“Hitotsubashi Game Theory Workshop 2009 ”

(2009/03/06 11:15-11:45)

Coevolution and Diversity in Evolutionary Game Theory
(進化ゲーム理論における共進化と多様性)

：Random Environment
(−確率的環境の場合−)

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OUTLINE

1. INTRODUCTION（Motivation, Purpose）
2. RELATED LITERATURES and PRELIMINARIES
3. OUR MODEL
   3-1. HARSANYI TYPE
   3-2. SELTEN TYPE
4. EXTENSION (Global Game)
5. APPLICATION (FINANCE : Black-Sholes eq.)
6. SUMMARY and FUTURE WORKS

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1. INTRODUCTION

OUR PROBLEM

- Q How does each player choose the action in stochastic environment ?

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- A.1：Each player randomly chooses the action. (mixed strategy) (Harsanyi , 1973)

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- A.1：Each player randomly chooses the action. (mixed strategy) (Harsanyi , 1973)

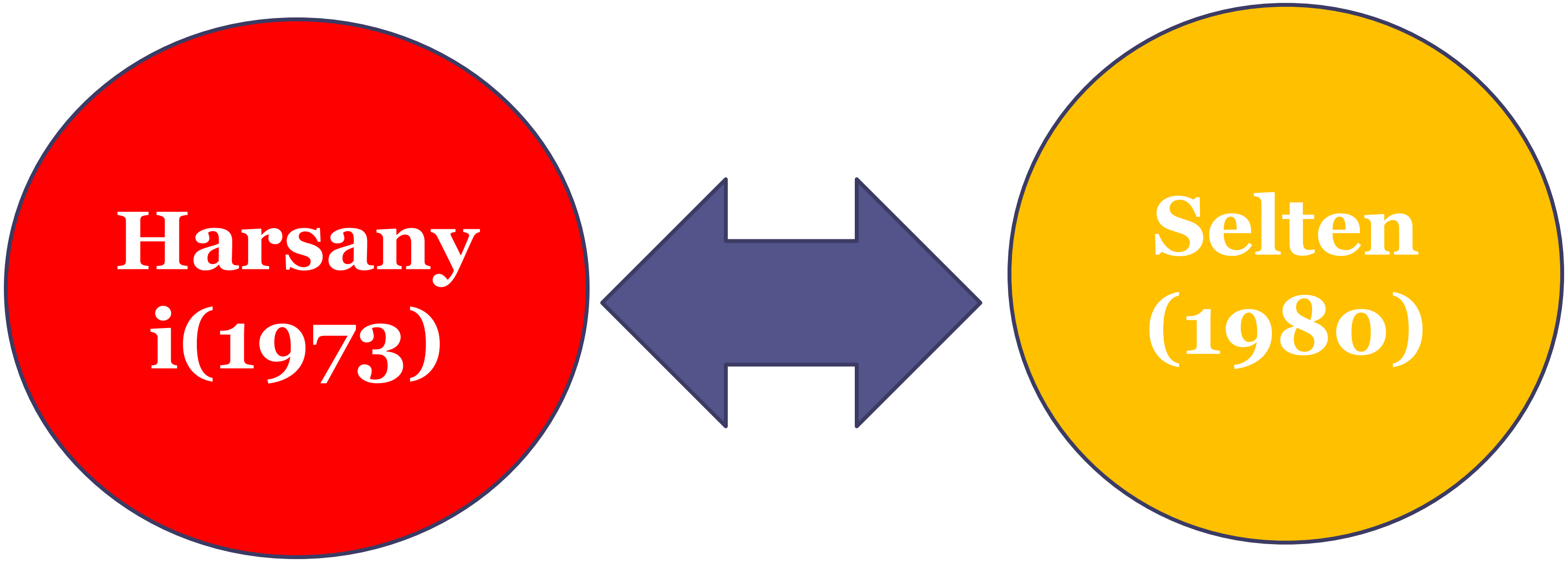
- A.2：Each player chooses the better action. (pure strategy) (Selten, 1980)

Research Fields (this study)

Harsany i(1973)

Selten (1980)

Research Fields (this study)



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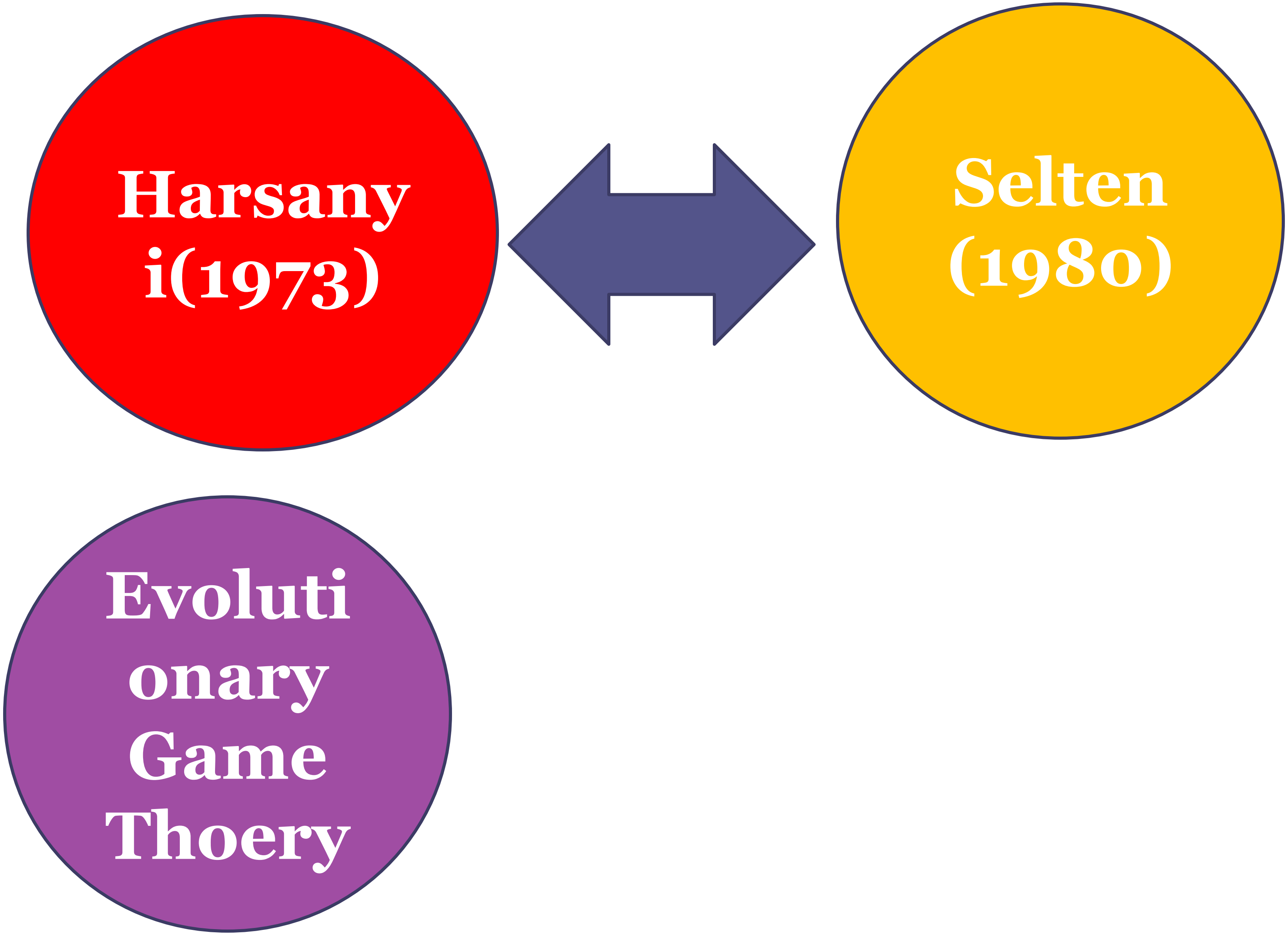

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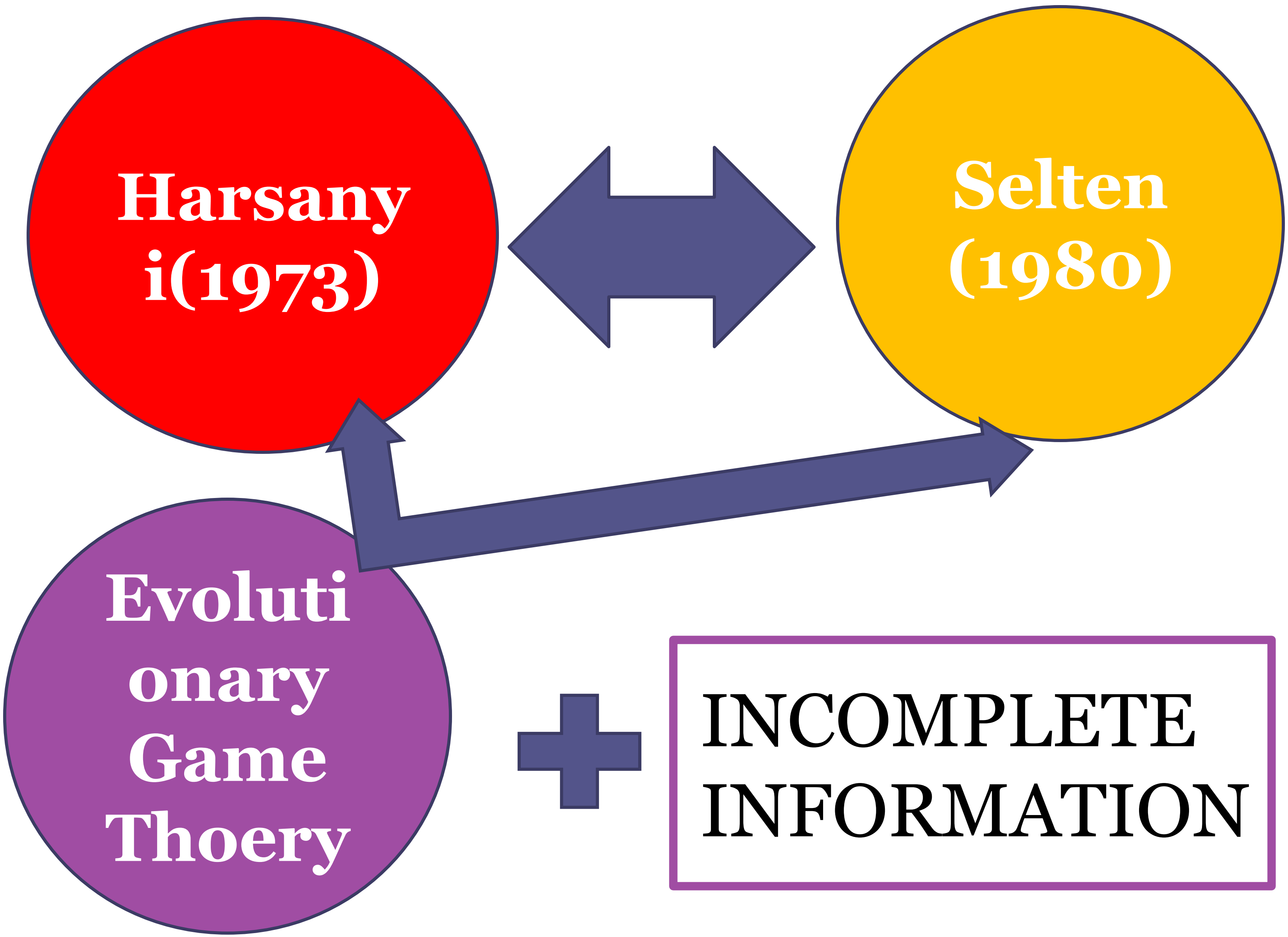
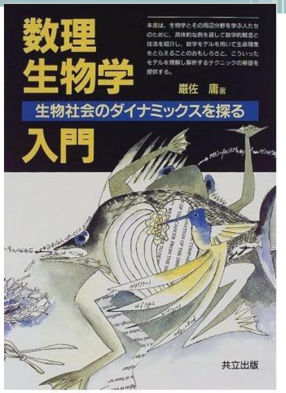


2. RELATED LITERATURES and PRELIMINARIES

MATHEMATICAL BIOLOGY

- Stochastic Environment
- Bet-Hedging Strategy (=Mixed Strategy)



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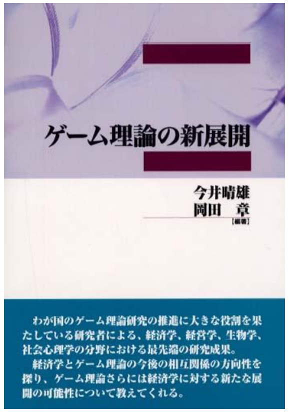
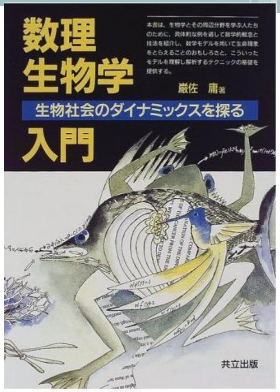


IMAI and OKADA (2002)

MATHEMATICAL BIOLOGY

- Stochastic Environment
- Bet-Hedging Strategy (=Mixed Strategy)

- Fitness Function is
- (i) Geometric mean
- (ii) Arithmetic average
-
-



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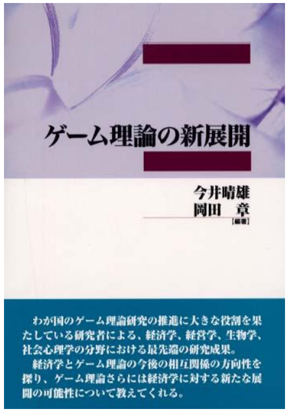

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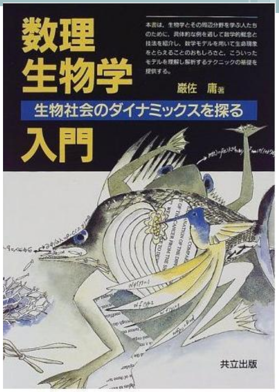

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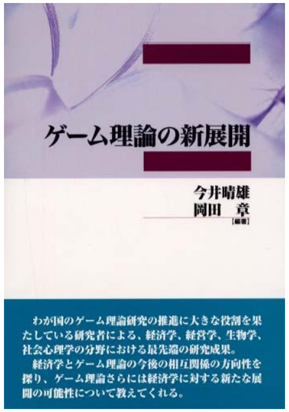

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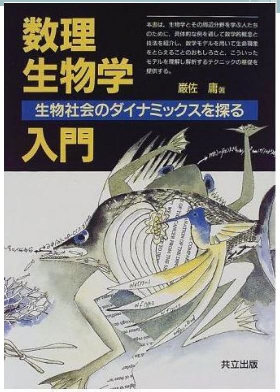

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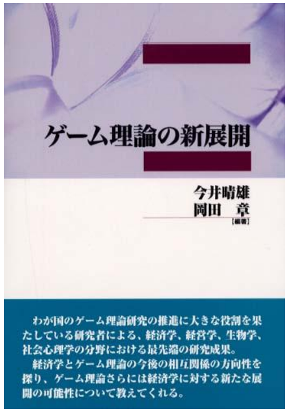

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Game Theory

- The fitness(utility) function is a von-Neumann-Morgenstern utility function .
- → No Bet-Hedging Strategy ?

Harsanyi (1973)

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- **Th.** Fix a set of I players and strategy spaces S_i. For a set of payoffs $\{u_i(s)\}_{i\in F, s\in S}$ of Lebesgue measure 1, for all independent, twice-differentiable distributions p_i on $\Theta_i=[-1,1]^{\#S}$, any equilibrium of the payoffs u_i is the limit as $\varepsilon \to 0$ of a sequence of pure-strategy equilibria of the perturbed payoffs $\tilde{u}_i$.

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→The probability distributions over strategies induced by the pure-strategy equilibria of the perturbed game converge to the distribution of the equilibrium of the unperturbed game.

EX. Battle of Sexes (BoS)

- Consider two-player games in which each player i has two pure strategies, a_i and b_i. Let δ_i for i=1,2 be independent random variables, each uniformly distributed on [-1, 1], and let the random variables $\varepsilon_i(a)$ for i=1,2 and $a \in A$ have the property that $\varepsilon_1(a_1,x)-\varepsilon_1(b_1,x)= \delta_1$ for $x=a_2,b_2$ and $\varepsilon_2(x,a_2)-\varepsilon_2(x,b_2)= \delta_2$ for $x=a_1,b_1$.
- All the equilibrium of BoS are approachable under ε.

	a2	b2
a1	$2+\gamma\delta_1$, $1+\gamma\delta_2$	$\gamma\delta_1$,0
b1	0, $\gamma\delta_2$	1,2

Proof Outline

(1) The pure equilibria are trivially approachable.

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(2) We consider the strictly mixed equilibrium.

For $i = 1, 2$ let pi be the probability that player i's type is one for which he chooses a_i in some Nash equilibrium of $G(\gamma\varepsilon)$.

(i) it is optimal for player 1 to choose a_1 if $(2 + \gamma\delta_1)p_2 \geqq (1 - \gamma\delta_1)(1 - p_2)$.

(ii) $-1 \leqq \delta_1 \leqq 1$

(i) + (ii) : $p_1 = 1/2(1 - (1 - 3p_2)/\gamma)$.

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Solving for p_1 and p_2 we find that $p_1 = (2 + \gamma)/(3 + 2\gamma)$ and $p_2 = (1 + \gamma)/(3 + 2\gamma)$ satisfies these conditions. Since $(p_1, p_2) \to (2/3, 1/3)$ as $\gamma \to 0$ the mixed strategy equilibrium is approachable.

Selten (1980)

- Selten, R. (1980): ”A Note on Evolutionary Stable Strategies in Asymmetric Animal Conflicts,” *Journal of Theoretical Biology*, Vol.**84**, pp.93-101.

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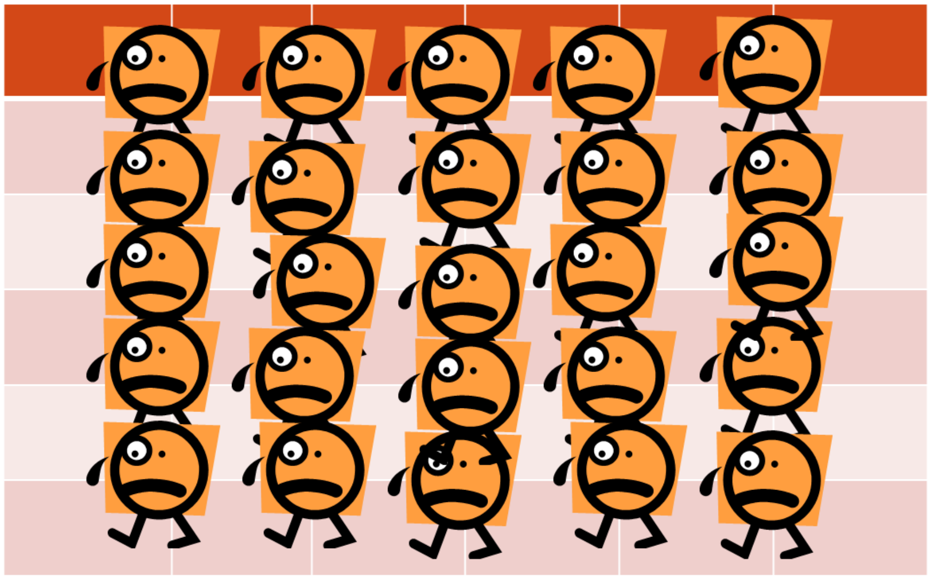
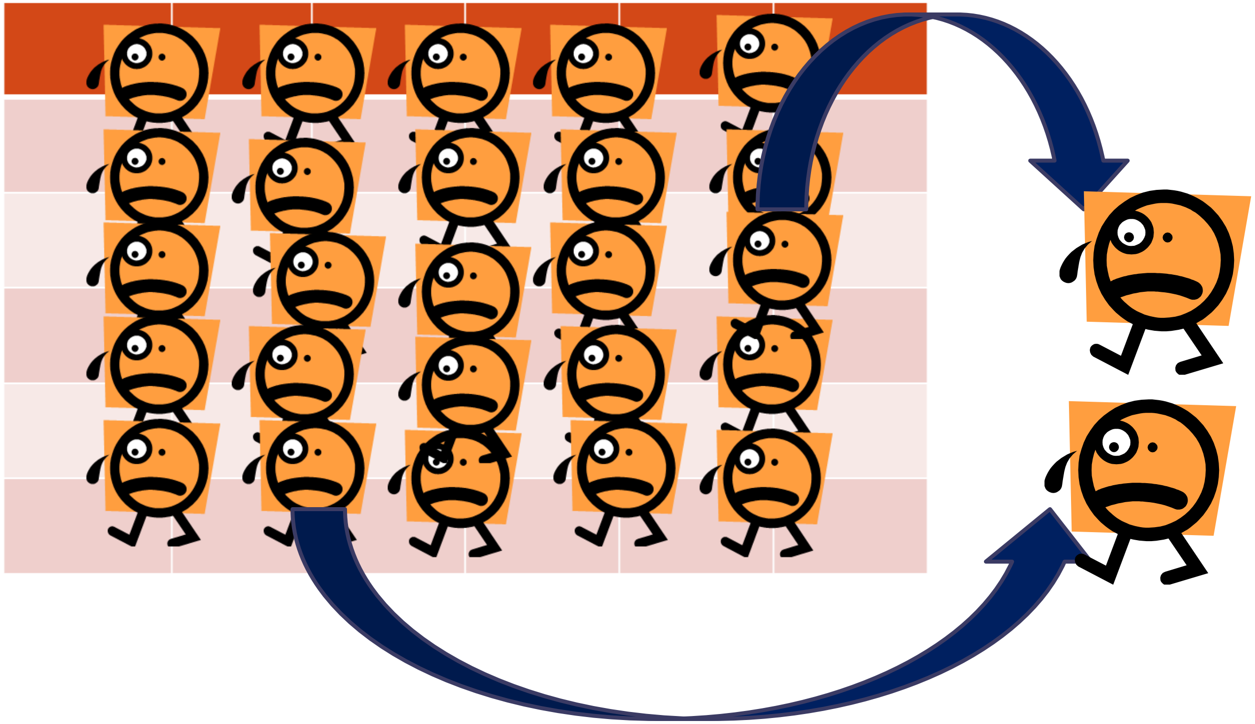
→No mixed equilibria are evolutionary stable when players can condition their strategies on their roles in a game.

Situation (Role Completed Game)



Situation (Role Completed Game)

At Random



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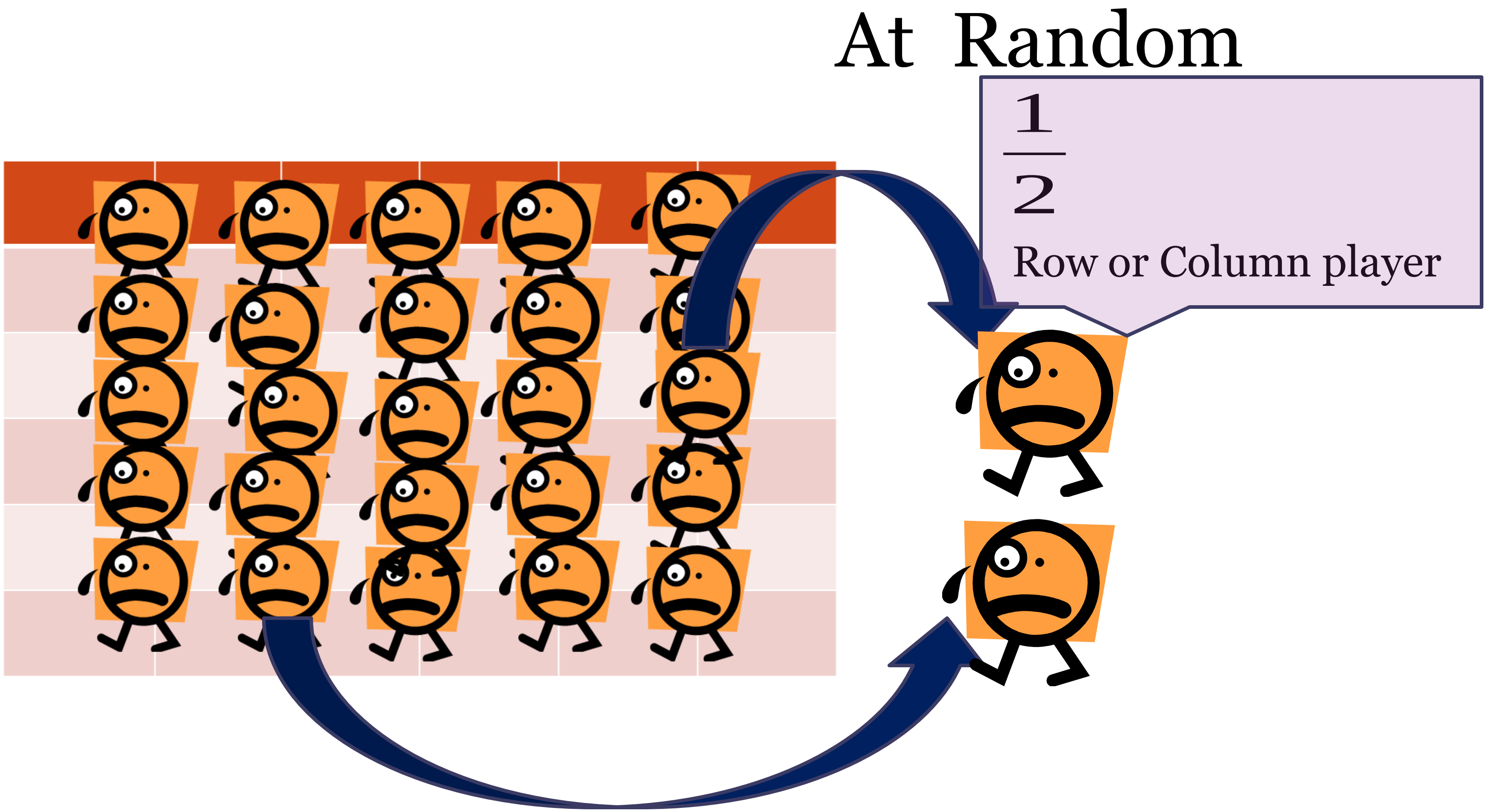

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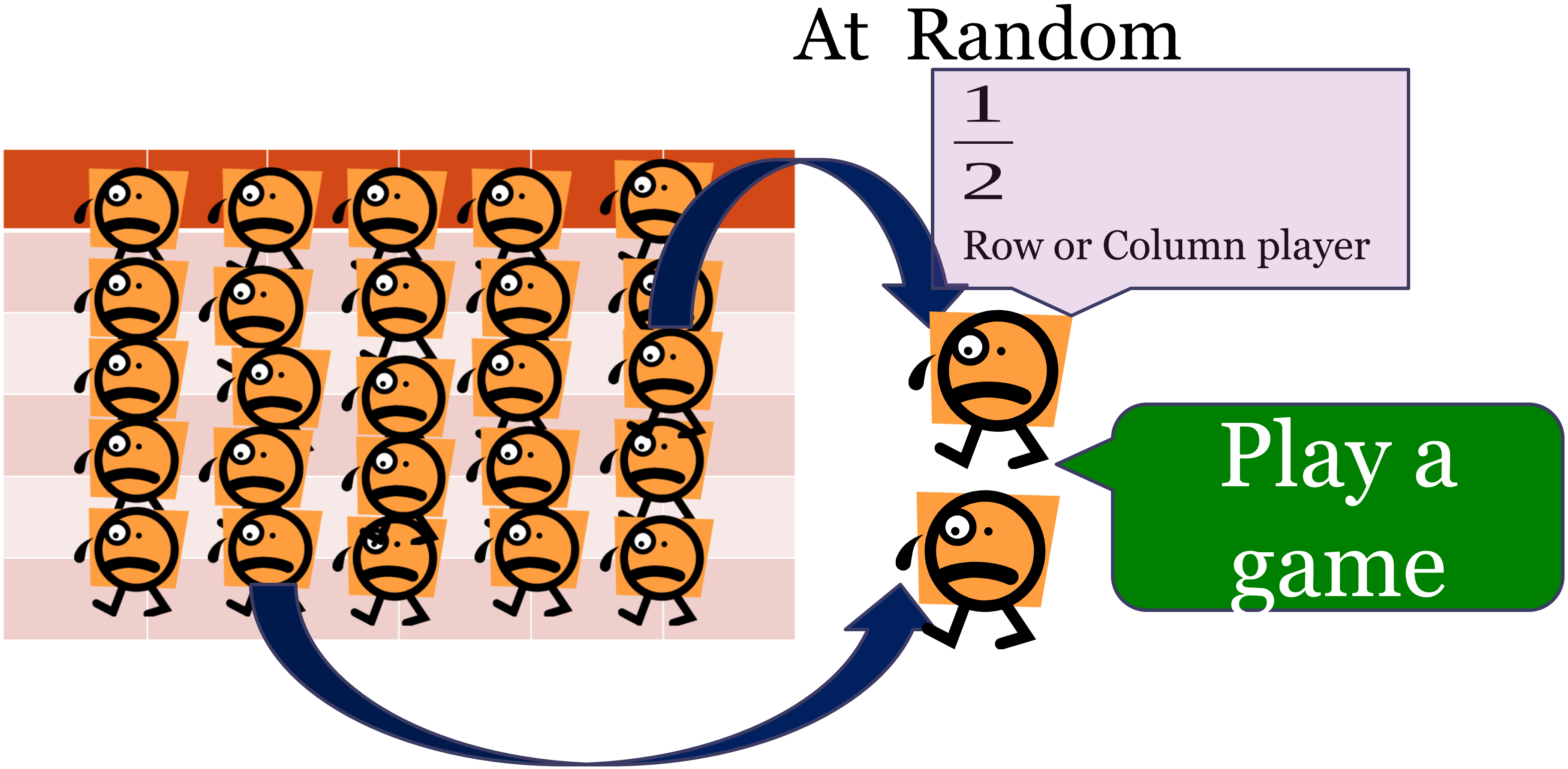
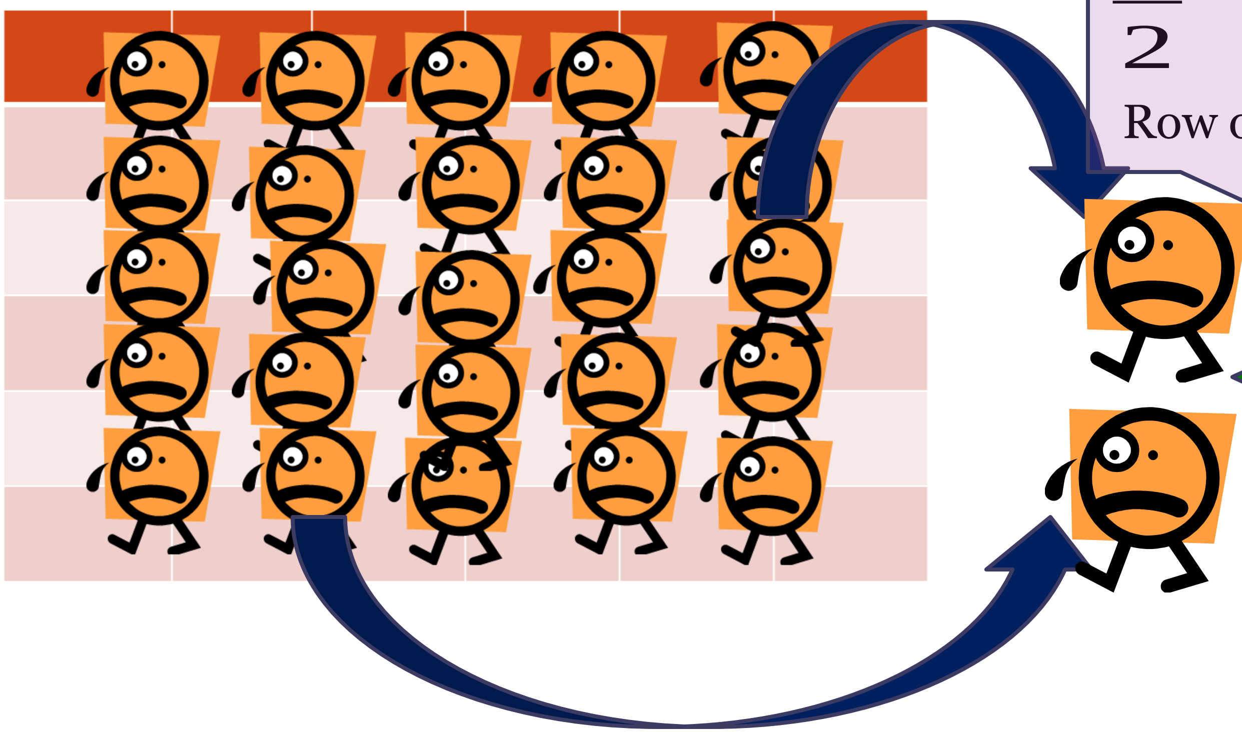


Situation (Role Completed Game)

At Random (infinitely)

$\frac{1}{2}$

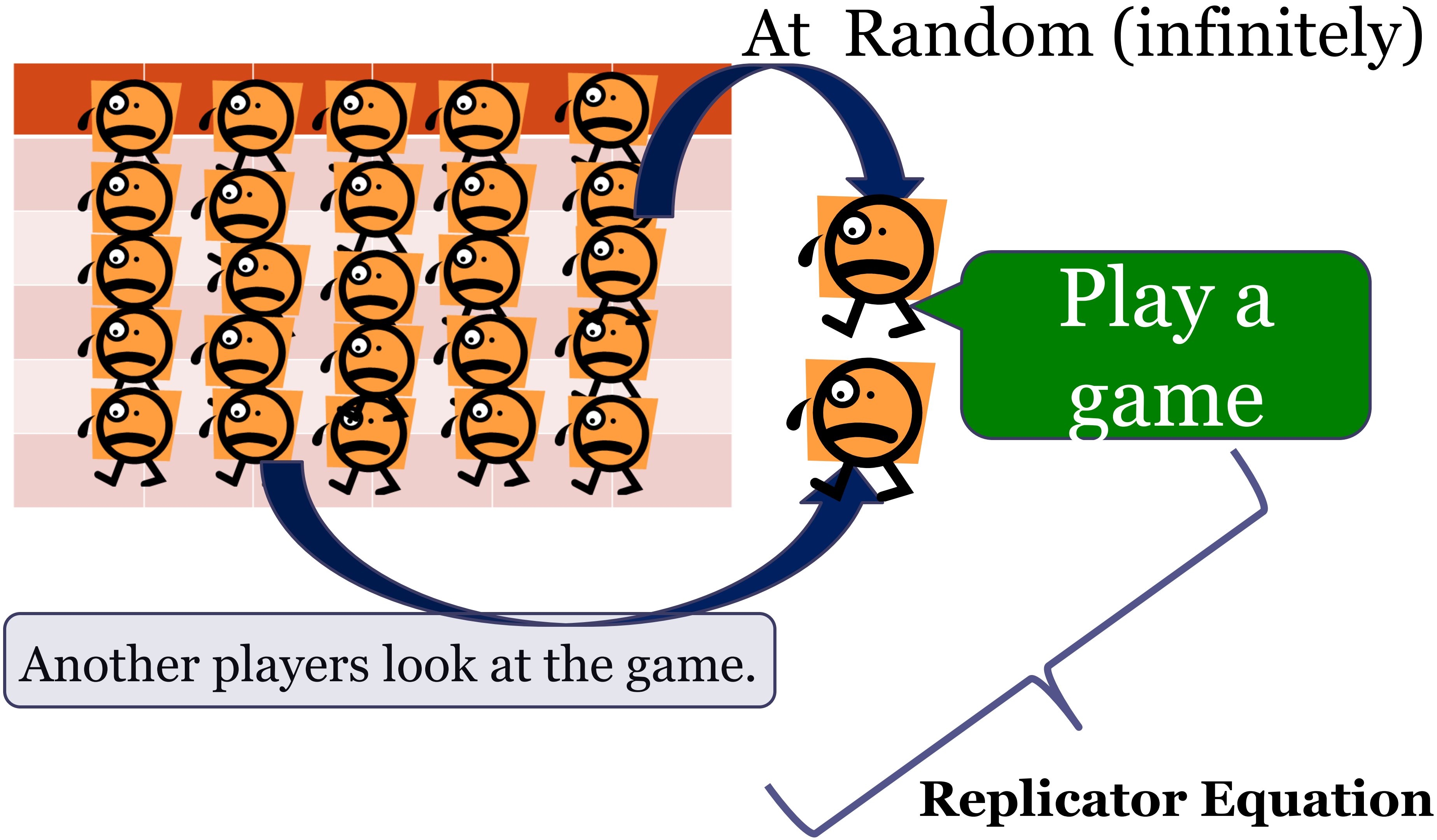
Row or Column player



Play a game

Another players look at the game.

Situation (Traditional Evolutionary Game Theory)



Ex. Hawk-Dove Game

	D	H
D	V/2, V/2	0,V
H	V, 0	V/2-C,V/2-C

H-D Game

- **Classical H-D Game** :
- **Strategy** : {Dove, Hawk}
- **Payoff** : $V>0$, $V<C$
- **Nash Eq.** : Pure {(H,D), (D, H)} + Mixed
-
-
-

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- **Nash Eq.** : Pure {(H,D), (D, H)} + Mixed
- **ESS :** Mixed (× strict Nash)
- **Stability** : Limit Cycle, Structurally Unstable.
- **Replicator Eq.**

$$\dot{x} = x(1-x)\{V/2 - C + Cx\}$$

- **Role Completed H-D Game**
- **Pure Strategy : {DD}, {DH}, {HD}, {HH}**
- **{DH} means play Dove if chosen to be a row player in the surface game and Hawk if chosen to be a column player.**
-

	DD	DH	HD	HH
DD	V/2,V/2	V/4,3V/4	3V/4,V/4	0,V
DH	3V/4,V/4	V/2,V/2	(V-C)/2, (V-C)/2	V/4-C/2, 3V/4-C
HD	V/4,3V/4	(V-C)/2, (V-C)/2	V/2,V/2	V/4-C/2, 3V/4-C
HH	V,0	3V/4-C,V/4-C/2	3V/4-C,V/4-C/2	V/2-C,V/2-C

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- **ESS :** (DH,DH) , (HD, HD)

	DD	DH	HD	HH
DD	V/2,V/2	V/4,3V/4	3V/4,V/4	0,V
DH	3V/4,V/4	**V/2,V/2**	(V-C)/2, (V-C)/2	V/4-C/2, 3V/4-C
HD	V/4,3V/4	(V-C)/2, (V-C)/2	**V/2,V/2**	V/4-C/2, 3V/4-C
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HH	V,0	3V/4-C,V/4-C/2	3V/4-C,V/4-C/2	V/2-C,V/2-C

Role Completed H-D Game

3. OUR MODEL

3-1. HARSANYI TYPE

Stochastic Environment (Harsanyi (1973) + Dynamics)

- Stochastic Environment = payoff variation

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- **Pro.** Let x be a strategy distribution. It satisfies :

$$P(x,t)dx = (2\pi\sigma^2 t)^{-1/2} \exp\left[-\frac{(\log x - \log x^*(t))^2}{2\sigma^2 t}\right]\frac{dx}{x}.$$

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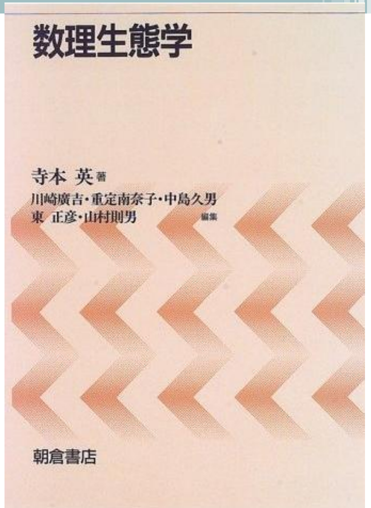
→Approachable under variance (σ^2)

PROOF OUTLINE

- Teramoto (1997)



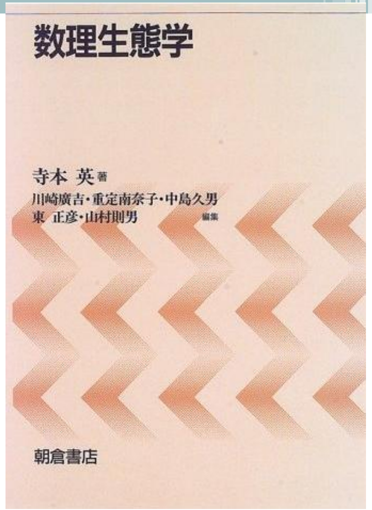
Teramoto(1997)

PROOF OUTLINE

- Teramoto (1997)
- (i) transformation

$$\dot{x}_i(t) = x_i(t)\{g_i(t) - g(t)\}, \quad g_i(t) = g_i + \zeta(t)$$

$$\log \frac{x_i(t)}{x(0)} - g_i t + \int_0^t g(t)\,dt = \sum_{k=1}^{n} \xi_k$$



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- (ii) apply central limit theorem
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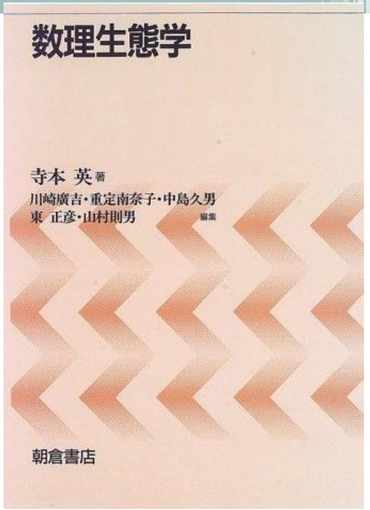

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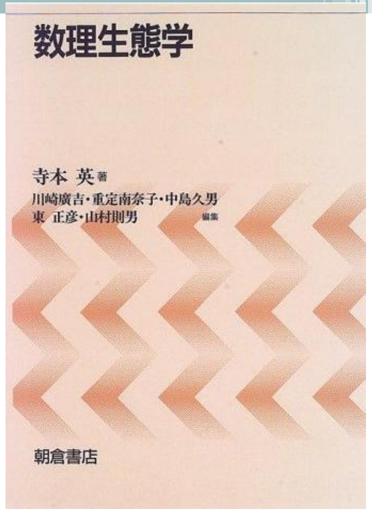


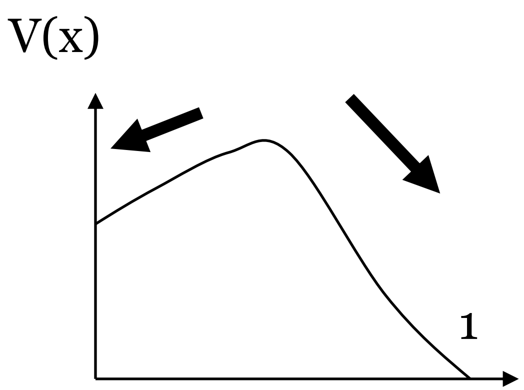
Teramoto(1997)

EX-1.

	a_2	b_2
a_1	$2+\gamma\delta_1$, $2+\gamma\delta_1$	$\gamma\delta_1$,0
b_1	0, $\gamma\delta_1$	1, 1

- **Coordination Game**
- Replicator Eq. : $\dot{x} = x(1-x)\{3x - 1 + \gamma\delta_1\}$
- Equilibrium : 0, 1, $\dfrac{1-\gamma\delta_1}{3}$
- Potential Func. :

$$V(x) = \frac{3}{4}x^4 - \frac{4-\gamma\delta_1}{3}x^3 + \frac{1-\gamma\delta_1}{2}x^2 + C$$



- The equilibrium of the mixed strategy is **unstable**.

EX-2.

	a_2	b_2
a_1	$2+\gamma\delta_1$, $1+\gamma\delta_1$	$\gamma\delta_1$,0
b_1	0, $\gamma\delta_2$	1, 2

- Battle of Sexes (BoS)
- Replicator Eq. :

$$\dot{x} = x(1-x)\{2-\gamma\delta_2 - 3y\}, \qquad \dot{y} = y(1-y)\{2+\gamma\delta_1 - 3x\}$$

- Equilibrium point : (y^*,x^*)=(0,0), (1,0), (0,1), (1,1),

$$\left(\frac{2-\gamma\delta_2}{3}, \frac{2+\gamma\delta_1}{3}\right)$$

- The stability of the Mixed Strategy is saddle point.

3. OUR MODEL

3-2. SELTEN TYPE

Stochastic Environment (Selten (1980) + Dynamics)

- “Role” = “Group”

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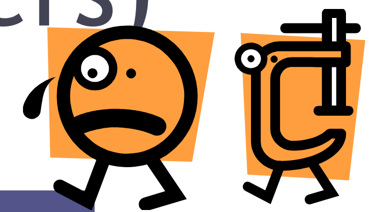
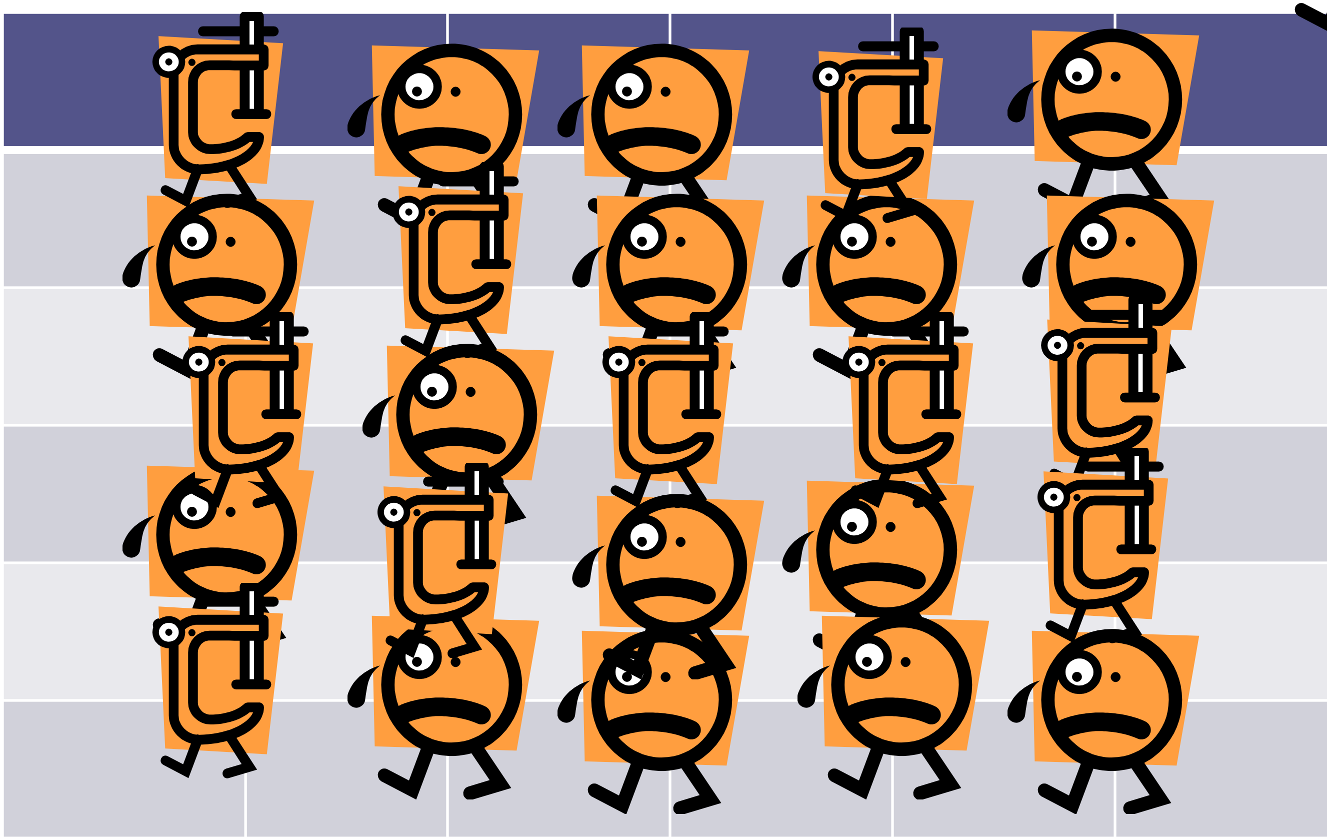
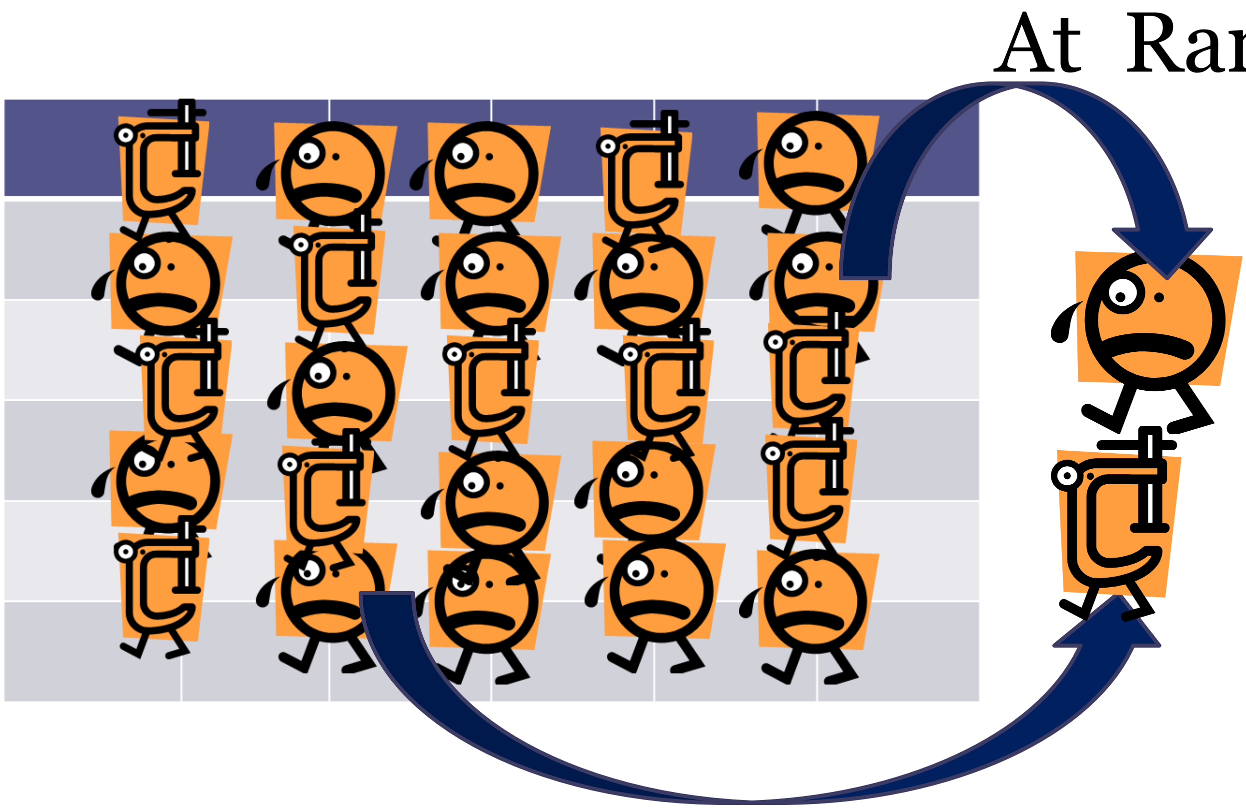
- **Situation** : see next slide.

Situation (two types players)





Situation (Evolutionary Game Theory with Group Structure)



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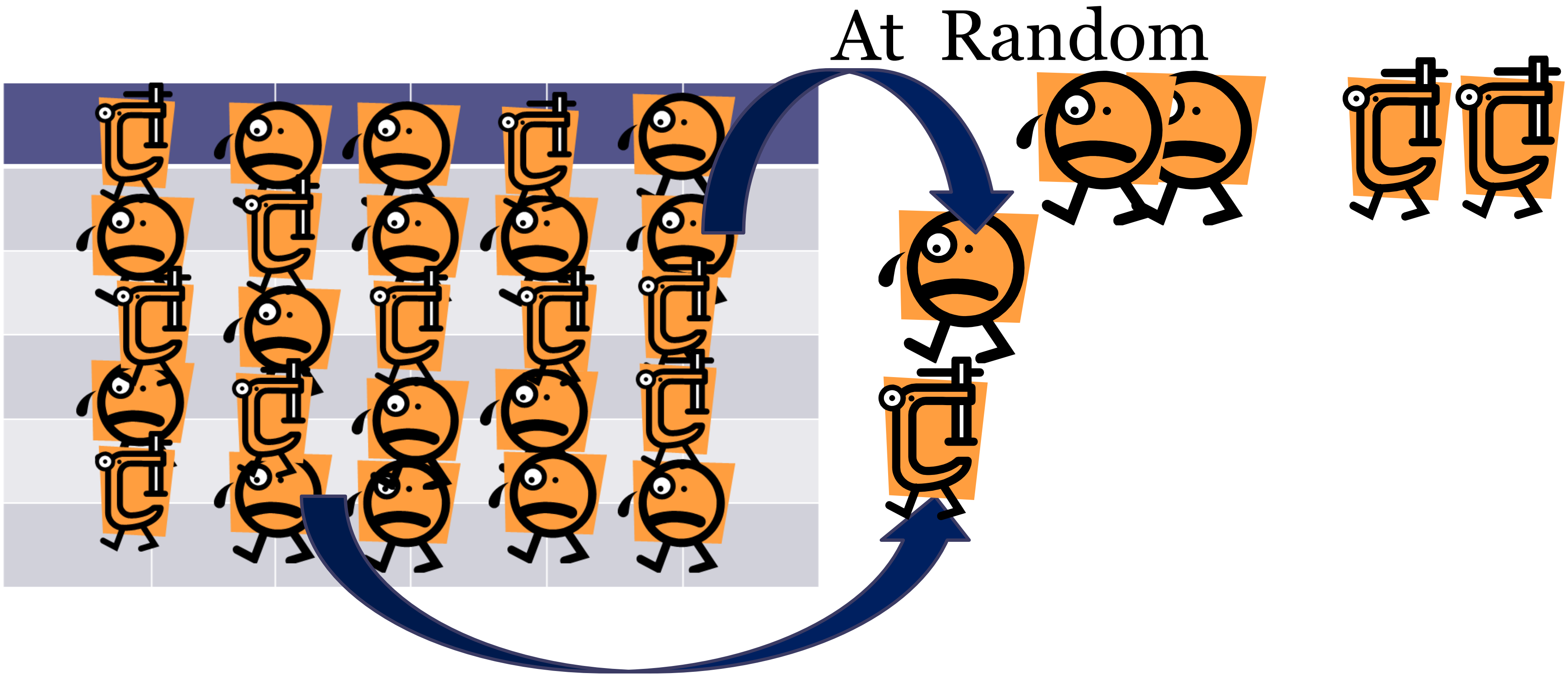
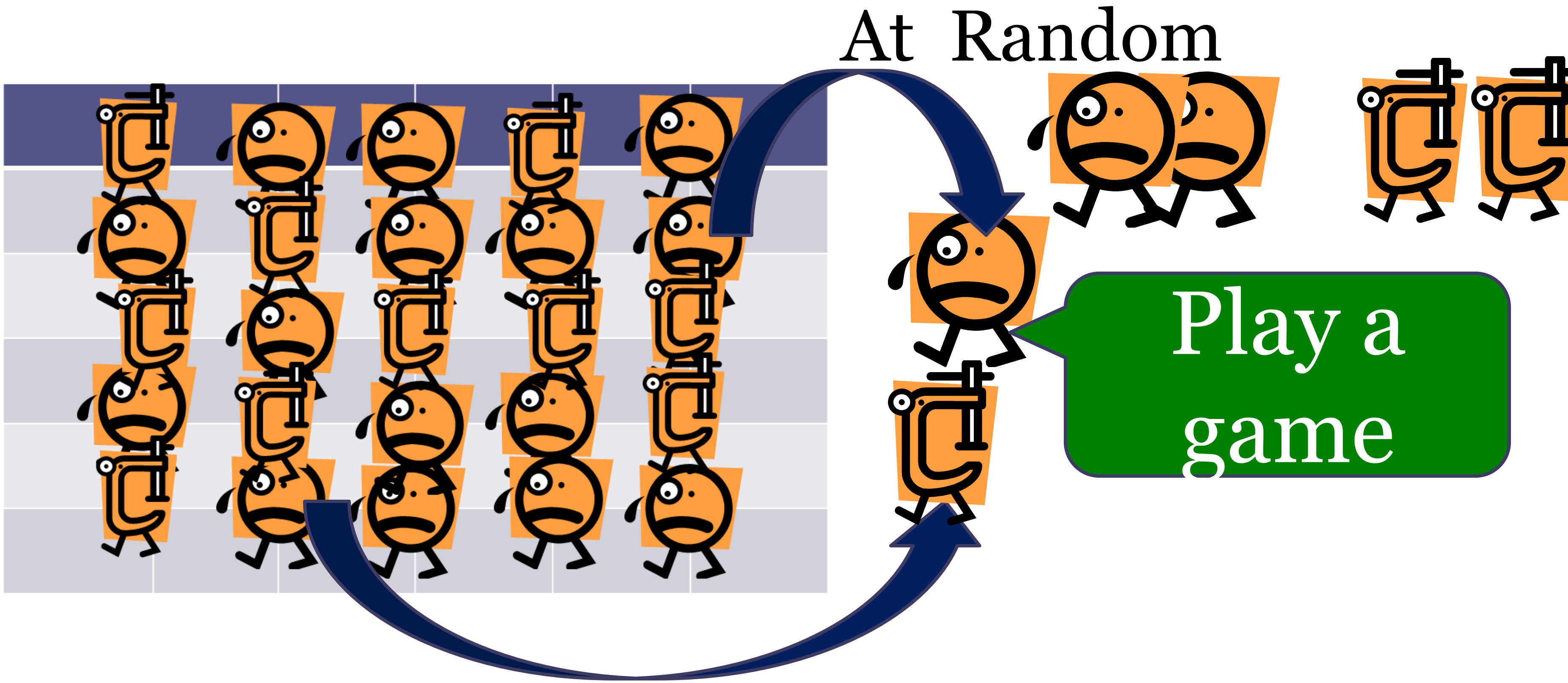


Situation (Evolutionary Game Theory with Group Structure)



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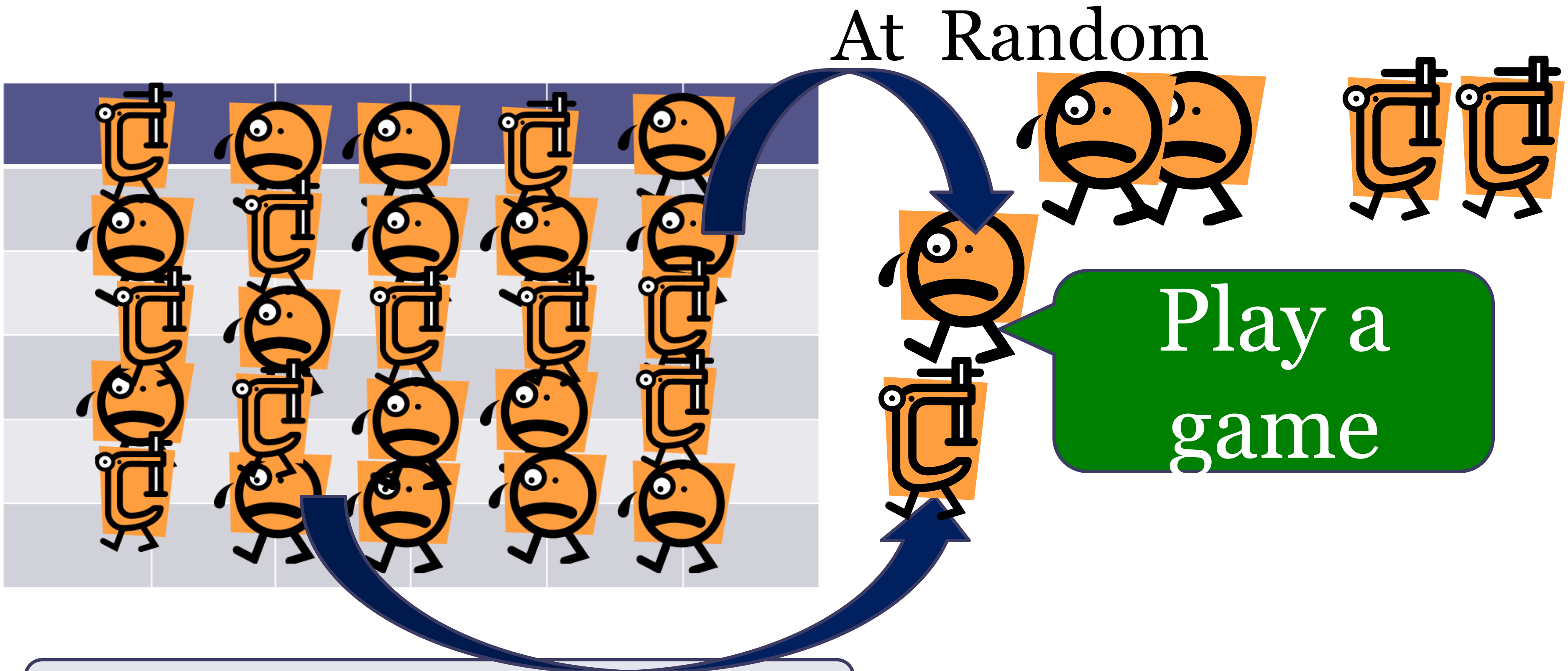
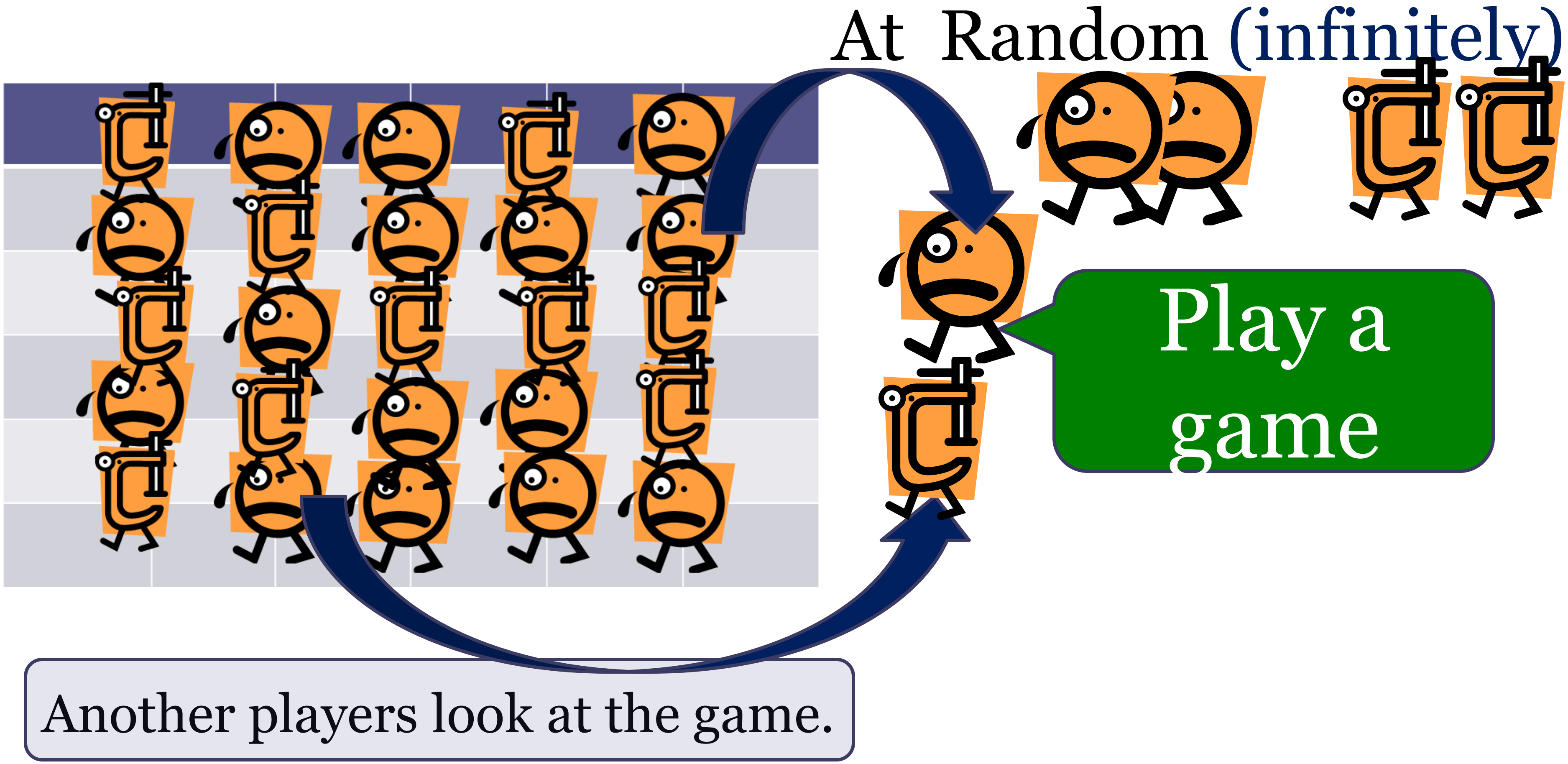


Another players look at the game.

Situation (Evolutionary Game Theory with Group Structure)



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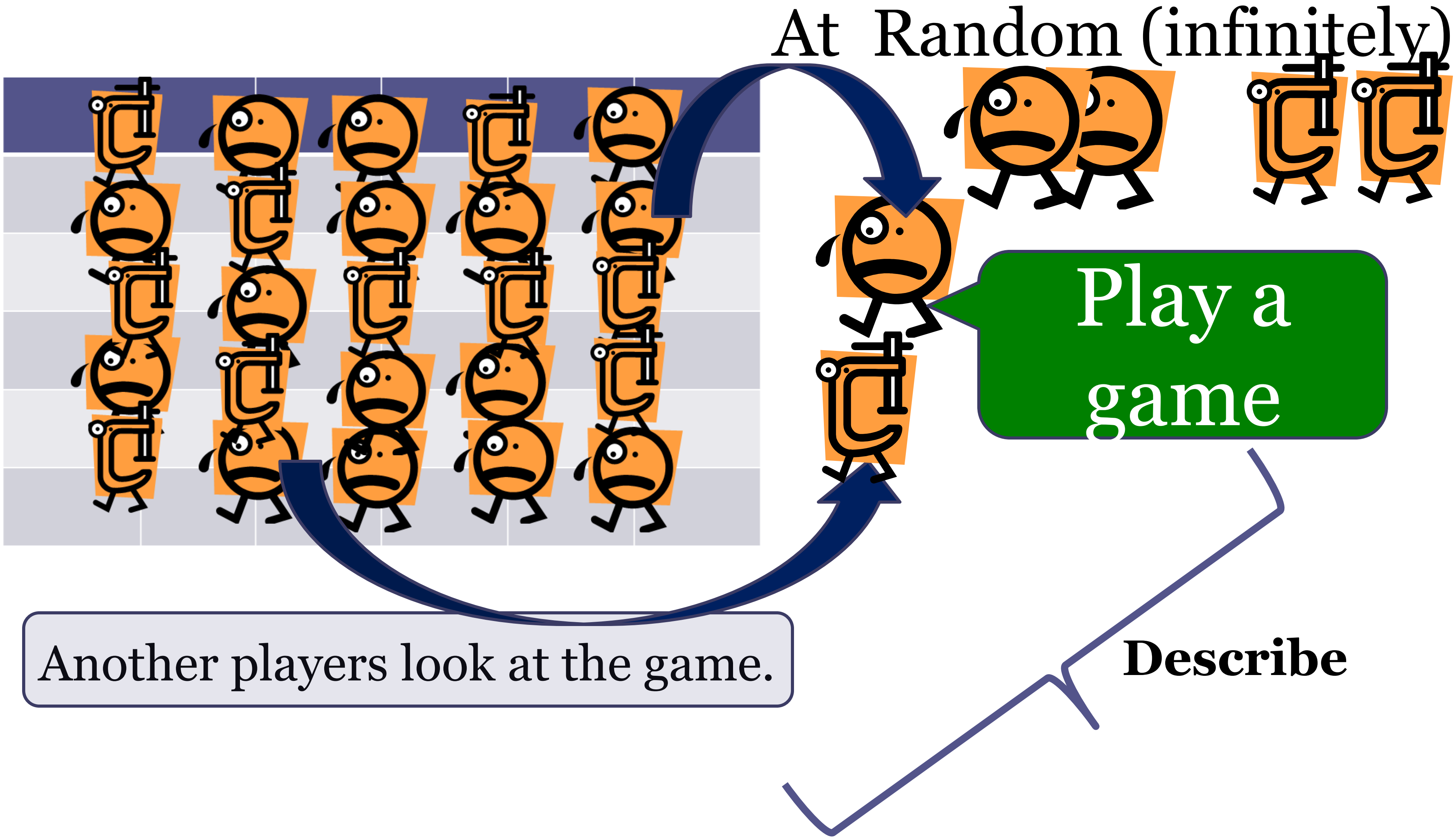

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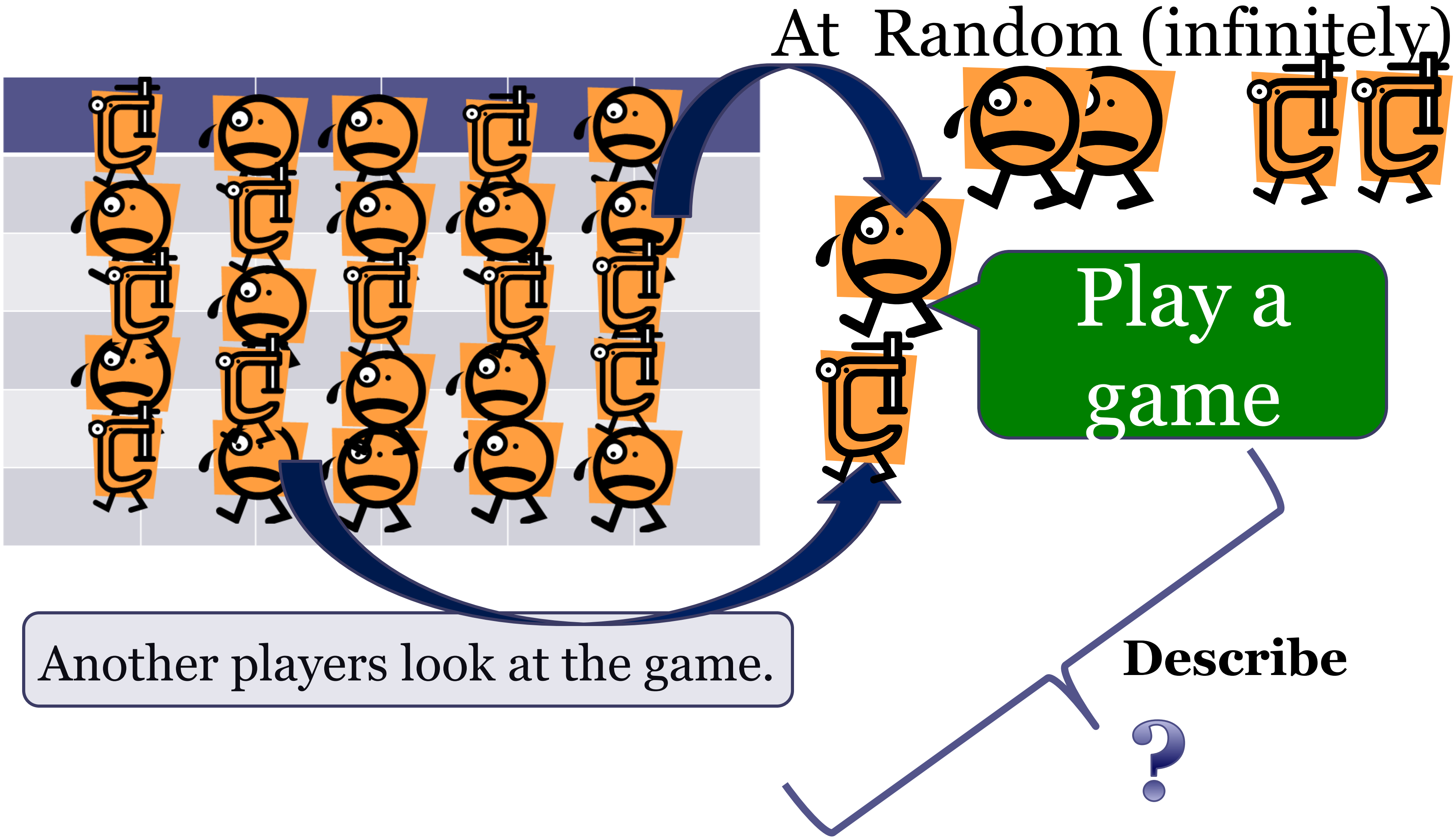
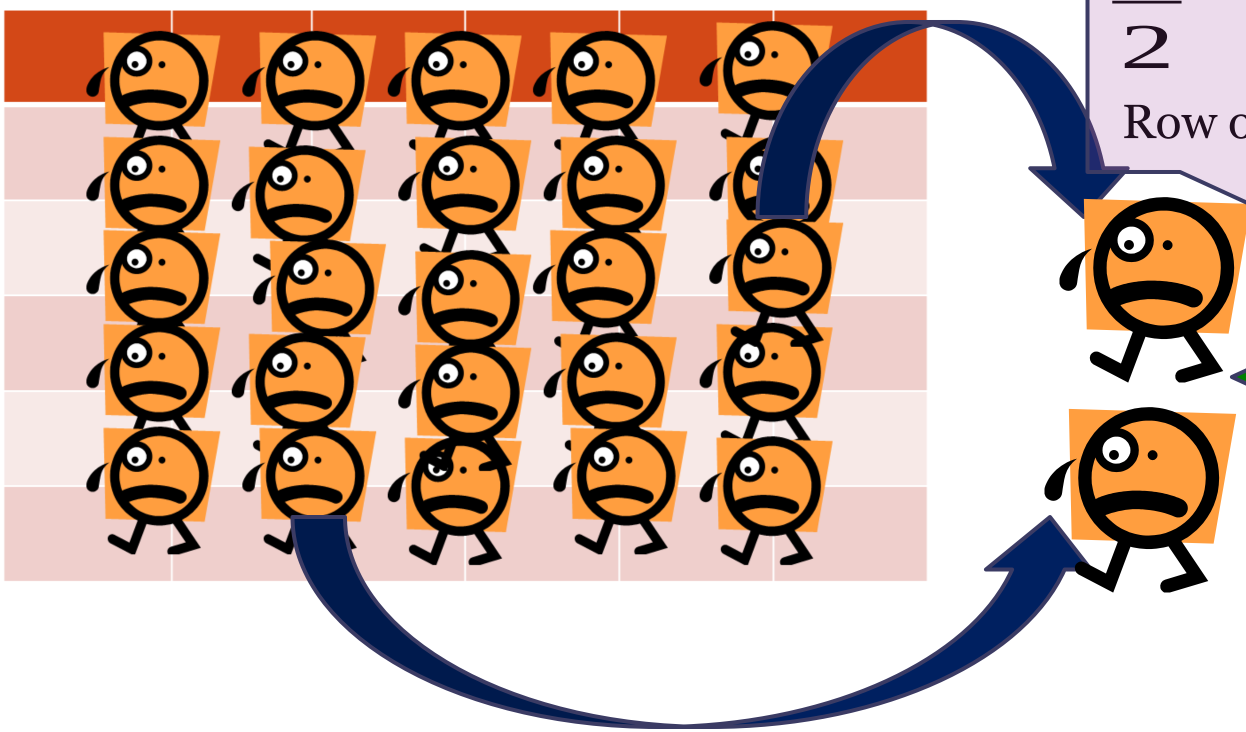


Situation (Role Completed Game)

At Random (infinitely)

$\frac{1}{2}$

Row or Column player

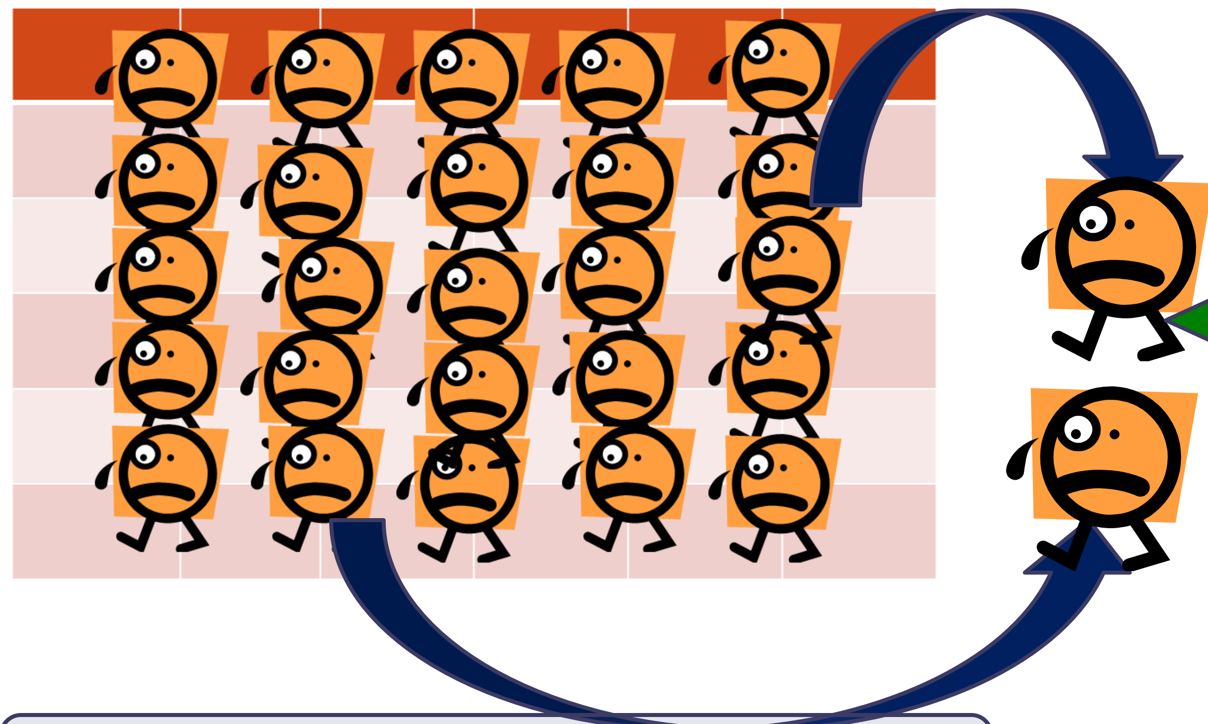


Play a game

Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



Play a game

Another players look at the game.

Replicator Equation

Stochastic Environment (Selten (1980) + Dynamics)

- “Role” = “Group”

- **Pro.** Group size and it’s fitness in a game with group structure are as follows :

  Price equation $\dot{E}(p) = Cov(f, p) + E\left(\dot{p}\right).$

PROOF OUTLINE

1) transformation

$$\bar{x}' - \bar{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\bar{\pi}} \right) x_i - \sum_i f_i \frac{\pi_i}{\bar{\pi}} \Delta x_i$$

where $\Delta x_i = x'_i - x_i$

PROOF OUTLINE

1) transformation

$$\bar{x}' - \bar{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\bar{\pi}} \right) x_i - \sum_i f_i \frac{\pi_i}{\bar{\pi}} \Delta x_i$$

where $\Delta x_i = x'_i - x_i$

2) $\bar{\pi} \Delta \bar{x} = \sum_i f_i (\pi - \bar{\pi}) x_i + \sum_i f_j \pi_i \Delta x_i$ where $\Delta \bar{x} = \bar{x}' - \bar{x}$

PROOF OUTLINE

1) transformation

$$\bar{x}'-\bar{x}=\sum_i f'_i \cdot x'_i-\sum_i f_i \cdot x_i=\cdots=\sum_i f'_i\left(\frac{\pi_i}{\bar{\pi}}\right)x_i-\sum_i f_i \frac{\pi_i}{\bar{\pi}}\Delta x_i$$

where $\Delta x_i = x'_i - x_i$

2) $\bar{\pi}\Delta\bar{x}=\sum_i f_i(\pi-\bar{\pi})x_i+\sum_i f_j\pi_i\Delta x_i$ where $\Delta\bar{x}=\bar{x}'-\bar{x}$

3) Definition

$$Cov[\pi,x]=\sum_i f_i(\pi_i-\bar{\pi})(x_i-\bar{x}),\ \sum_i f_i(\pi_i-\bar{\pi})\bar{x}=0$$

$$\bar{\pi}\Delta\bar{x}=Cov[\pi,x]+E[\pi\Delta x].$$

PROOF OUTLINE

1) transformation

$$\bar{x}' - \bar{x} = \sum_i f'_i \cdot x'_i - \sum_i f_i \cdot x_i = \cdots = \sum_i f'_i \left(\frac{\pi_i}{\bar{\pi}} \right) x_i - \sum_i f_i \frac{\pi_i}{\bar{\pi}} \Delta x_i$$

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3) Definition

$$Cov[\pi, x] = \sum_i f_i (\pi_i - \bar{\pi})(x_i - \bar{x}), \sum_i f_i (\pi_i - \bar{\pi}) \bar{x} = 0$$

$$\bar{\pi} \Delta \bar{x} = Cov[\pi, x] + E[\pi \Delta x].$$

- **Remark** : Price equation is equivalent to Replicator equation.

	H	D
H	a, a	0,0
D	0, 0	b, b

Payoff matrix

EX.

- Two type agent : {S,A}
- Random Matching : {SS}, {SA}, {AA}

$$Cov[\pi, x] = \sum_{i \in \{AA, AS, SS\}} f_i (\pi_i - \overline{\pi})(x_i - \overline{x}) = f(1-f)\{f(a+b) - b\}.$$

- Price Eq. = Replicator Eq.

- H-D game ($a, b < 0$)
- $Cov[\pi,x]=0 \Leftrightarrow f=0,1, b/(a+b).$

4. EXTENSION

GLOBAL GAME

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Global Game

(1) Complete information about x

(i) unique Nash eq.

$x<0$: strategy "D", $x > a$: strategy "C"

(ii) Multiple eq. $x \in [0,a]$: strategy "C" and "D"

•

Global Game

	C	D
C	x, x	x,0
D	0, x	a, a

$a > 0$

(1) Complete information about x

(i) unique Nash eq.

$x < 0$: strategy “D”, $x > a$: strategy “C”

(ii) Multiple eq. $x \in [0,a]$: strategy “C” and “D”

(2) Incomplete information about x

- Player i observes a private signal $s = x + \varepsilon_i$.

Pro. (Carlsson and van Damme, 1993) Let $\gamma \in \{\alpha, \beta\}$. If x lies on a continuous curve C such that $C \subseteq \Theta$, $g(C) \subseteq R^{\gamma}$, and $g(C) \cap D^{\gamma} \neq \emptyset$, then γ is iteratively dominant at x in Γ^{ε} if ε is sufficiently small.

	C	D
C	x, x	x,0
D	0, x	a, a

a>o

Global Game

(1) Complete information about *x*

(i) unique Nash eq.

$x<o$: strategy "D", $x > a$: strategy "C"

(ii) Multiple eq. $x\in[o,a]$: strategy "C" and "D"

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→ unique equilibrium : $x\in[o,a]$

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)
(2) Group Structure : {S,A}

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)
(2) Group Structure : {S,A}
(3) Price eq.

$$Cov[\pi, x] = f(1-f)\{af-(a-x)+r(x-af)\}$$

$Cov[\pi,x] = 0 \Leftrightarrow f=0, 1$, $\dfrac{a\{f(r-1)+1\}}{1+r}$.

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Dynamic Global Game

(1) Observation noise = assortative matching
($0 \leqq r \leqq 1$, $r=0$: random matching)
(2) Group Structure : {S,A}
(3) Price eq.

$$Cov[\pi, x] = f(1-f)\{af-(a-x)+r(x-af)\}$$

$Cov[\pi,x] = 0 \Leftrightarrow f=0, 1, \dfrac{a\{f(r-1)+1\}}{1+r}.$

$r \rightarrow 1$: $x>a/2$, $Cov[\pi,x]>0$, $x<a/2$, $Cov[\pi,x]<0$
$\rightarrow$ ESS Unique.

	C	D
C	x, x	x,0
D	0, x	a, a

$a>0$

Global Game

(1) Complete information about x

(i) unique Nash eq.

$x<0$: strategy "D", $x > a$: strategy "C"

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→ unique equilibrium : $x \in [0,a]$

5. Application

利得が確率的に変動→ファイナンス理論へ応用

売り手と買い手の行動から考えるとどのようなBlack-Sholes (ヨーロピアンコールオプション)の公式が導かれるのか？

OPTION

- **オプション(option)** とは売買を行う権利のことであり、買い付ける権利を**コール(call)**、売り付ける権利を**プット(put)**と言います。
-
-

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- 日本で取引されている株価指数オプションには、**日経225オプション、日経300オプション**(大阪証券取引所)、**TOPIXオプション**(東京証券取引所)。

[DATA] 大阪証券取引所日報
http://www.nippo.ose.or.jp/pdf.html

-

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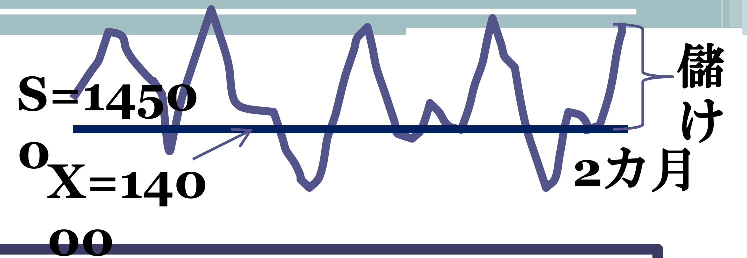
http://www.nippo.ose.or.jp/pdf.html

- **Black-Sholesの公式**・・・ヨーロピアン・オプションの価格評価公式

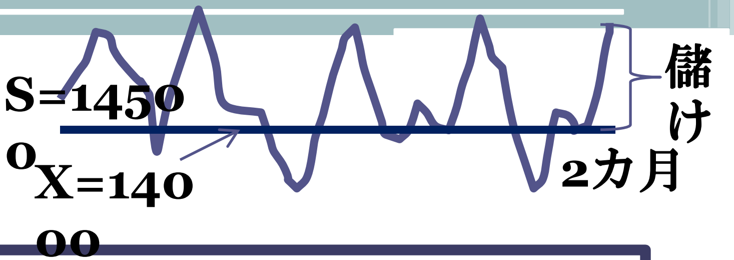
PROBLEM



- 次のヨーロピアン・コールオプションの価格を求めよ。

  現在の株価 S=14500円, 権利行使価格 K=14000円, オプションの期間=2ヵ月, ボラティリティ σ =38%, 非危険利子率 r=6%

PROBLEM



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→あなたは上記の条件でいくら支払い購入する権利をするのか？

SOLUTION.

- **Black-Sholes Formula**

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{x}} + \sigma\sqrt{x}\right) - K \cdot \exp(-rx) \cdot N\left(\frac{u}{\sigma\sqrt{x}}\right).$$

SOLUTION.

- **Black-Sholes Formula**

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{x}} + \sigma\sqrt{x}\right) - K \cdot \exp(-rx) \cdot N\left(\frac{u}{\sigma\sqrt{x}}\right).$$

- オプションの期間 T-t=2/12=0.1667
- u=log(S/K)+(r-σ^2/2)(T-t)=0.0331
- u/σ√x+σ√x=0.3685, u/σ√x=0.2133.
- 標準正規分布の数表から
- N(u/σ√x+σ√x)=0.6437, N(u/σ√x)=0.5845.
- 以上から, ヨーロピアンコールオプション価格は f(S,t)=14500×0.6437-14000×exp(-0.06×0.1667)×0.5845 = **1232.0884**円

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- **Black-Sholes Formula**

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{x}} + \sigma\sqrt{x}\right) - K \cdot \exp(-rx) \cdot N\left(\frac{u}{\sigma\sqrt{x}}\right).$$

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⇒2ヶ月後の株価 ＞ **15232**円「儲け」

＜ 「損」

Model

- **連続時間**
- 財：安全資産(金利r)、危険資産: 幾何ブラウン運動
- 主体：売り手と買い手 (非対称2人ゲーム)
- δt の間に, 現在の株価を見て、自分と相手の利得を勘定し、自らの戦略を決定するということを行っている。
- 戦略：2つ。例：{bear, bull} など
- 利得：売り手：K(t)-S(t), 買い手: S(t)-K(t)
- →ゼロ＝サム型
- **+仮定**：無裁定条件

- 利得表

	戦略1	戦略2
戦略1	a(t), -a(t)	0,0
戦略2	0, 0	b(t),-b(t)

- ↑の利得表は相対的な「利得差」を表している。

- 利得表

	戦略1	戦略2
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- ↑の利得表は相対的な「利得差」を表している。

このときのReplicator 方程式

$$\dot{s}_1 = s_1(1-s_1)\{a(t)-(a(t)+b(t))s_2\},$$

$$\dot{s}_2 = s_2(1-s_2)\{-a(t)+(a(t)+b(t))s_1\},$$

s_1を主体1が戦略1を採用する確率, s_2 を主体2が戦略2を採用する確率

平衡点とその安定性

- このときの平衡点は純粋戦略の組み4つと、内点解(=混合戦略)の5つ存在する.
-

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平衡点とその安定性

- このときの平衡点は純粋戦略の組み4つと、内点解(=混合戦略)の5つ存在する.

- ESSは内点解。

補題 ノイズがない場合の内点の安定性はリミットサイクルであり, またノイズがある場合もリミットサイクルである.

この場合のBlack-Sholesの公式

- 前に取り上げたBlack-Sholesモデルにおいて、行使価格の影響があるのは、境界条件を使用するとき。
- よって、K:= $\overline{K}$ とすればよい. つまり

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{\tau}} + \sigma\sqrt{\tau}\right) - \overline{K} \cdot e^{-r\tau} \cdot N\left(\frac{u}{\sigma\sqrt{\tau}}\right).$$

ただし$\overline{K}$ ＝ 平衡時の戦略1における行使価格 $\cdot s_1^*$

＋ 平衡時の戦略2における行使価格 $\cdot (1 - s_1^*)$

s_1^* は混合戦略を採用する場合の確率.

注)　離散時間の場合は確率的進化ゲーム理論を応用させることによって、同様にBlack-Sholesの公式を導出することができる.

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6. SUMMARY and FUTURE WORKS

OUR PROBLEM

- Q How does each player choose the action in stochastic environment ?

- A.1 ： Each player randomly chooses the action. (mixed strategy) (Harsanyi , 1973)

- A.2 ： Each player chooses the better action. (pure strategy) (Selten, 1980)

OUR ANSWER

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→ uncertainty of the payoff

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→ uncertainty of the payoff

- A.2 ： Each player chooses the better action. (pure strategy) (Selten, 1980)

→ incomplete information

Summary

1. Harsanyi(1973)+ Dynamics :

2. Selten (1980)+ Dynamics :

3. Global Game + Dynamics :

Summary

1. Harsanyi(1973)+ Dynamics :

→ log-normal distribution (central limit theorem)

2. Selten (1980)+ Dynamics :

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1. Harsanyi(1973)+ Dynamics :

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→Approachable under variance (σ^2)

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1. Harsanyi(1973)+ Dynamics :

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2. Selten (1980)+ Dynamics :

→ Bayesian Game = Game with Group Structure

3. Global Game + Dynamics :

Summary

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→ log-normal distribution (central limit theorem)

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Summary

1. Harsanyi(1973)+ Dynamics :

→ log-normal distribution (central limit theorem)

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2. Selten (1980)+ Dynamics :

→ Bayesian Game = Game with Group Structure

→ Price eq. = Replicator eq.

3. Global Game + Dynamics :

Thank you for your attention.

Mitsuru KIKKAWA (mitsurukikkawa@hotmail.co.jp)

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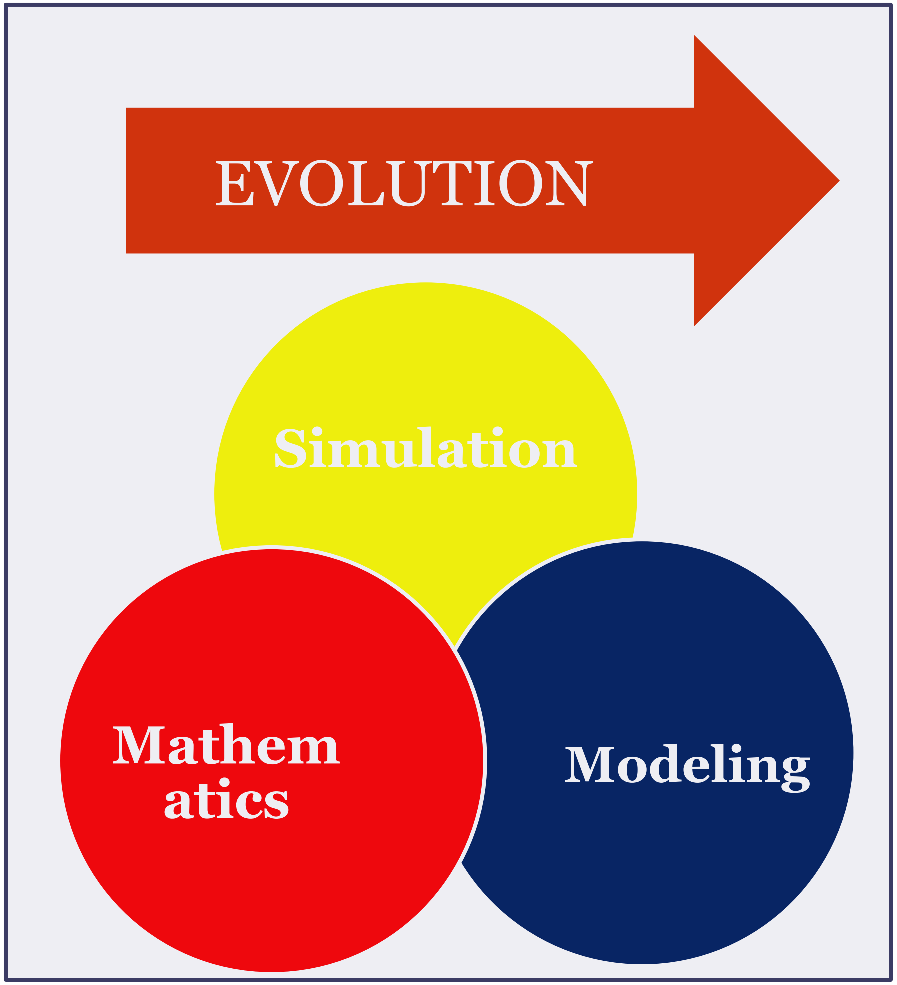

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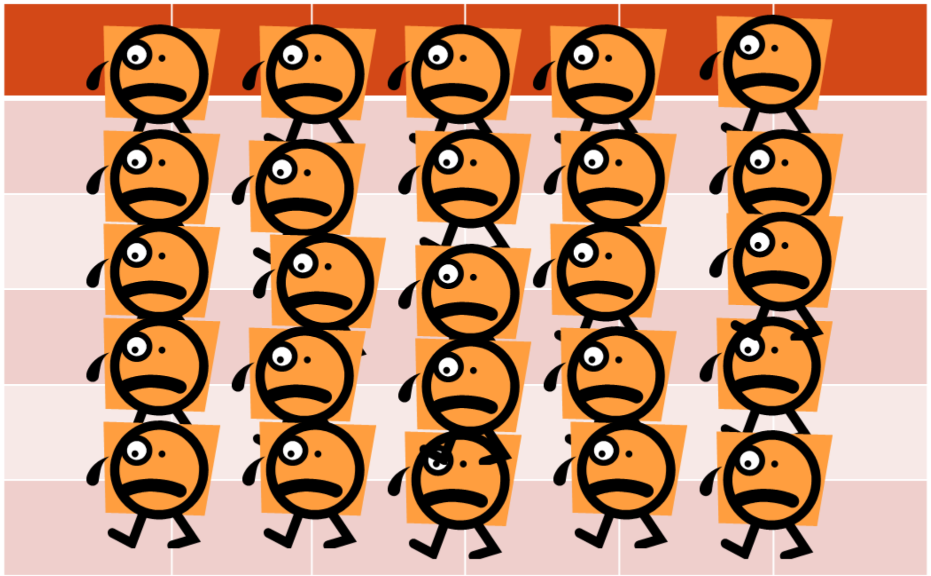
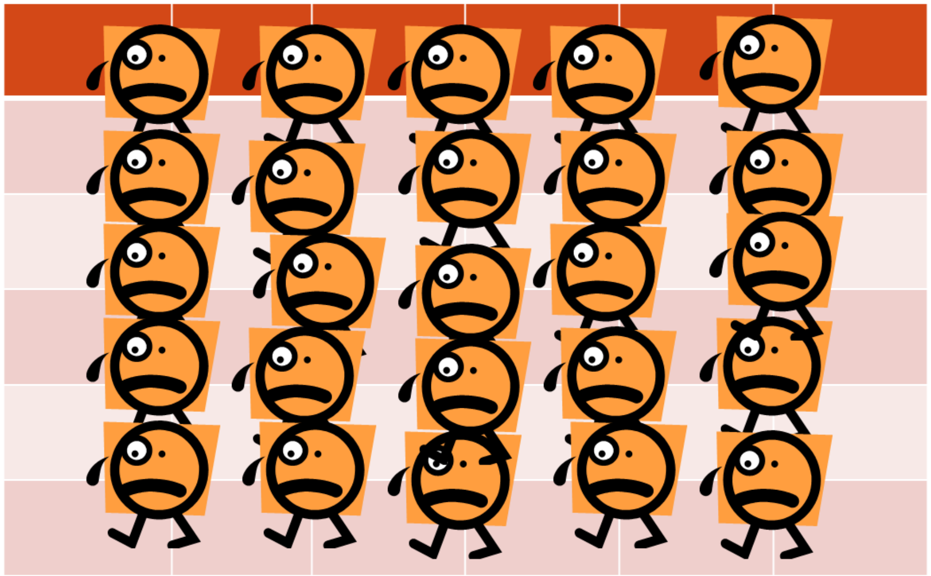
明治大学
グローバルCOEプログラム
現象数理学の形成と発展

PRELIMINARIES (EVOLUTIONARY GAME THEORY)

Situation (Traditional Evolutionary Game Theory)



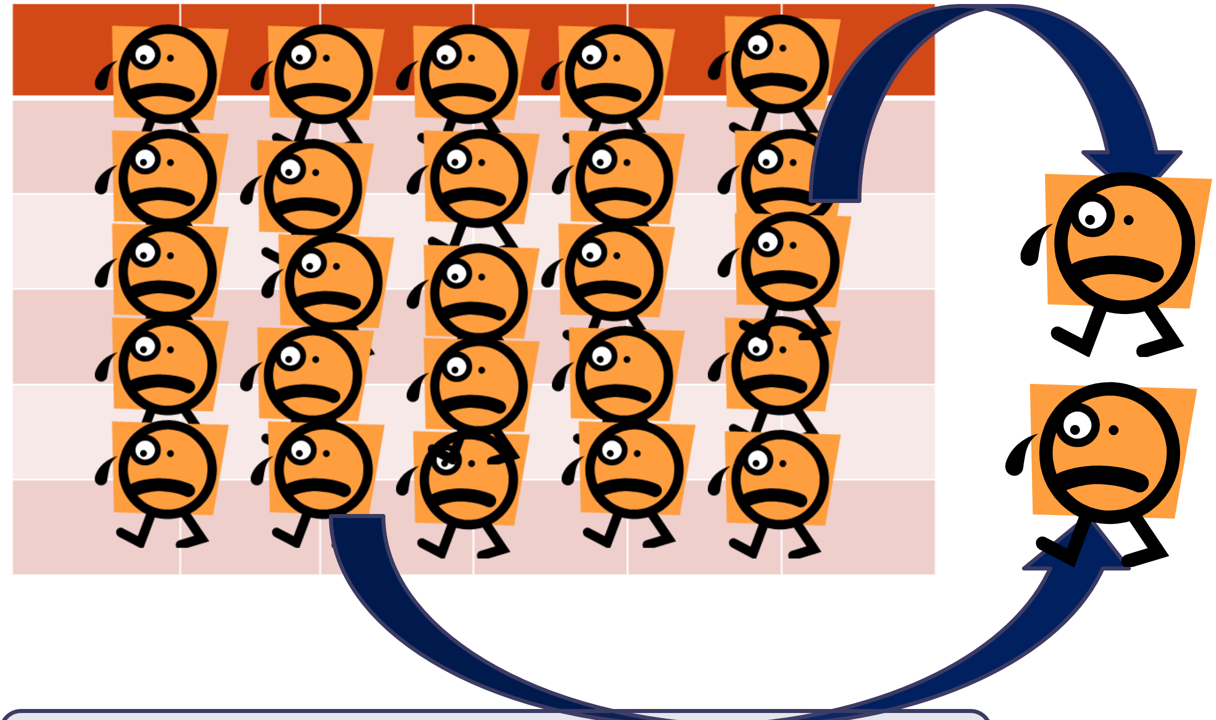
Situation (Traditional Evolutionary Game Theory)



Another players look at the game.

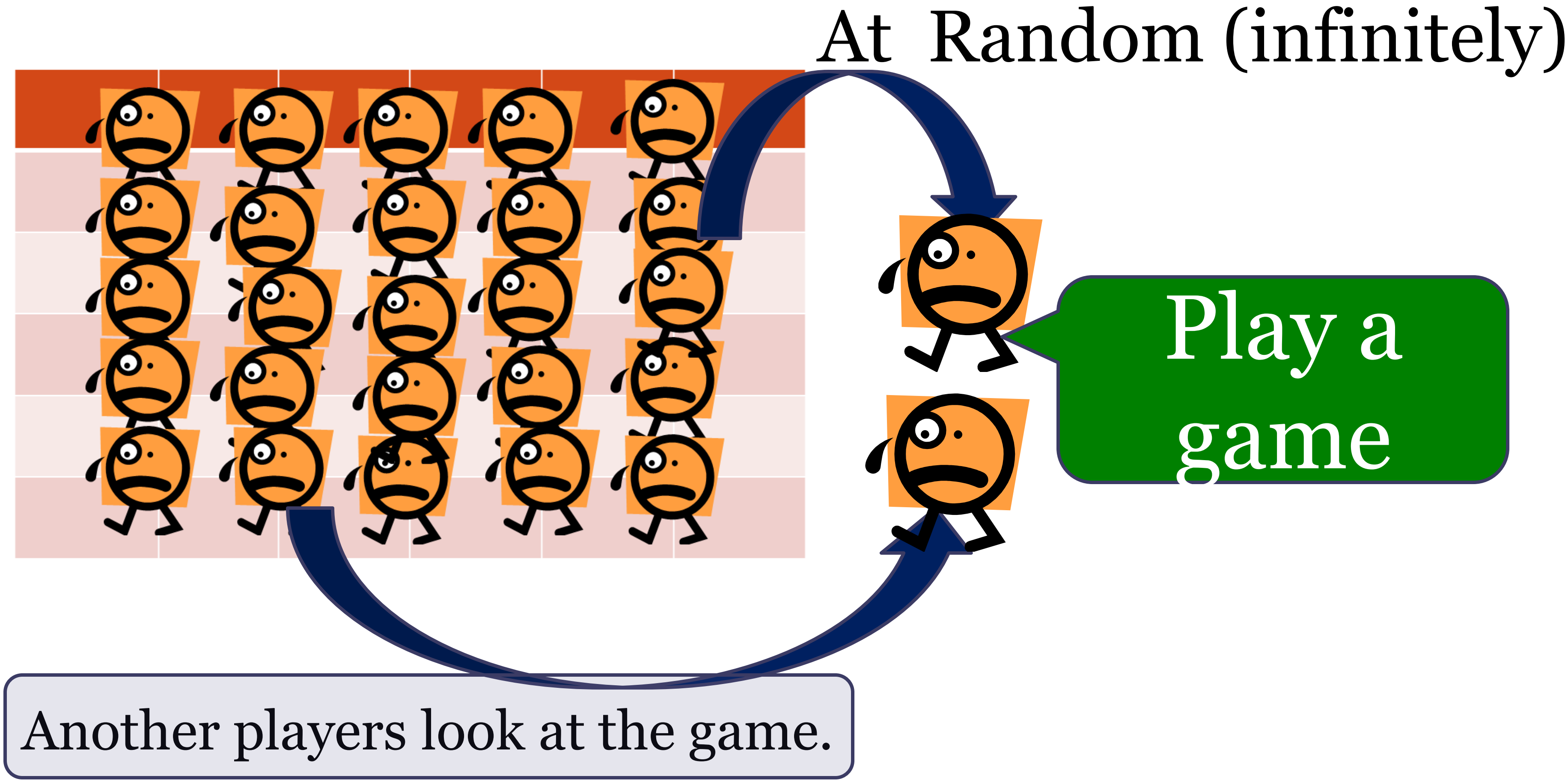
Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)

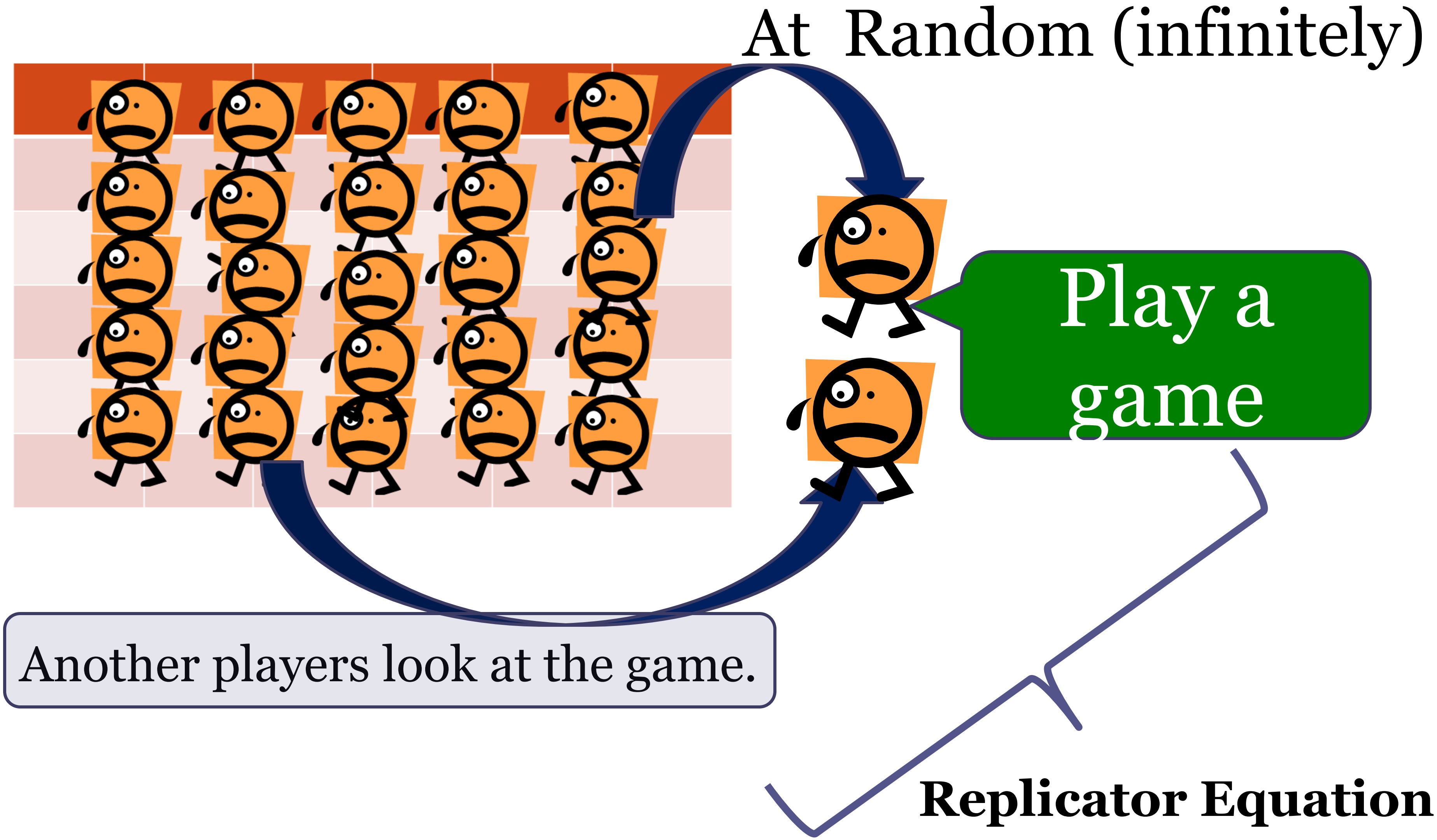


Another players look at the game.

Situation (Traditional Evolutionary Game Theory)



Situation (Traditional Evolutionary Game Theory)



REVIEW: Replicator Equation

REPLICATOR EQ. $\dot{x}_i = x_i\left((Ax)_i - x \cdot Ax\right), i = 1, \cdots, n.$

If the player's payoff from the outcome *i* is greater than the expected utility $x\,Ax$, the probability of the action *i* is higher than before.

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Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \quad \cdots (*)$$

Classification

- (I) Non-dilemma: a > 0. b < 0, ESS : one
- **(II)** Prisoner's dilemma **:** a < 0. b > 0, ESS :one
- **(III) Coordination** : a>0,b>0, ESS two
- **(IV) Hawk-Dove** : a<0,b < 0, ESS one (mixed strategy)

1 \ 2	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

EVOLUTIONARY STABLE STRATEGY (ESS)

DEF.： Weibull(1995): $x \in \Delta$ is an ***evolutionary stable strategy (ESS)*** if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$.

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INTERPRETATION： incumbent payoff (fitness) is higher than that of the post-entry strategy

（ESS : ①the solution of the Replicator equation + ② asymptotic stable.）

PROPOSITION

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Asymptotic Stable Conditon