VER.9/6

Kyoto Econophysics 9/8 11:40-12:00

Option Market Analysis with Evolutionary Game Theory (進化ゲーム理論を用いたオプション市場分析)

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This Talk (本報告)

- ANALYSIS the financial market with **Evolutionary game theory**.(金融市場において、進化ゲーム理論を用いて、分析する)
- **PREDICT** the next market state with Stability Analysis.(安定性の概念を用いることによって、次期の市場の状態を予測する)
- EXAMINE the **Real Market** (Future Market) to apply this model. (構築したモデルをもとに、実際の市場を分析する)
- MOVIE (avi)

OUTLINE

1. Introduction (Motivation)
2. Related Literatures and Review
3. Model
4. Apply this model to the Future market (Nikkei 225)
5. Option Market (Black-Sholes Eq.)
6. Summary (Future works)

1. INTRODUCTION

Motivation (動機)

- For **Practical Use** (実務への応用を目指して)

More **Detail** (より具体的で), More **Useful** (より役に立つ)

→We construct the market from the **market depth**. (板情報に着目)

- Econophysics (経済物理学)
- Use the "Real Data" (実際のデータを取り扱う)

Motivation　(動機)

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本音：Motivation　(動機)

- 経済物理学 III (2007年12月)に参加して、PUCK 動画 (real player)

- PUCK関連の研究を聞き、
　　　　「これだったら、いろいろいけるかも」
と感じた。

ということで進化ゲーム理論を使って、似たようなことをやってみよう。

Market Micro Structure (市場のマイクロストラクチャー)

- Roughly Speaking, We analysis the agents' behavior from the financial data.(データから市場参加者の行動を探る)
- Method: Evolutionary Game Theory (進化ゲーム理論)

→ Esaley and O'hara (1992) **[HP]**

- ＋ In this talk,

we use the "Real Data." (実際のデータを扱う)

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2. RELATED LITERATURES

Related Literatures(先行研究)

- **Micro Structure**

- **Applied Evolutionary Game Theory**
- **川西 (2008)**

- **PUCK (Econophysics)**
- Takayasu, et al. (2006) **[HP]**, Yamada, et al. (2008) **[HP]**, Yamada, et al. (2009) **[HP]**

3. MODEL

Model (モデル)

- **Players**… large population : seller and buyer, potentially (大人数の潜在的な売り手と買い手)

- **Goods** (財) … 1財
- **Strategy** (戦略)… **n** ($<\infty$) 個

Here, the price : how much do you buy or sell a goods. (ここでは購入、売却価格)

Market Depth (板情報)

（売 呼 値）	銘柄（値段）	（買 呼 値）
2 4 H I	成行呼値	1　3 K M
○○○	503円	
○○○	502円	1 T
○○	501円	5 2 P N
1 1 1 G F E	500円	4 3 2 1 A B C D
2 S	499円	○○○
4 R	498円	○○○
	497円	○○○

東京証券取引所のホームページより

a.まず、成行の売呼値 6,000株(H2,000株、I4,000株)と、成行の買呼値 4,000株(K1,000株、M3,000株)を対当させます。この時点では、成行の売呼値が2,000株残ります。

b.次に、始値を500円と仮定して、成行の売呼値の残りの2,000株及び499円以下の売呼値 6,000株(S2,000株、R4,000株)と、501円以上の買呼値8,000 株(P5,000株、N2,000株、T1,000株)を対当させます。

以上の結果、売呼値が12,000株、買呼値が12,000株で、株数が合致します。

略

Model (モデル)

- **Payoff (利得) … Buyer : S(t)-K, Seller : K-S(t)**

where S(t) : Brownian Motion.

- **Replicator Equation**

$$\frac{dx_i(t)}{dt} = x_i(t)\left(g_i(t) - \bar{g}(t)\right), g_i(t) = g_i + \zeta(t)$$

$$\frac{dy_i(t)}{dt} = y_i(t)\left(h_i(t) - \bar{h}(t)\right), h_i(t) = h_i + \zeta'(t)$$

where x_i, y_i : the probability of choosing the strategy 1 for each player. g_i, h_i : the payoff when each player chooses the strategy 1.

Two Strategies Case (戦略の数が2つ) :

- Replicator equation

$$\dot{x} = x(1-x)\left\{-b(t) + \left(a(t)+b(t)\right)y\right\},$$

$$\dot{y} = y(1-y)\left\{b(t) - \left(a(t)+b(t)\right)x\right\},$$

where x, y is the probability of choosing the strategy 1, 2 for each player.

Player 2

player1

	S1	S2
S1	a(t),-a(t)	0,0
S2	0,0	b(t),-b(t)

Prediction (予測)

- Replicator equation divided by xy(1-x)(1-y) :

$$\dot{x} = -\frac{b(t)}{y} + \frac{a(t)}{1-y}, \dot{y} = \frac{b(t)}{x} - \frac{a(t)}{1-x}.$$

- Discrete the above equations:

$$x(t+\varepsilon) = x(t) - \left(\frac{b(t)}{y} + \frac{a(t)}{1-y} \right) \varepsilon,$$

$$y(t+\varepsilon) = y(t) + \left(\frac{b(t)}{x} - \frac{a(t)}{1-x} \right) \varepsilon.$$

Digression (余談): Physics

- Canonical Equation (正準方程式) :

$$\dot{x} = \frac{\partial H}{\partial y}, \dot{y} = -\frac{\partial H}{\partial x}.$$

- We obtain the Hamiltoninan:

$$H = -b(t)\left(\log x + \log y\right) \\ - a(t)\left(\log(1-x) + \log(1-y)\right).$$

- We can discuss the **Nearly Integrability** (Hamiltonian + Noise). See 吉川(**2005**,**2006**) for detail.

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s1,s2),(s2,s2)

	S 1	S 2
S 1	+, -	0,0
S 2	0,0	0,0

ii) ↓ (Down)

N.E. (s1,s1),(s1,s2)

	S 1	S 2
S 1	0,0	0,0
S 2	0,0	-,+

iii) → (No change)

N.E. (s1,s2)

	S 1	S 2
S 1	+,-	0,0
S 2	0,0	-,+

i),ii),iii) → (s1,s2) (x→1, y→0)

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S 2	0,0	-,+

Is This OK?

iii) → (No change)

N.E. (s1,s2)

	S 1	S 2
S 1	+,-	0,0
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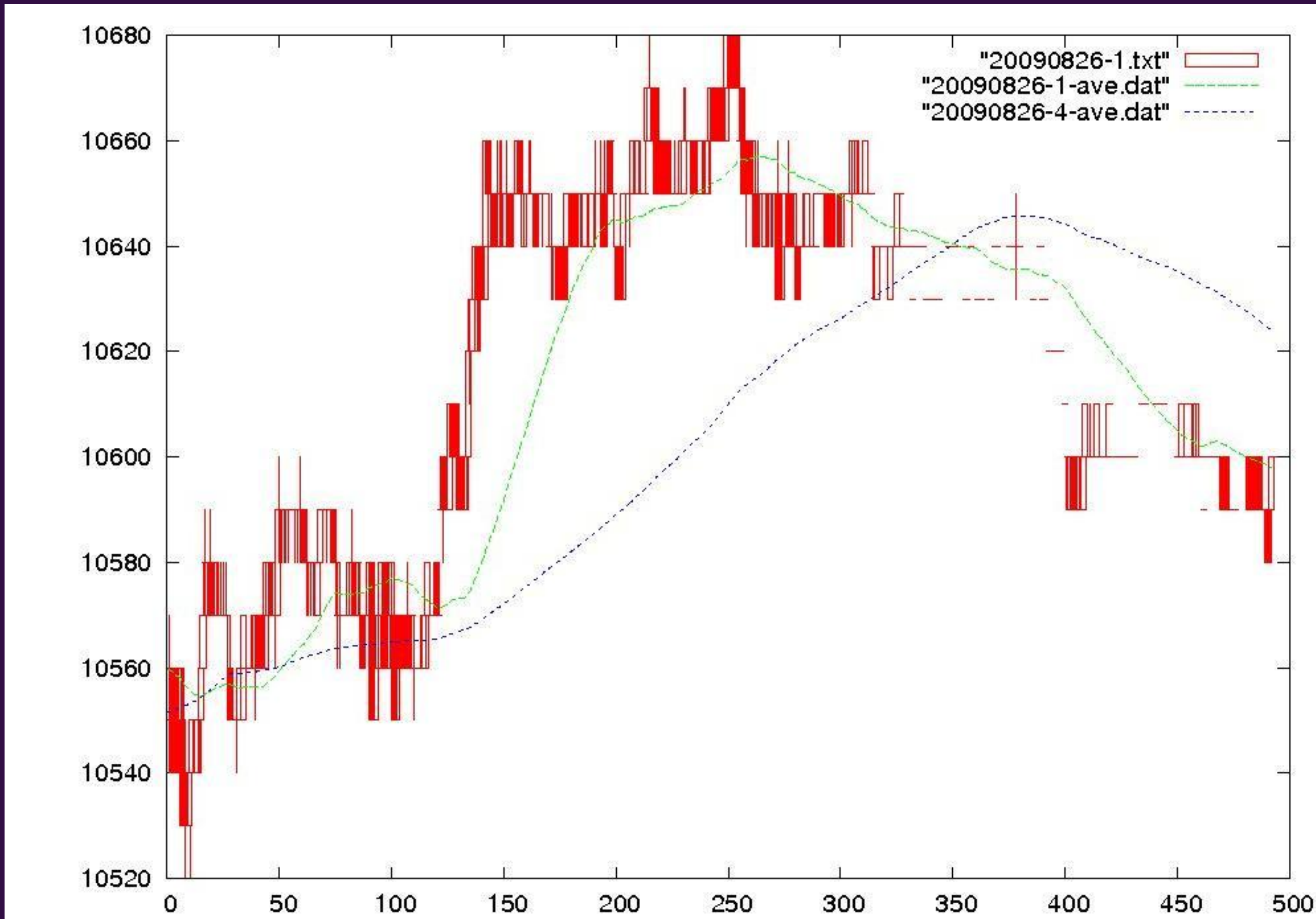
i),ii),iii) → (s1,s2) (x→1, y→0)

4. Application:

NIKKEI 225 FUTURE MARKET (日経225先物市場)

EX: 20090826



10680
10660
10640
10620
10600
10580
10560
10540
10520
0
50
100
150
200
250
300
350
400
450
500
"20090826-1.txt"
"20090826-1-ave.dat"
"20090826-4-ave.dat"

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

	S 1	S 2
S 1	+, -	0,0
S 2	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

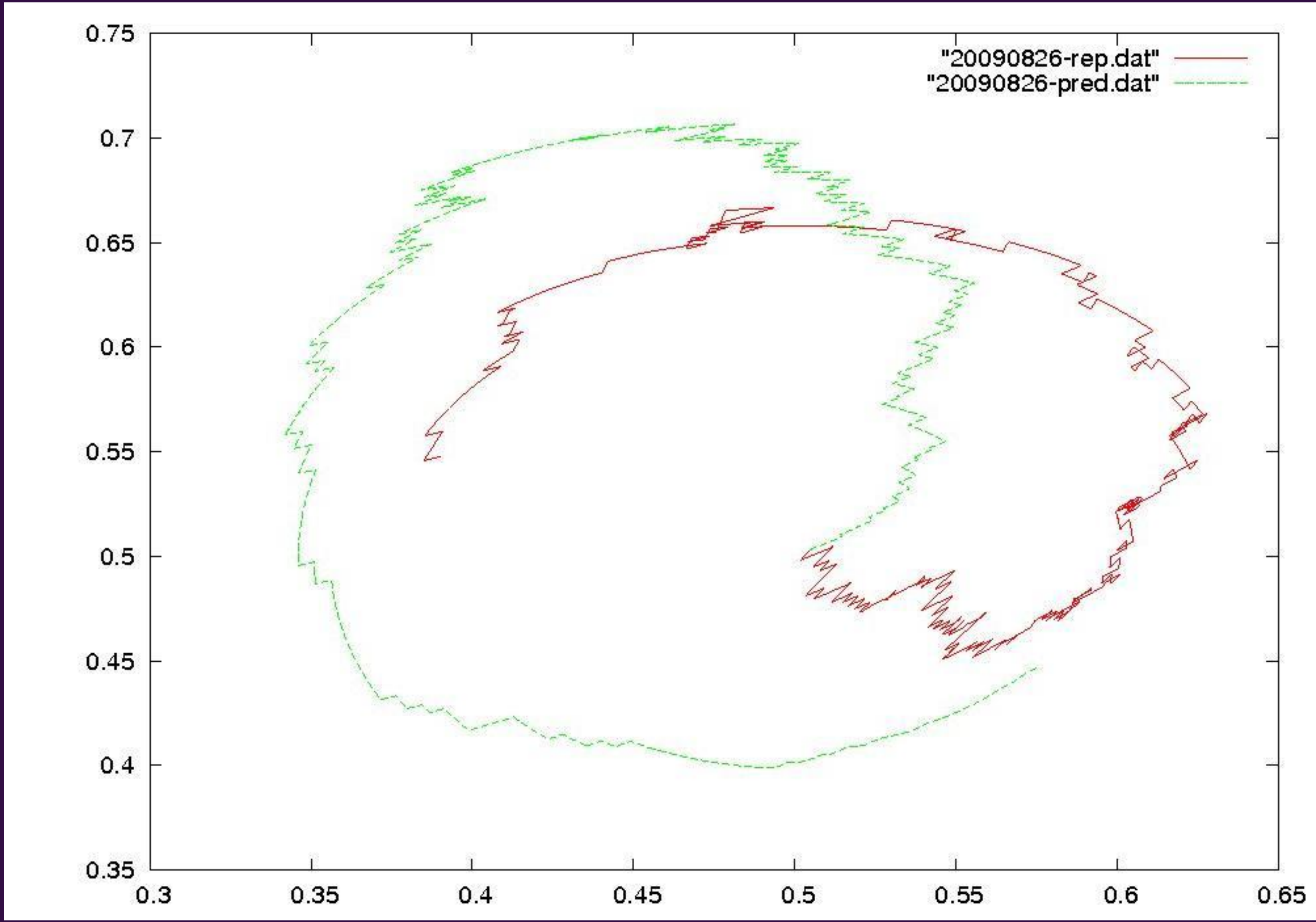
	S 1	S 2
S 1	+,+	0,0
S 2	0,0	-,+

iii) → (No change)

N.E. Mixed Strategy.

	S 1	S 2
S 1	-,+	0,0
S 2	0,0	-,+

EX: 20090826



"20090826-rep.dat"
"20090826-pred.dat"
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65

5. Application:

OPTION MARKET
(オプション市場)

この場合のBlack-Sholesの公式

- Black-Sholesモデルにおいて、行使価格の影響があるのは、**境界条件**を使用するとき。
- よって、K := $\overline{K}$ とすればよい. つまり

$$f(S,t)=S\cdot N\left(\frac{u}{\sigma\sqrt{\tau}}+\sigma\sqrt{\tau}\right)-\overline{K}\,e^{-r\tau}\cdot N\left(\frac{u}{\sigma\sqrt{\tau}}\right).$$

ただし $\overline{K}=$ 平衡時の戦略1における行使価格 $\cdot s_1^*$

\+ 平衡時の戦略2における行使価格 $\cdot(1-s_1^*)$

s_1^* は混合戦略を採用する場合の確率.

6. SUMMARY AND FUTURE WORKS

Summary and Future Works

Summary

- **MODELING** the Financial Market.
- **DERIVE** the payoff matrix for each player.
- **APPLY** the Real Market.
- **DERIVE** the Optimal Behavior for each player.

Future Works

- **GET** the Online Financial Data, **CALCULATE** and **DISPLAY**. (オンラインでデータを入手し、計算し、それを表示する)
- **MAKE** the software like a PUCK based on the Evolutionary Game Theory. (PUCKの進化ゲーム理論版の構築)

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Thank You For Your Attention

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This File is available at

http://kikkawa.cyber-ninja.jp/

REFERENCE

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EVOLUTIONARY STABLE STRATEGY（ESS)

DEF.： Weibull(1995): $x \in \Delta$ is an ***evolutionary stable strategy (ESS)*** if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$.

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

INTERPRETATION：incumbent payoff (fitness) is higher than that of the post-entry strategy

（ESS : ①the solution of the Replicator equation + ② asymptotic stable.）

Replicator Equation

REPLICATOR EQ. $$\dot{x}_i = x_i\left((Ax)_i - x\cdot Ax\right), i=1,\cdots,n.$$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before. And this equation shows that the probability of the action *i* chosen by another players is also higher than before (**externality**). Furthermore, the equation is derived uniquely by the **monotonic** (that is if one type has increased its share in the population then all types with higher profit should also have increased their shares).

Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \quad \cdots(*)$$

Classification

(I) Non-dilemma: a > 0, b < 0, ESS : one

(II) Prisoner's dilemma : a < 0, b > 0, ESS :one

(III) Coordination : a>0,b>0, ESS two

(IV) Hawk-Dove : a<0,b < 0, ESS one (mixed strategy)

2

1	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

- 本研究の一部は，平成20年度採択, 文部科学省 グローバルCOEプログラム**「現象数理学の形成と発展」** 現象数理若手プロジェクト「人間特有の現象に対する学習の影響 - 進化ゲーム理論による分析 -」に関する研究拠点形成費の助成を受けて行われた.