.

Mitsuru KIKKAWA (mitsurukikkawa@hotmail.co.jp)

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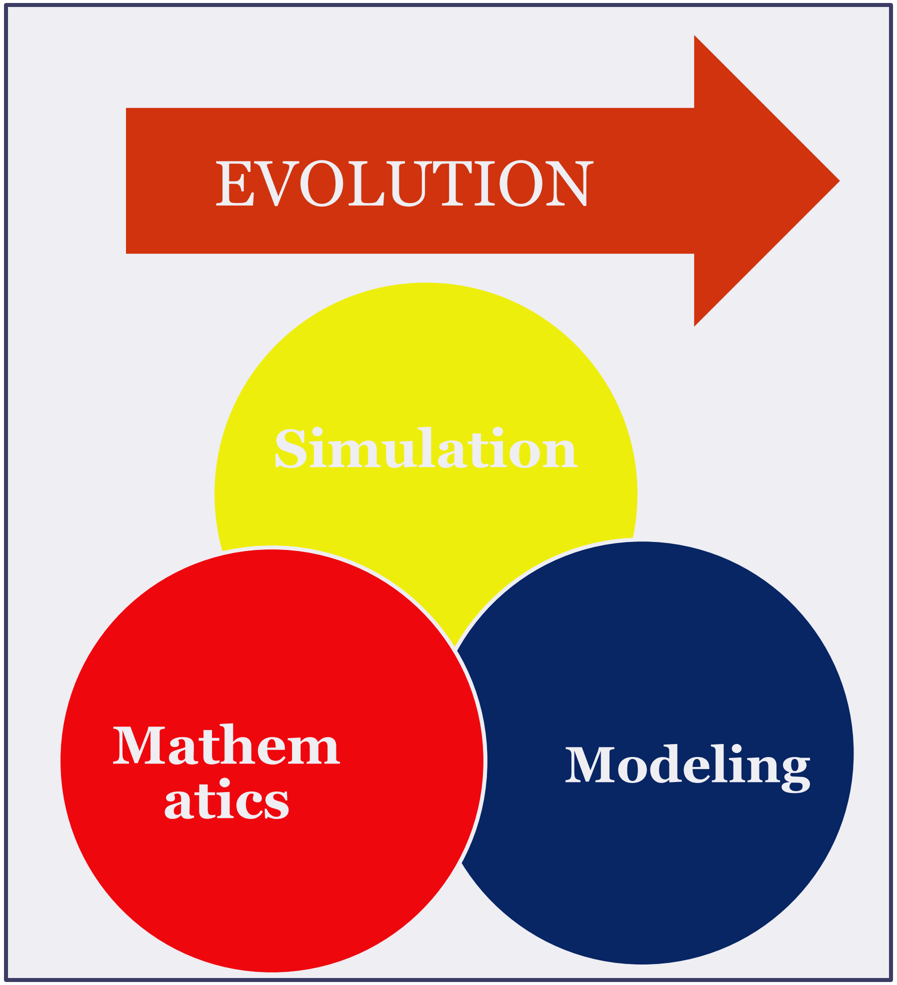

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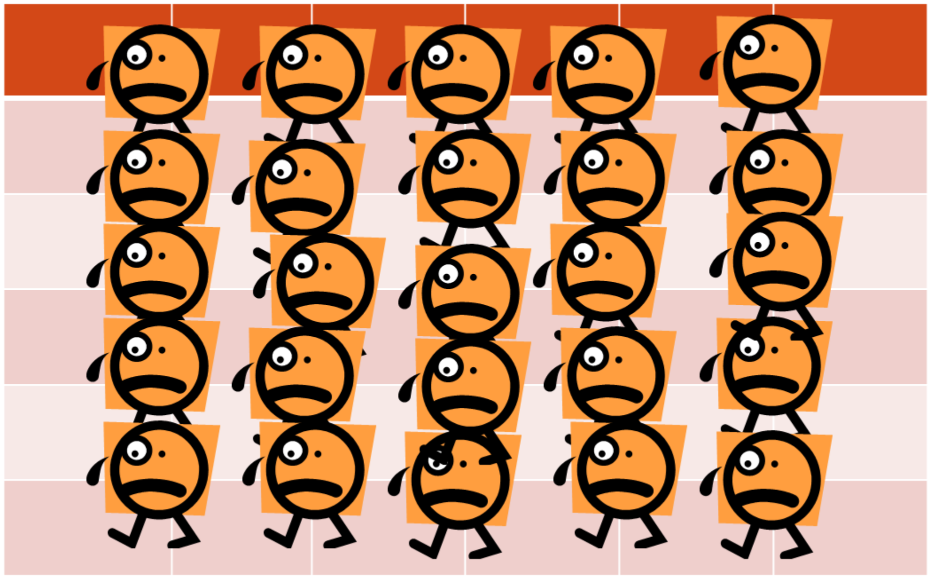
明治大学
グローバルCOEプログラム
現象数理学の形成と発展

PRELIMINARIES (EVOLUTIONARY GAME THEORY)

Situation (Traditional Evolutionary Game Theory)



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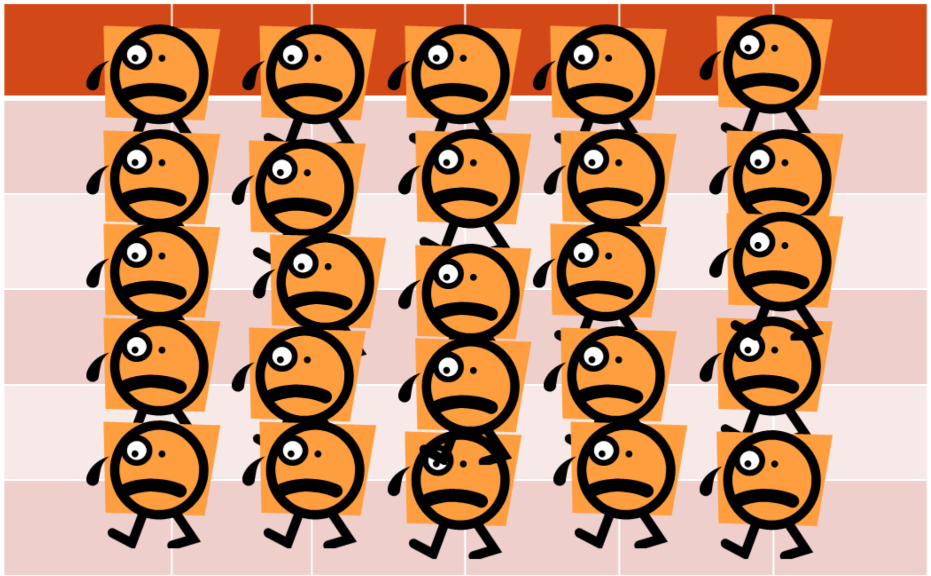
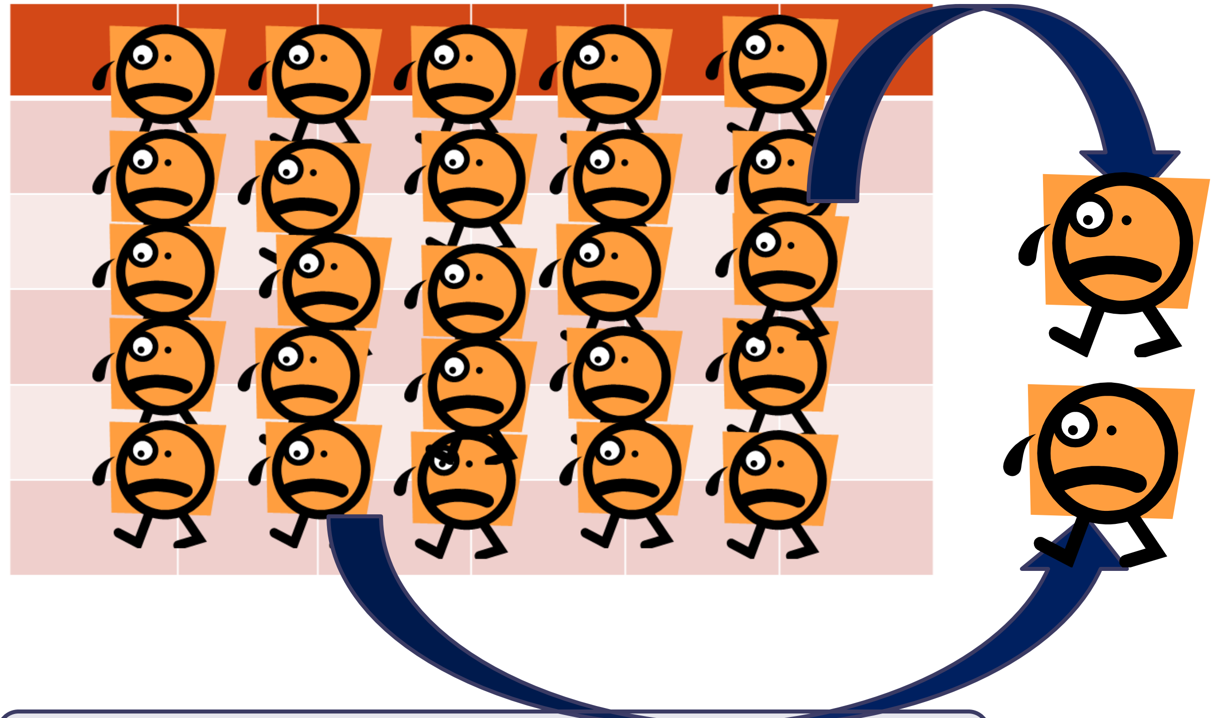


Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



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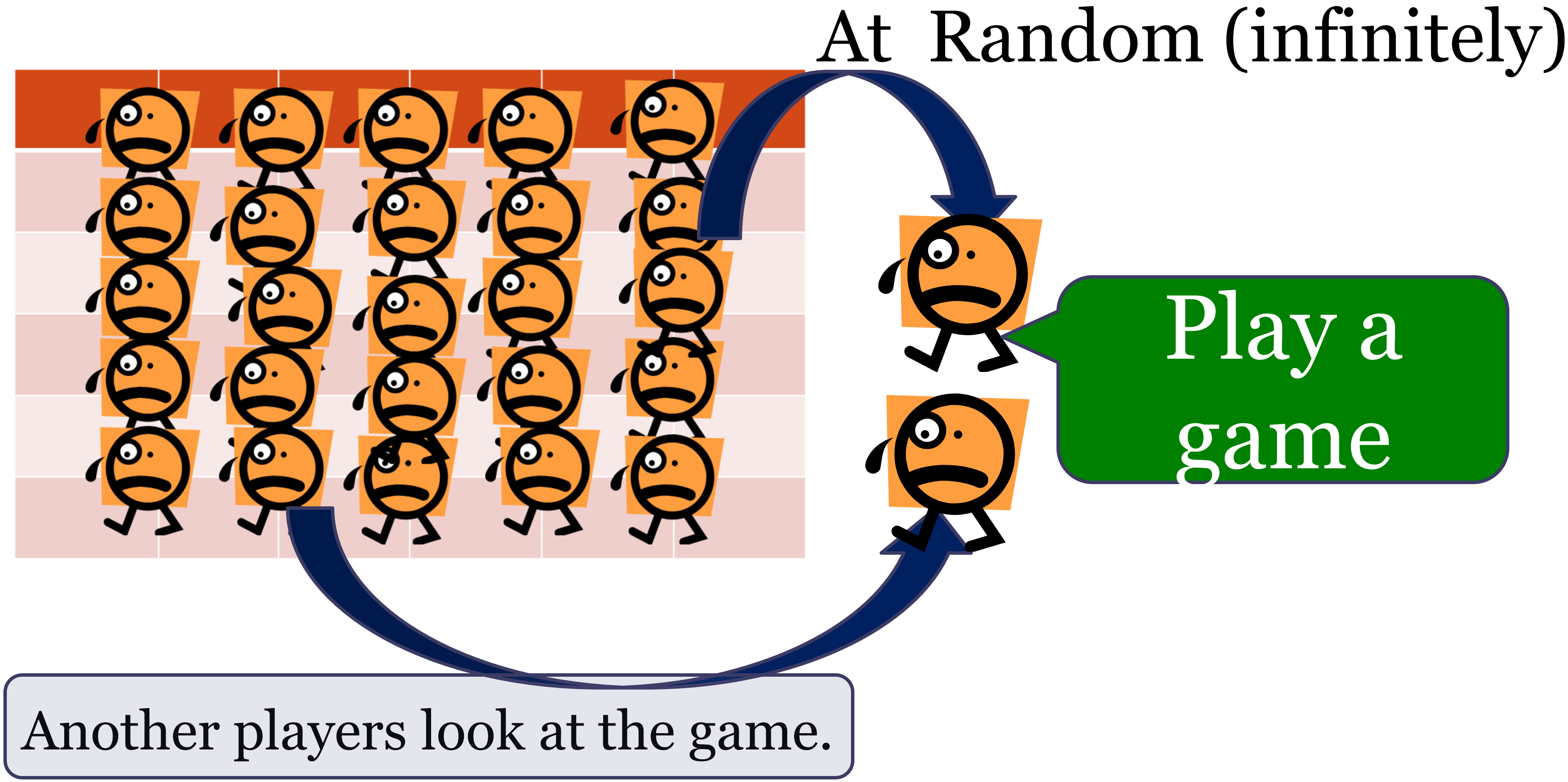

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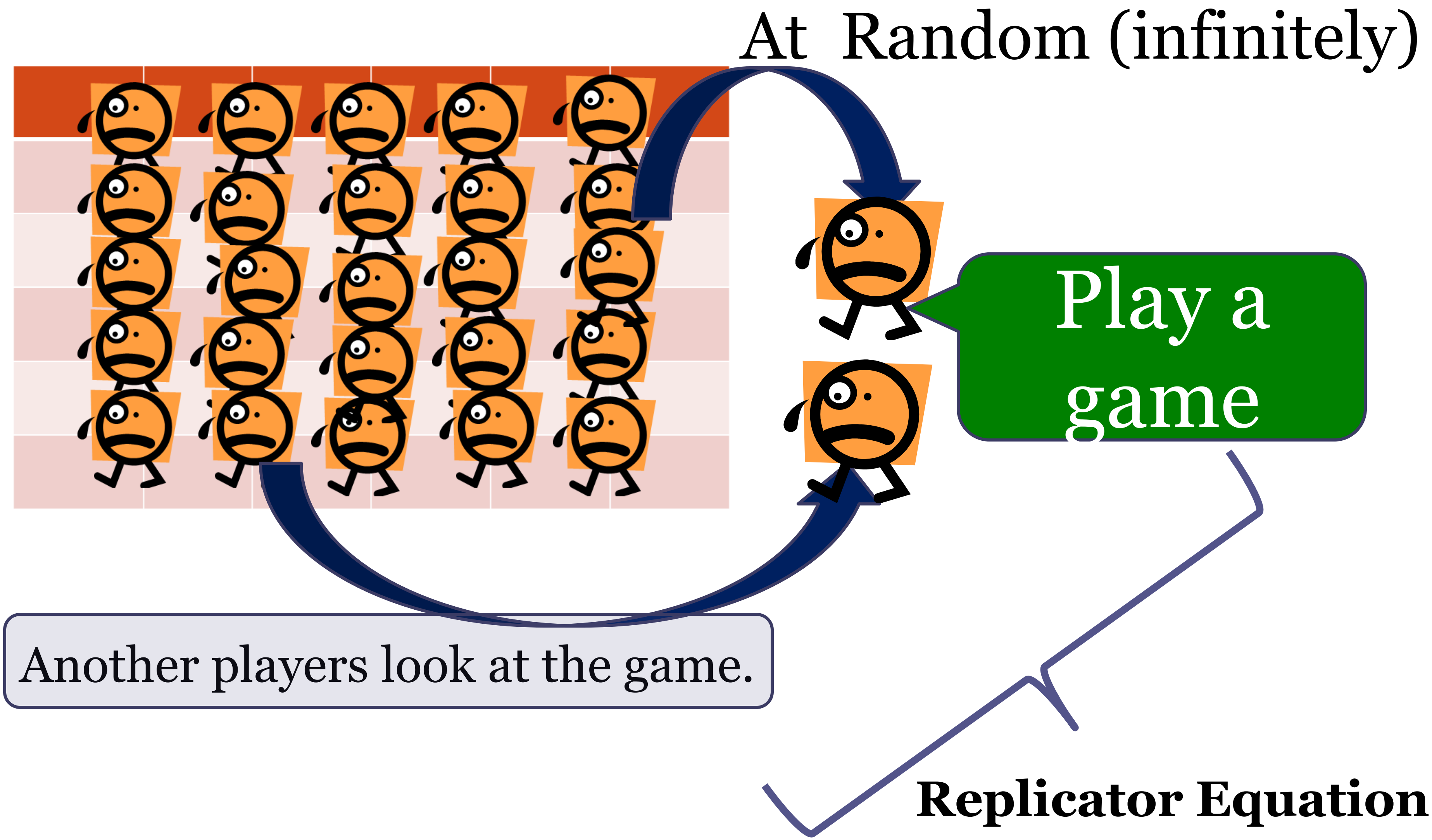

REVIEW: Replicator Equation

REPLICATOR EQ. $\dot{x}_i = x_i\left((Ax)_i - x \cdot Ax\right), i = 1, \cdots, n.$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before.

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Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \quad \cdots (*)$$

Classification

- (I) Non-dilemma: a > 0. b < 0, ESS : one
- **(II)** Prisoner's dilemma **:** a < 0. b > 0, ESS :one
- **(III) Coordination** : a>0,b>0, ESS two
- **(IV) Hawk-Dove** : a<0,b < 0, ESS one (mixed strategy)

1 \ 2	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

EVOLUTIONARY STABLE STRATEGY （ESS)

DEF.： Weibull(1995): $x \in \Delta$ is an ***evolutionary stable strategy (ESS)*** if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$.

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

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$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

INTERPRETATION: incumbent payoff (fitness) is higher than that of the post-entry strategy

(ESS : ①the solution of the Replicator equation + ② asymptotic stable.)

PROPOSITION

PRO.(Bishop and Cannings (1978)): $x \in \Delta$ is evolutionary stable strategy if and only if it meets these first-order and second-order best-reply :

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Nash Eq.

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Asymptotic Stable Conditon