MEE Seminar Oct 13. 2009 14:40~16:10 (Meiji Univ. (Ikuta) A207)

Option Market Analysis with Evolutionary Game Theory

(進化ゲーム理論を用いたオプション市場分析)

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http://kikkawa.cyber-ninja.jp/

This Talk (本報告)

- ANALYZES the financial market with **Evolutionary game theory**.(金融市場において、進化ゲーム理論を用いて、分析する)
- **PREDICTS** the next market state with Stability Analysis.(安定性の概念を用いることによって、次期の市場の状態を予測する)
- EXAMINES the **Real Market** (Future Market) to apply this model. (構築したモデルをもとに、実際の市場を分析する)
- MOVIE (avi)

OUTLINE

1. Introduction (Motivation)
2. Related Literatures and Review
3. Model
4. Apply this model to the Future market (Nikkei 225)
5. Option Market (Black-Sholes Eq.)
6. Summary (Future works)

1. INTRODUCTION

Motivation　(動機)

- For **Practical Use** (実務への応用を目指して)

  More **Detail** (より具体的で), More **Useful** (より役に立つ)

→We construct the market from the **order book**. (板情報に着目)

- ＋Use the “Real Data” (実際のデータを取り扱う)

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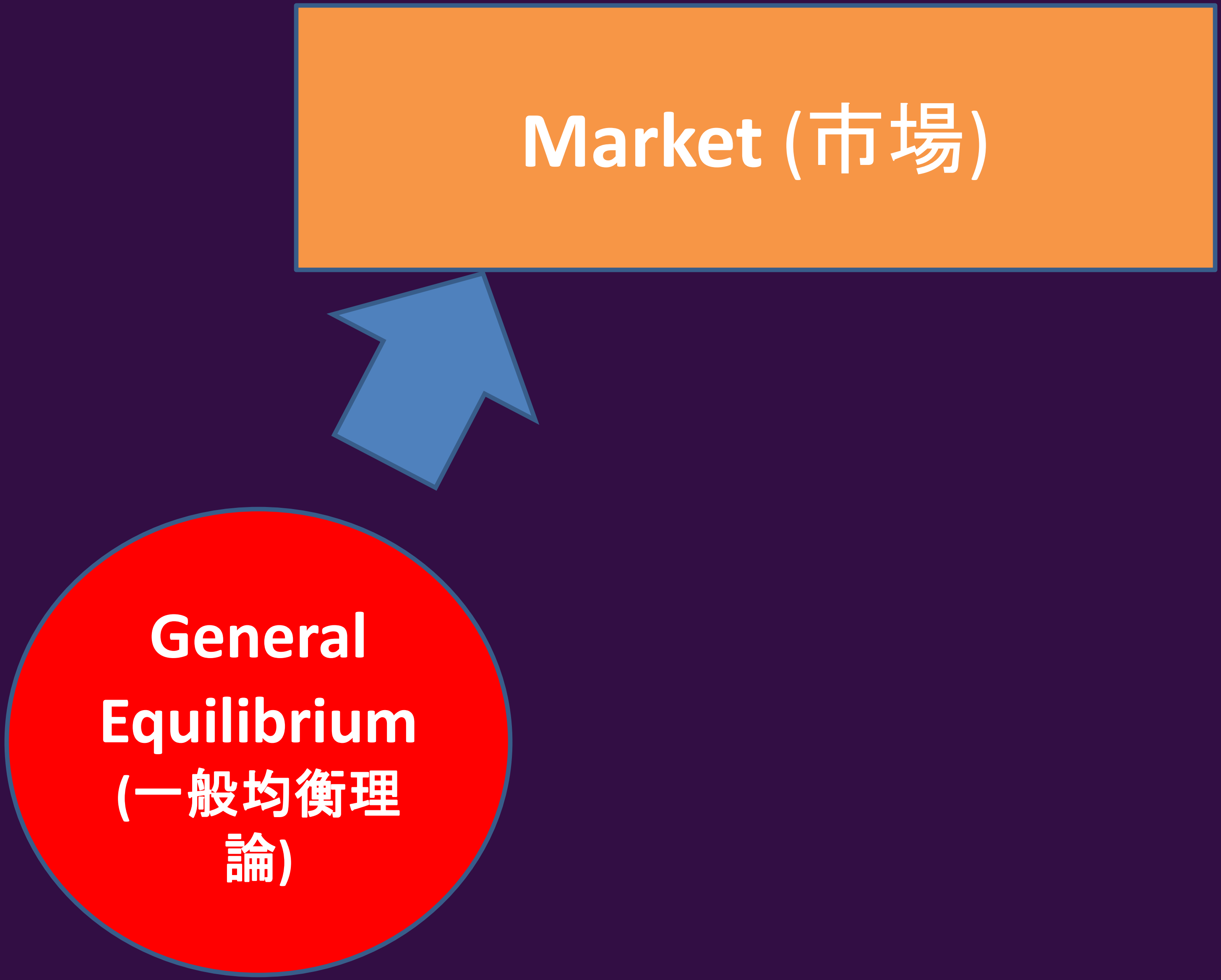
Research Fields (this study)

Market (市場)

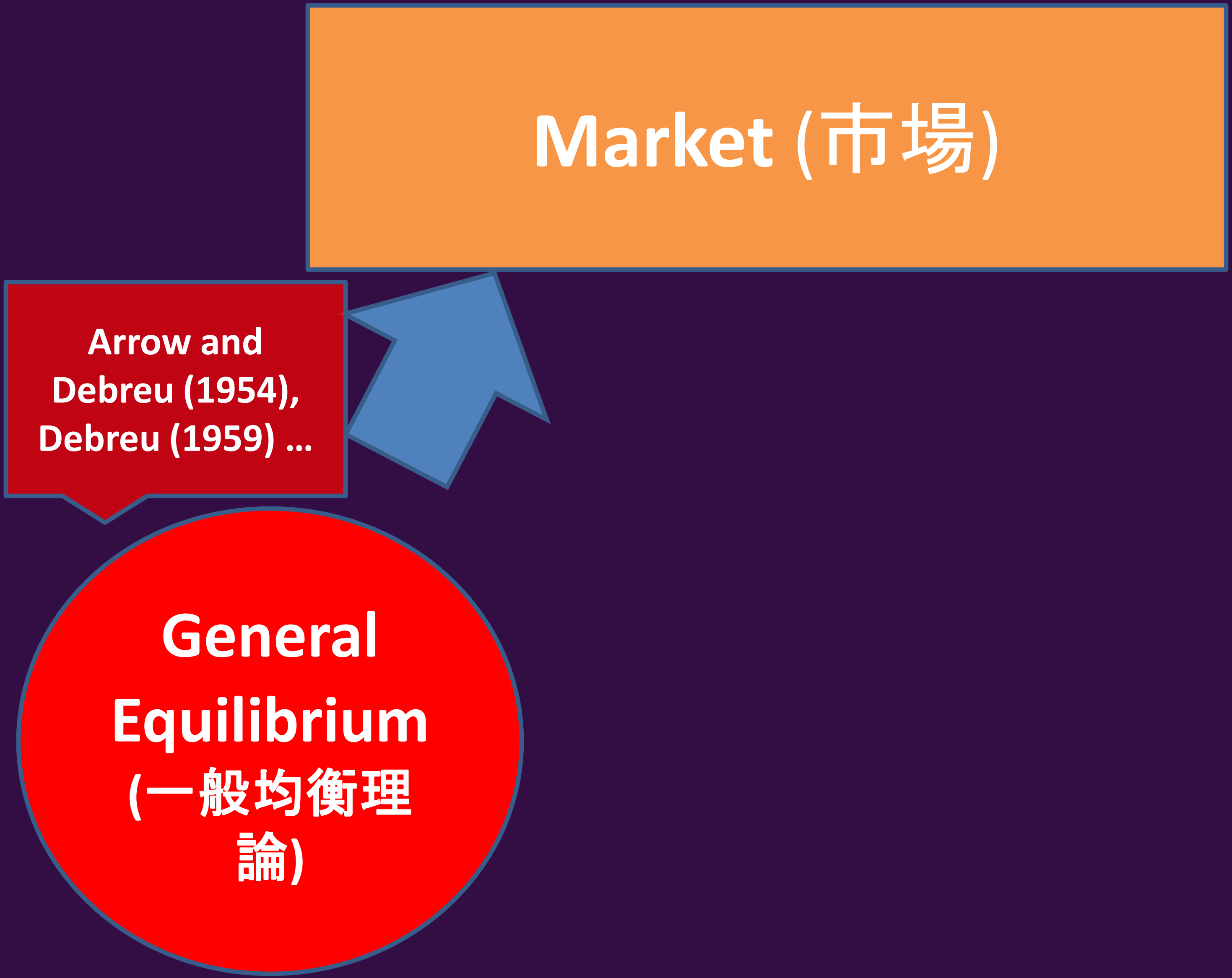
Research Fields (this study)



Market (市場)

General Equilibrium (一般均衡理論)

Research Fields (this study)



Market (市場)

Arrow and Debreu (1954), Debreu (1959) …

General Equilibrium (一般均衡理論)

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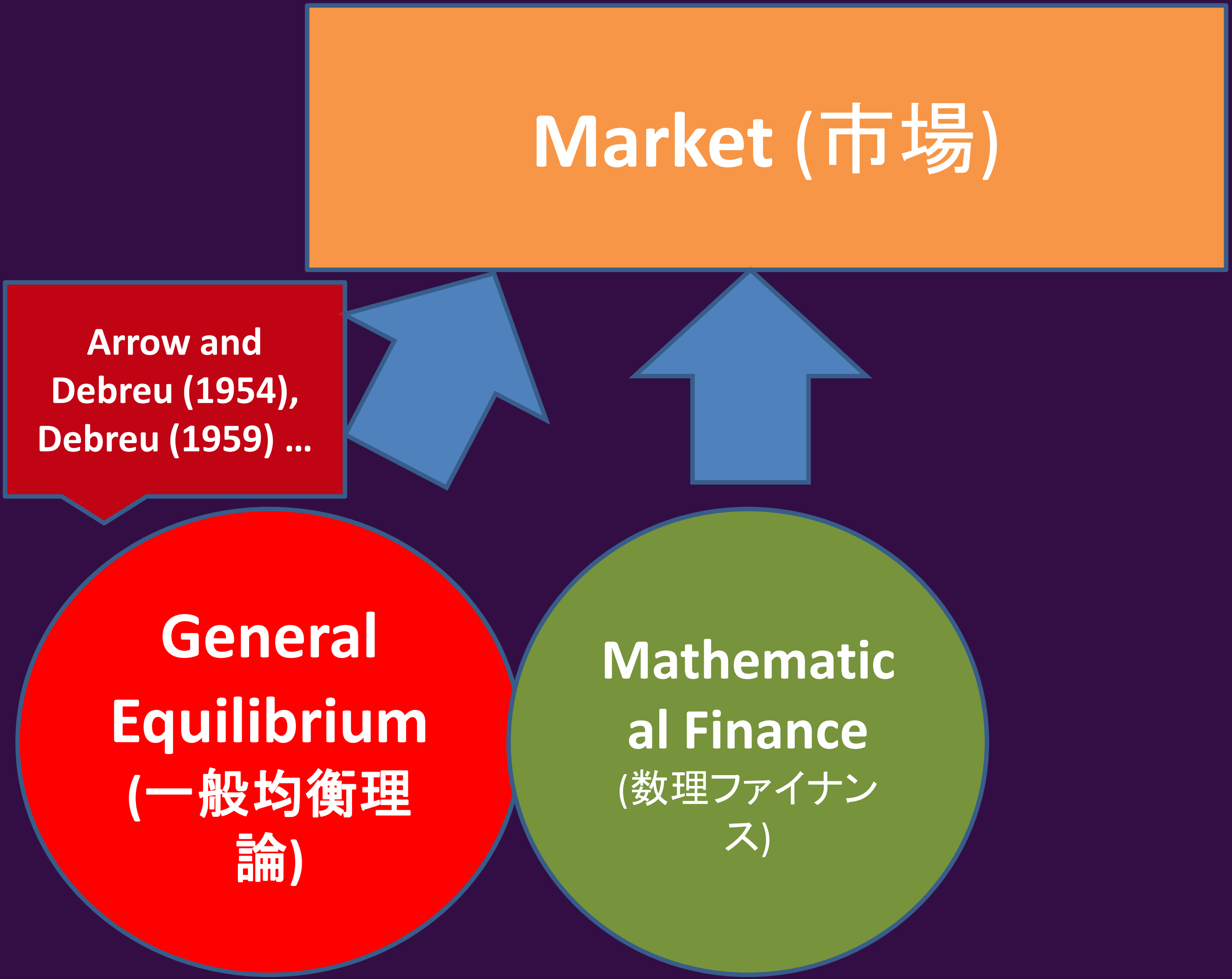

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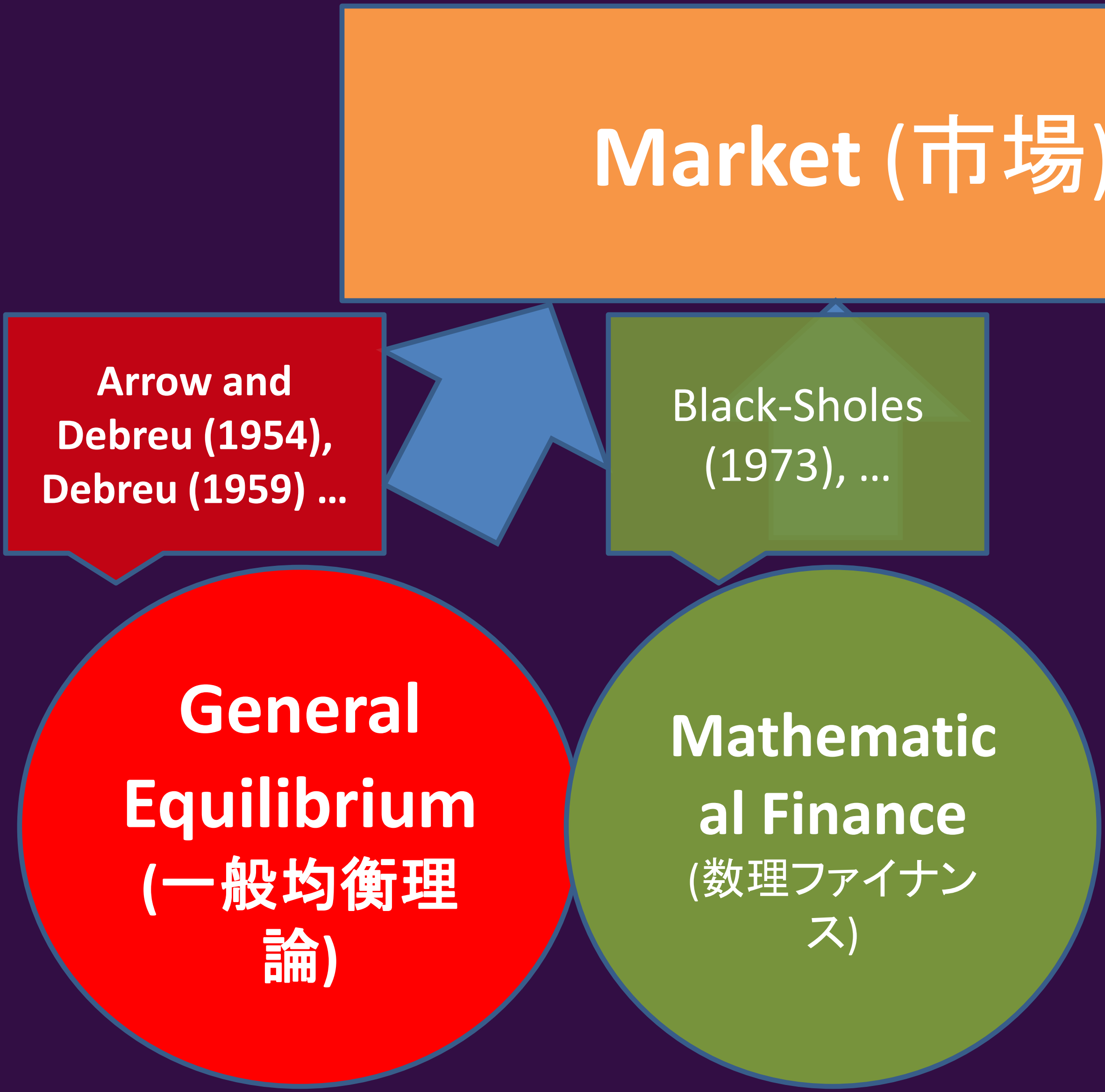

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Arrow and Debreu (1954), Debreu (1959) …

Black-Sholes (1973), …

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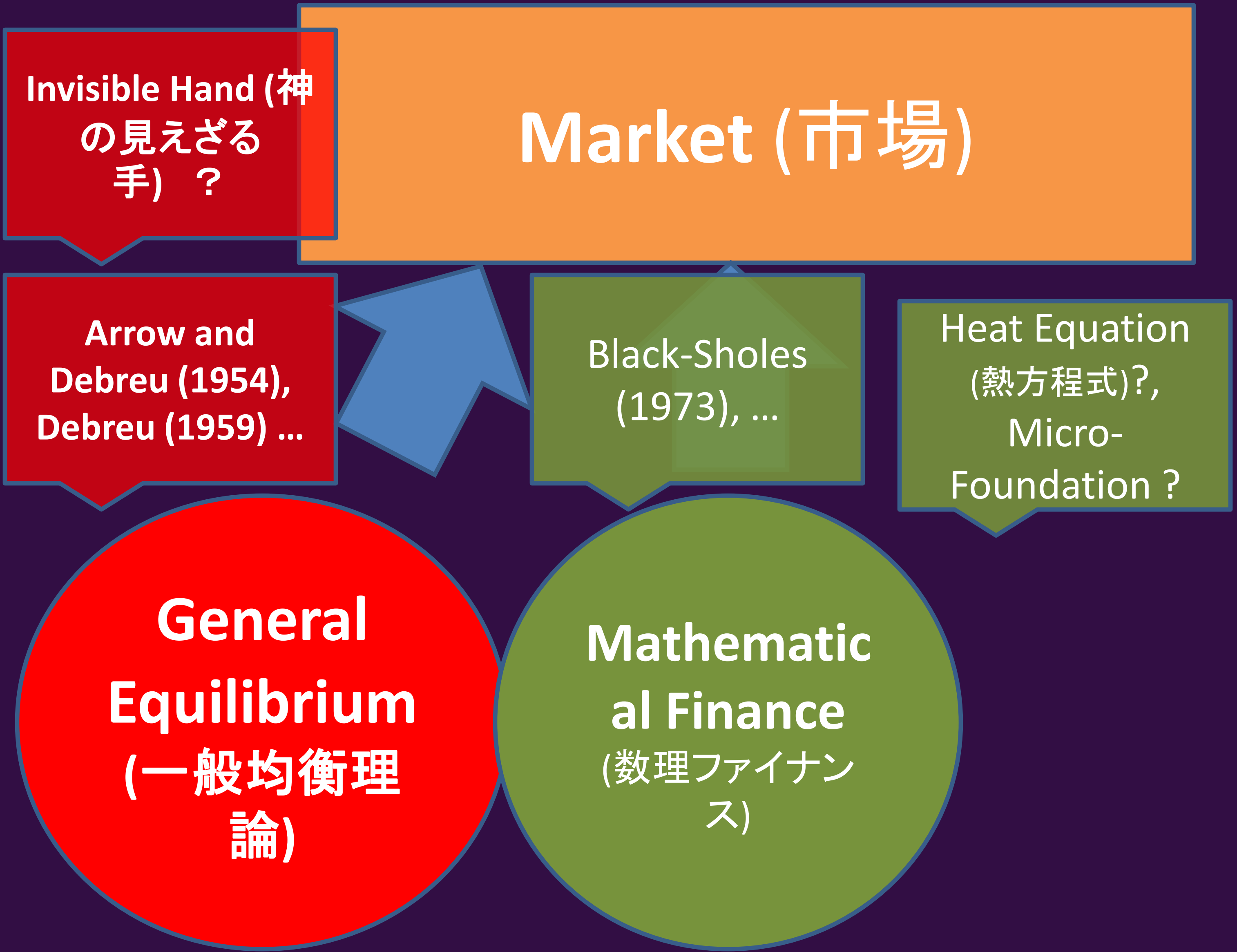


Market (市場)

Invisible Hand (神の見えざる手) ？

Arrow and Debreu (1954), Debreu (1959) …

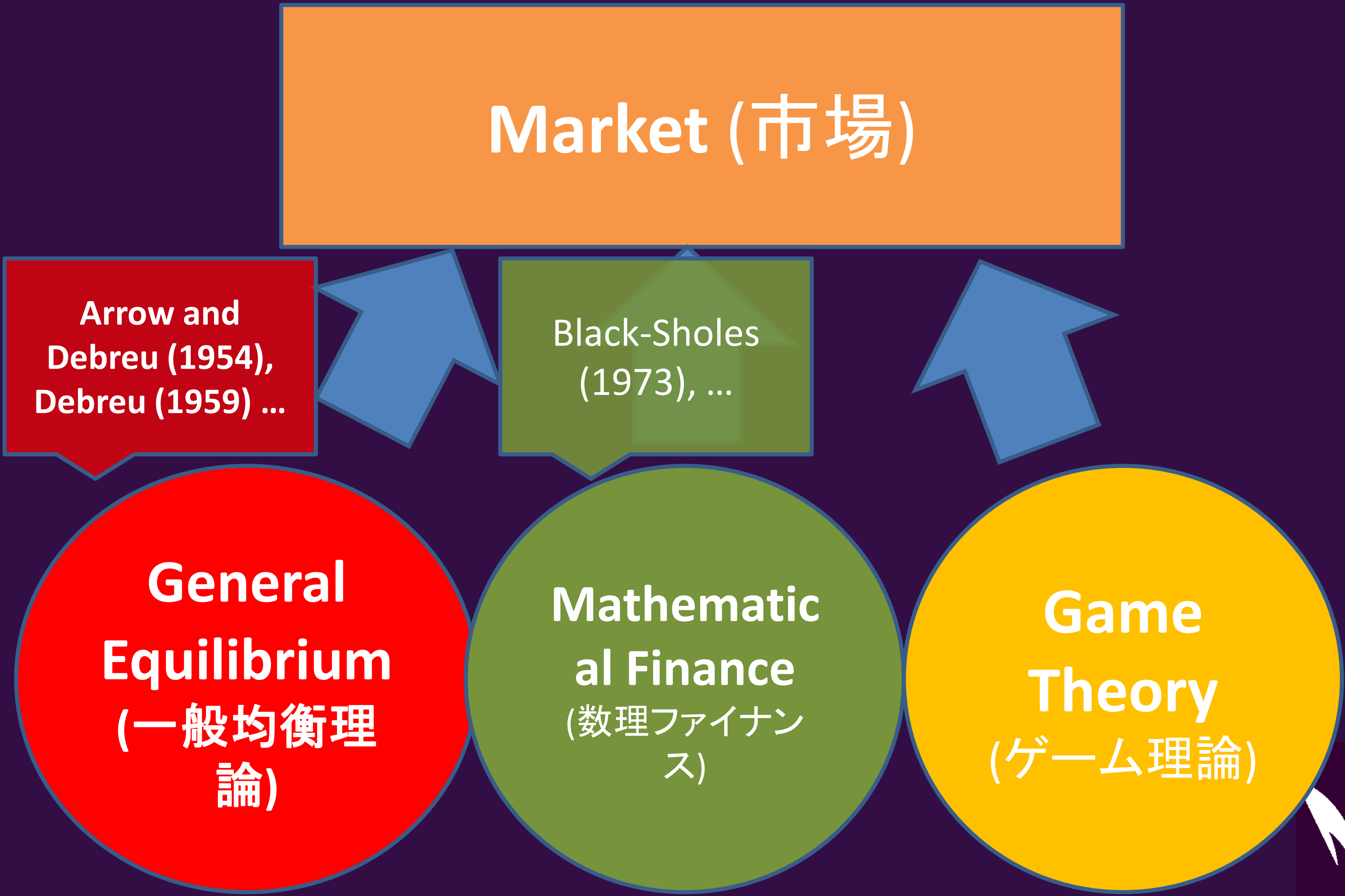
Black-Sholes (1973), …

Heat Equation (熱方程式)?, Micro-Foundation ?

General Equilibrium (一般均衡理論)

Mathematical Finance (数理ファイナンス)

Research Fields (this study)



Market (市場)

Arrow and Debreu (1954), Debreu (1959) …

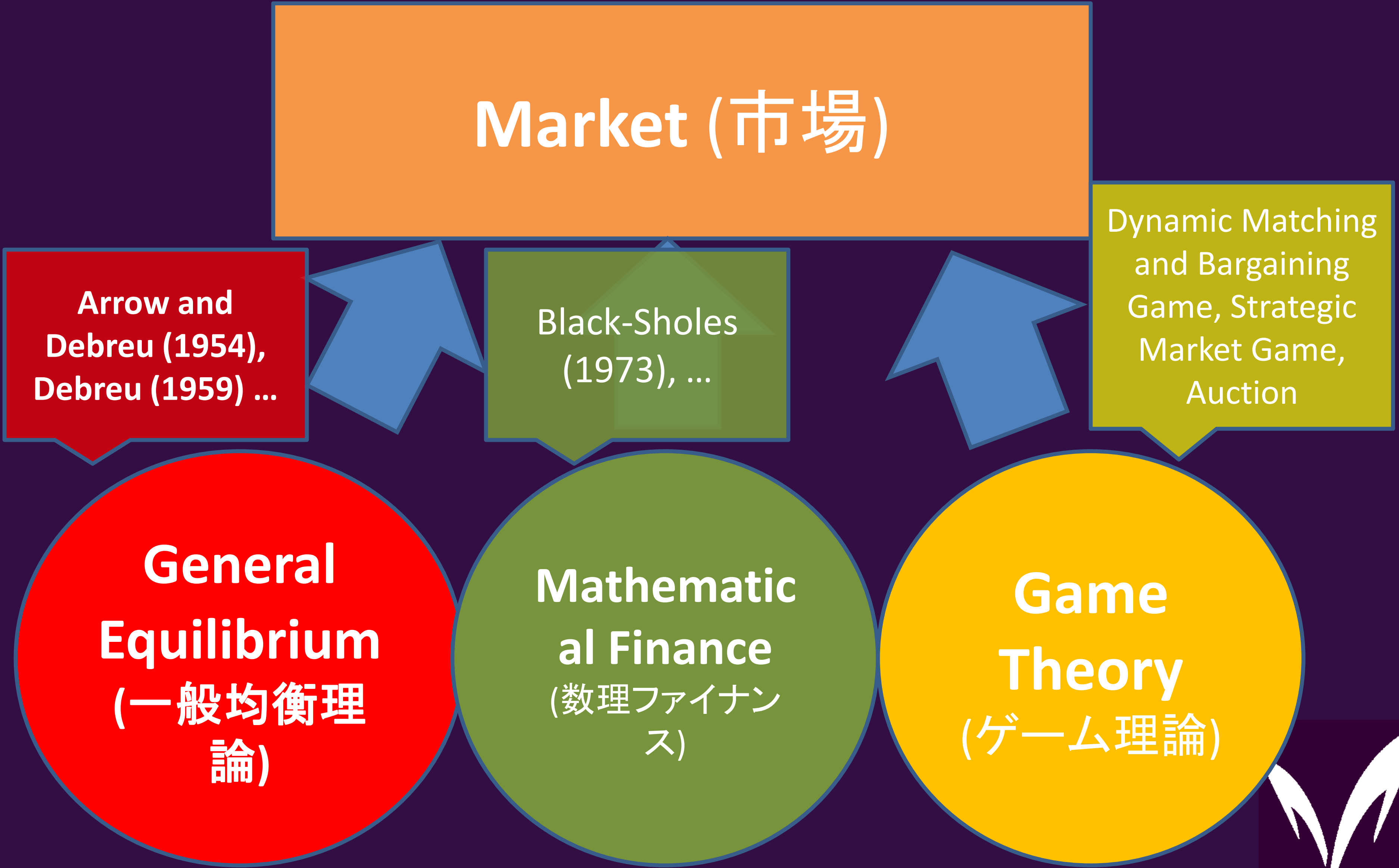
Black-Sholes (1973), …

General Equilibrium (一般均衡理論)

Mathematical Finance (数理ファイナンス)

Game Theory (ゲーム理論)

Research Fields (this study)



Market (市場)

Arrow and Debreu (1954), Debreu (1959) …

Black-Sholes (1973), …

Dynamic Matching and Bargaining Game, Strategic Market Game, Auction

General Equilibrium (一般均衡理論)

Mathematical Finance (数理ファイナンス)

Game Theory (ゲーム理論)

2. RELATED LITERATURES AND PRELIMINARIES

Related Literatures(先行研究)

- **Micro Structure**

  Roughly speaking, we analysis the agents' behavior from the financial data.(データから市場参加者の行動を探る)
- Method: Evolutionary Game Theory (進化ゲーム理論)

→ Esaley and O'hara (1992) **[HP]**

- **Applied Evolutionary Game Theory**
- **川西 (2008) [amazon]**

PUCK (Econophysics)

- **PUCK** ･･･Potential Unbalanced Complex Kinetics PUCK 動画 (real player)
- USE: Discrete Langevin Equation

→Derive the potential energy in the market and Predict the next states from Potential Function’s Form .

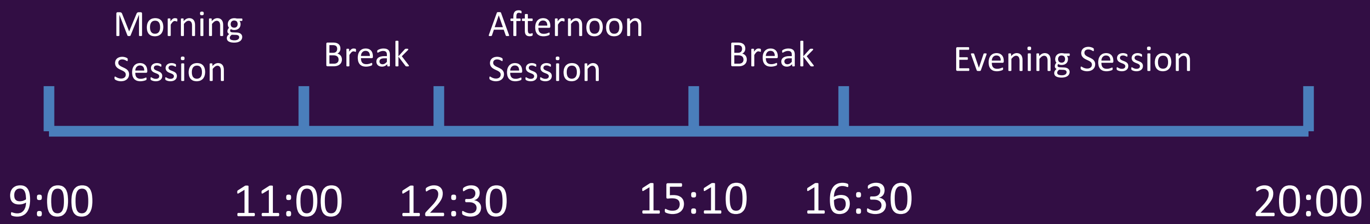
- Takayasu, et al. (2006) **[HP]**, Yamada, et al. (2008) **[HP]**, Yamada, et al. (2009) **[HP]**

How are stock prices determined ?

- Stock prices are determined by two methods, the *Itayose(板寄せ)* and *Zaraba(ザラバ)* methods. The *Itayose* method is mainly used to decide opening and closing prices; the *Zaraba* method is used during continuous auction trading for the rest of the trading session.

→ The stock price are determined by **Rule**.

[Nikkei 225 Future Market(日経225先物)] [1day]



Morning Session

Break

Afternoon Session

Break

Evening Session

9:00 11:00 12:30 15:10 16:30 20:00

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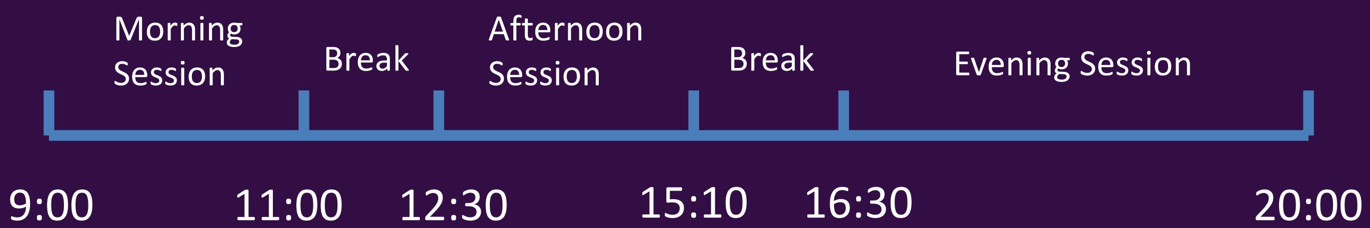


Two Principles (2つの原則)

1) **Price Priority** (価格優先の原則)

means that the lowest sell and highest buy orders take precedence over other orders.

2) **Time Priority** (時間優先の原則)

means that among orders at the same price, the order placed earliest takes precedence.

Offer(sell)	Price	Bid (buy)
A 3000(5), C 4000(4)	502	early ← → late
D 10000(3), E 9000(2), F 5000(1)	501	
	500	H 80000(1), B 1000(2), J 4000(3)
late← → early	499	H 1000(4), B 150000(5)

The Order Book (板情報)

（Offer(sell)）	Price	（Bid (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

In this case, opening price is 500 or 501.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

Assume: opening price is 500.

（Offer(sell)）	Price	（Bid (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

- The market orders of 4000 shares to buy and 6000 shares to sell are matched, leaving sell orders of 2000 shares.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

Second Step

（Offer(sell)）	Price	（Bid (buy)）
2000	Market orders	
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- The market sell orders of 2000 shares and sell orders 6000 shares at limit prices of 499 or less are matched with the buy orders of 8000 shares at limit prices of 501 or more. Thus far, 12000 shares have been matched in total.

Third Step

（Offer(sell)）	Price	（Bid (buy)）
Market orders		
8000	502	
20000	501	
4000	500	10000
	499	8000
	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- Finally, the sell orders of 4000 shares at a limit price of 500 are matched with the buy orders of 10000 shares at a limit price of 500. Although this still leaves buy orders of 6000 shares at 500.

Fourth Step

（Offer(sell)）	Price	（Bid (buy)）
	Market orders	
8000	502	
20000	501	
	500	6000
	499	8000
	498	30000

- Thus the opening price is determined at 500 and transactions of 16000 shares are completed at 500.

The stock price and the trade depends on the **order book**. (価格は板情報によって決定する。)

3. MODEL

Model (モデル)

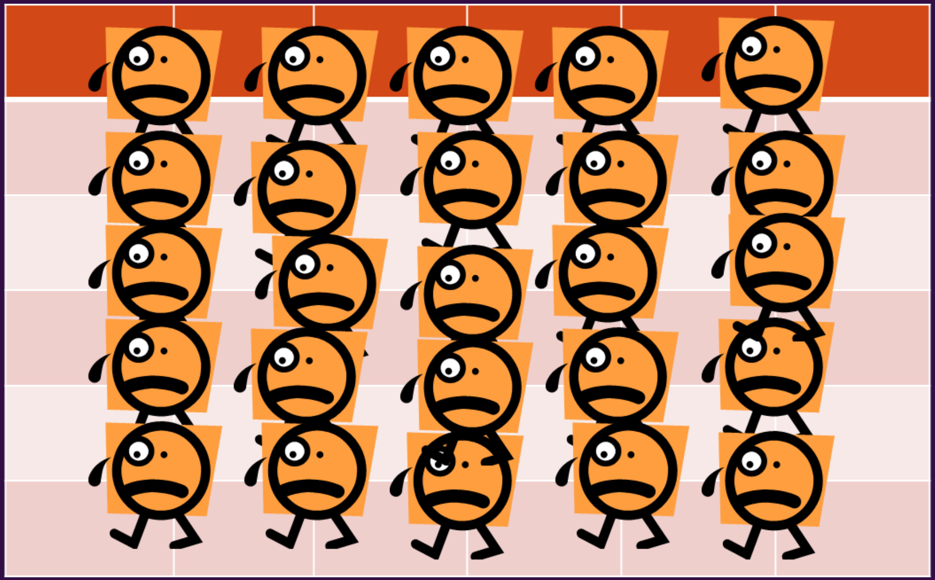
- **Players**… large population : seller and buyer, potentially (大人数の潜在的な売り手と買い手)

Seller and Buyer trade an asset.

- **Goods** (財) … 1財
- **Strategy** (戦略)… **n** (<∞) 個

Here, the price : how much do you buy or sell an asset. (ここでは購入、売却価格)

Situation (Traditional Evolutionary Game Theory)



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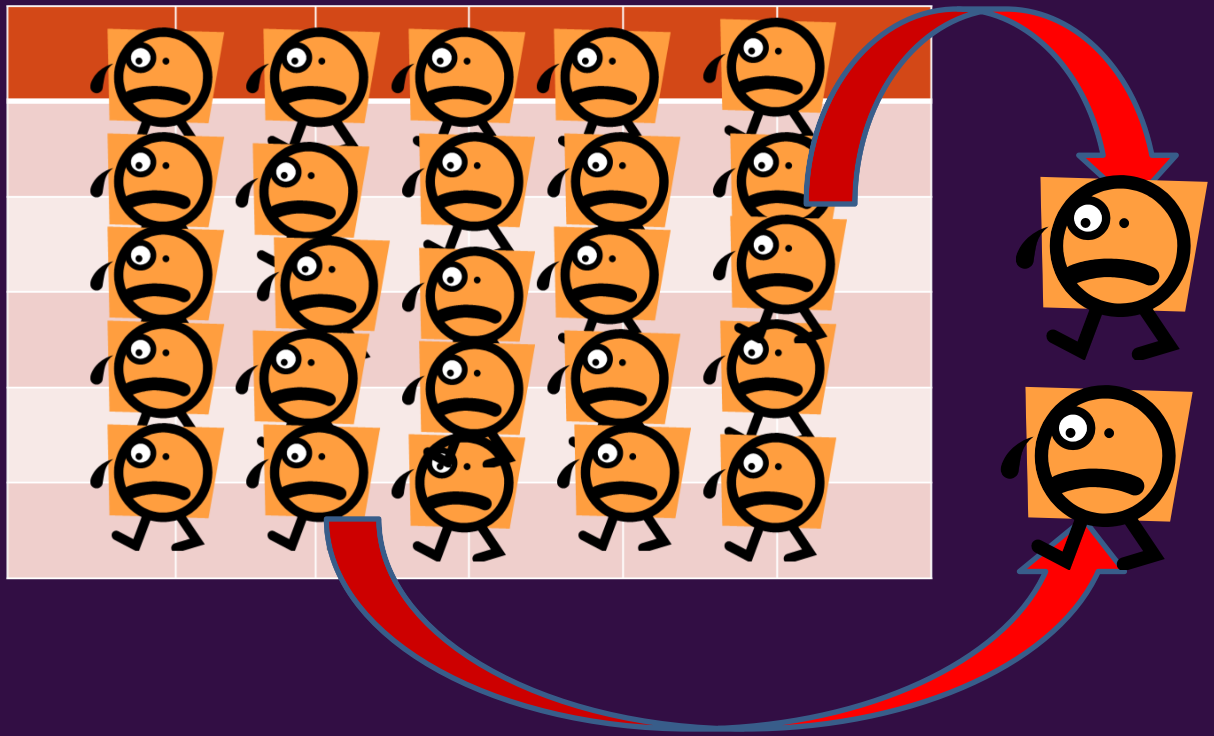
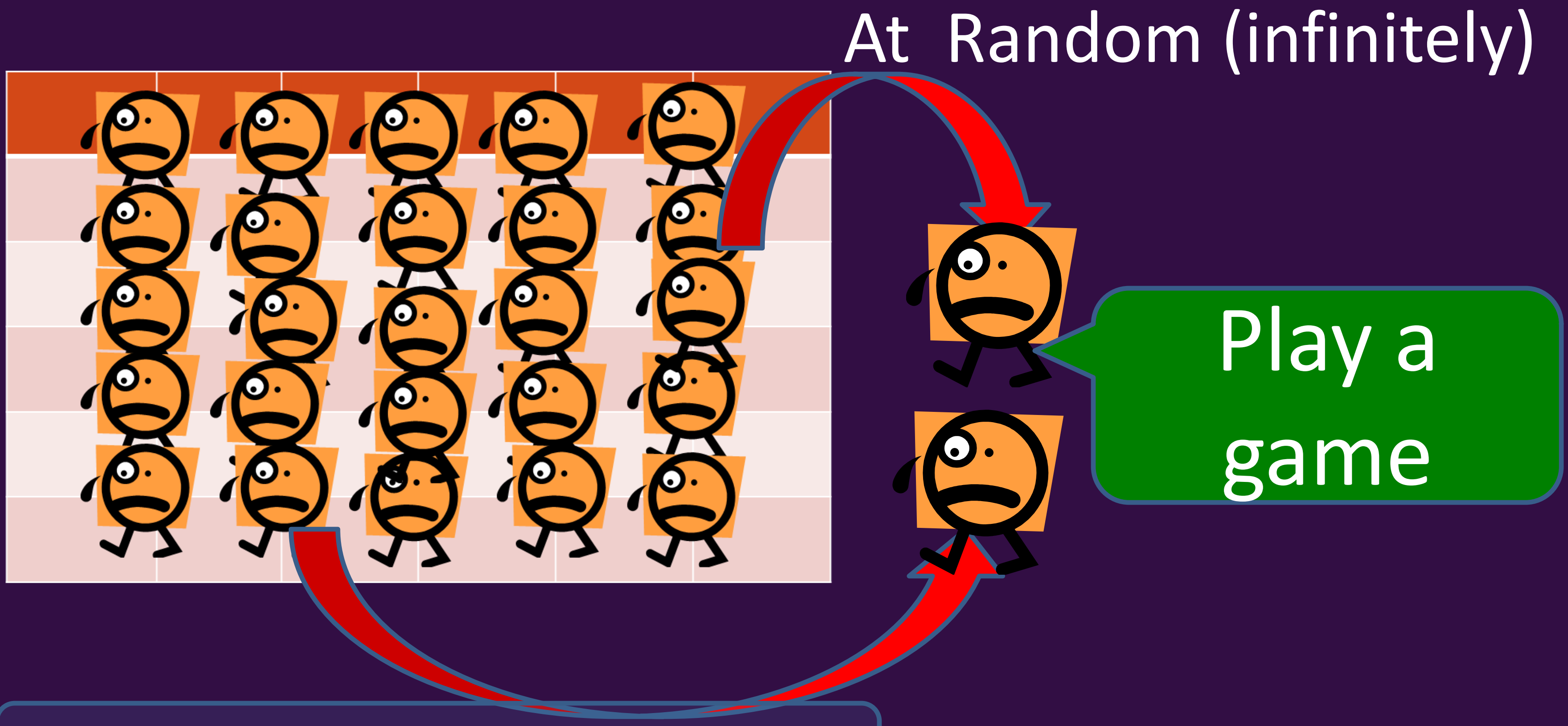


At Random (infinitely)

Situation (Traditional Evolutionary Game Theory)

Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

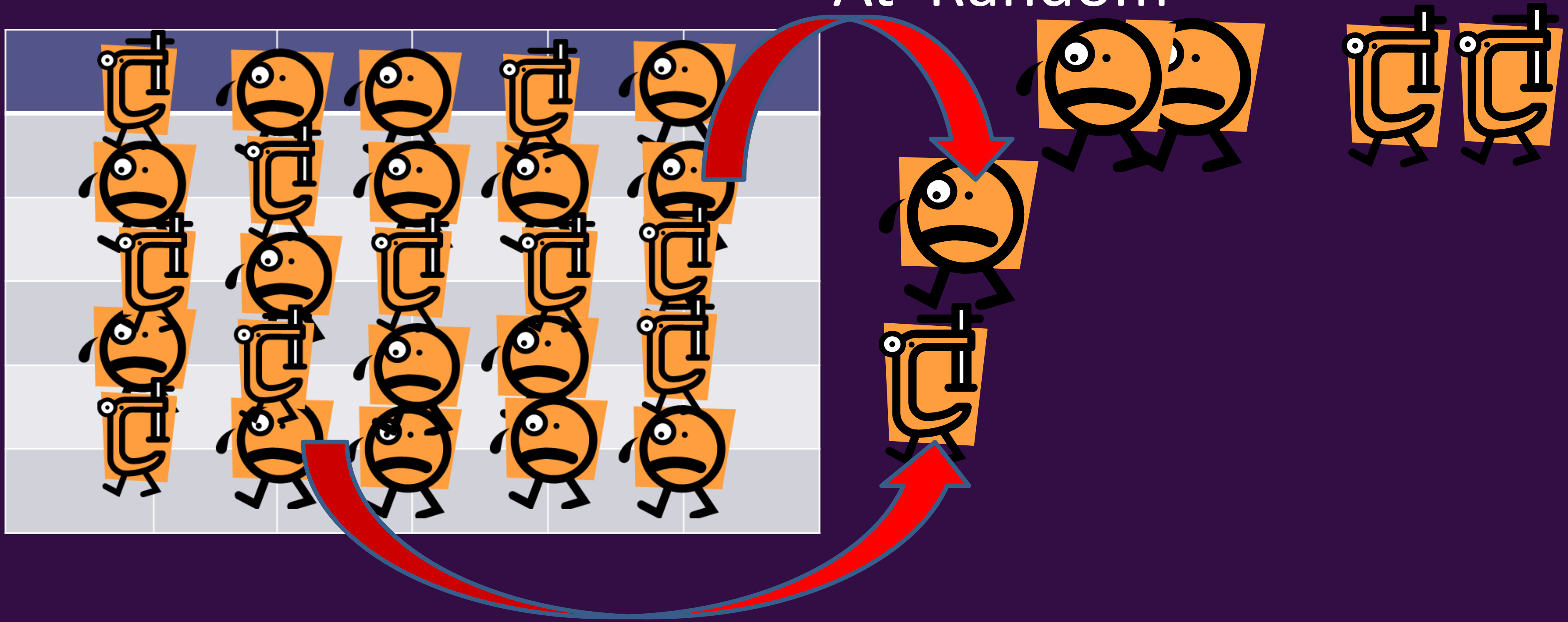


Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

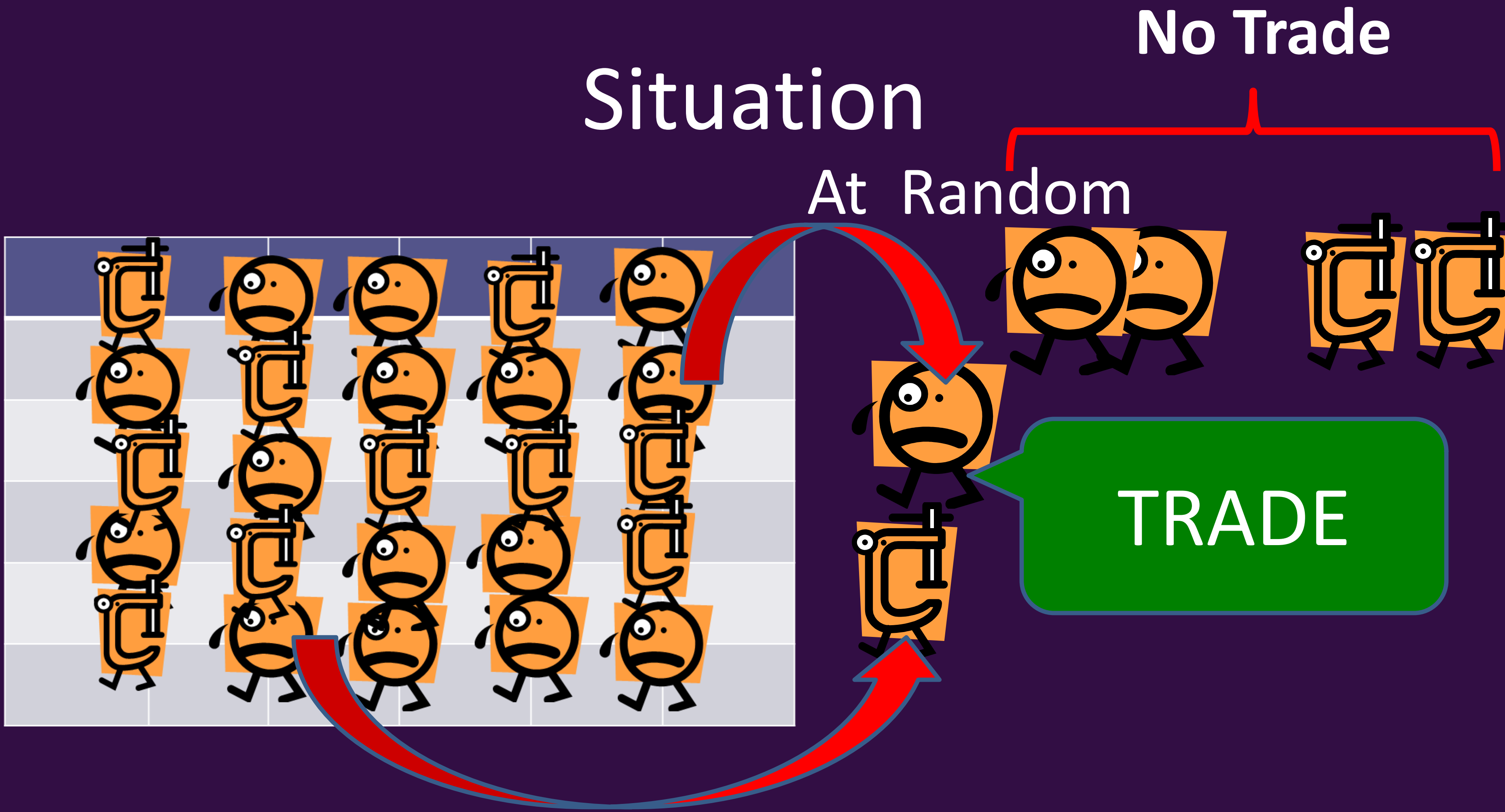
Situation (two types players)

Situation

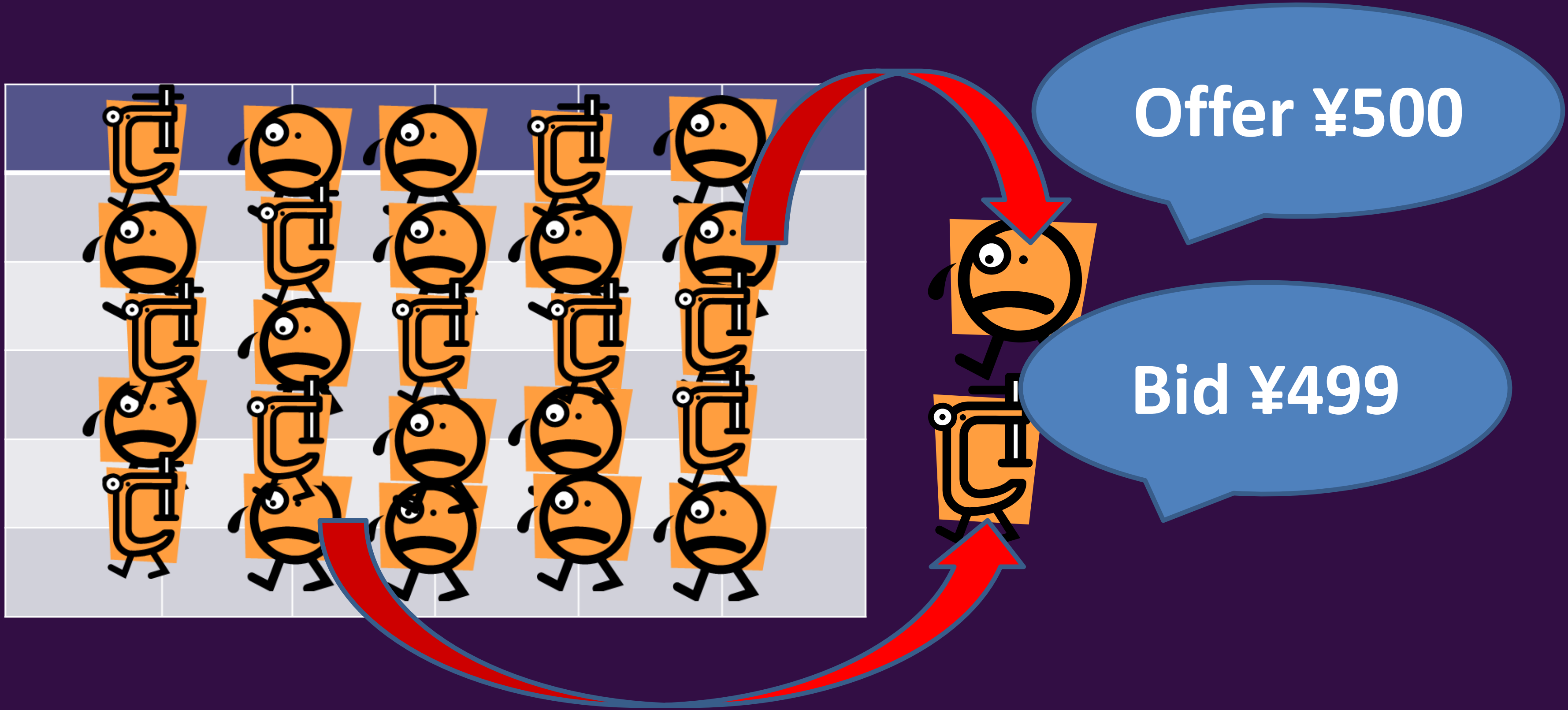


At Random

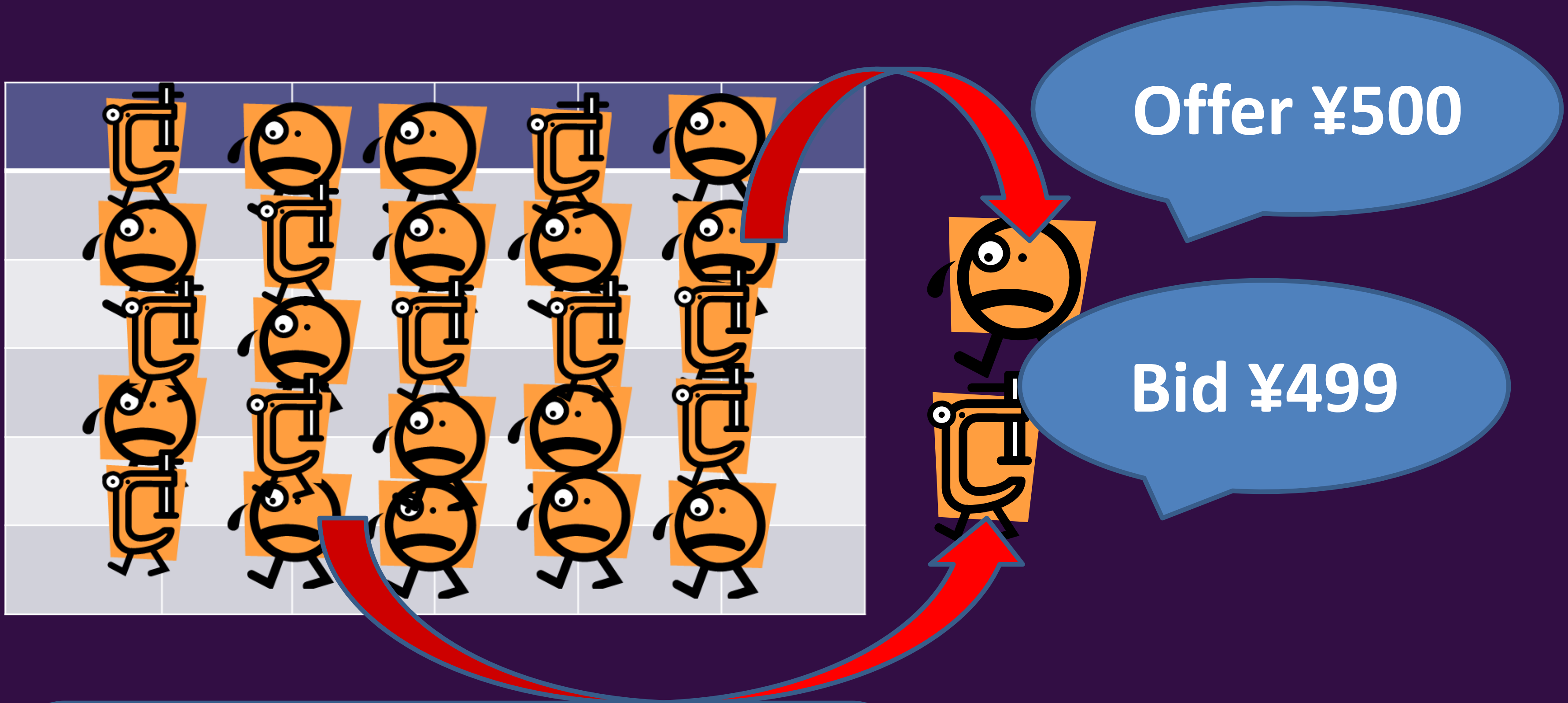
Situation



Situation

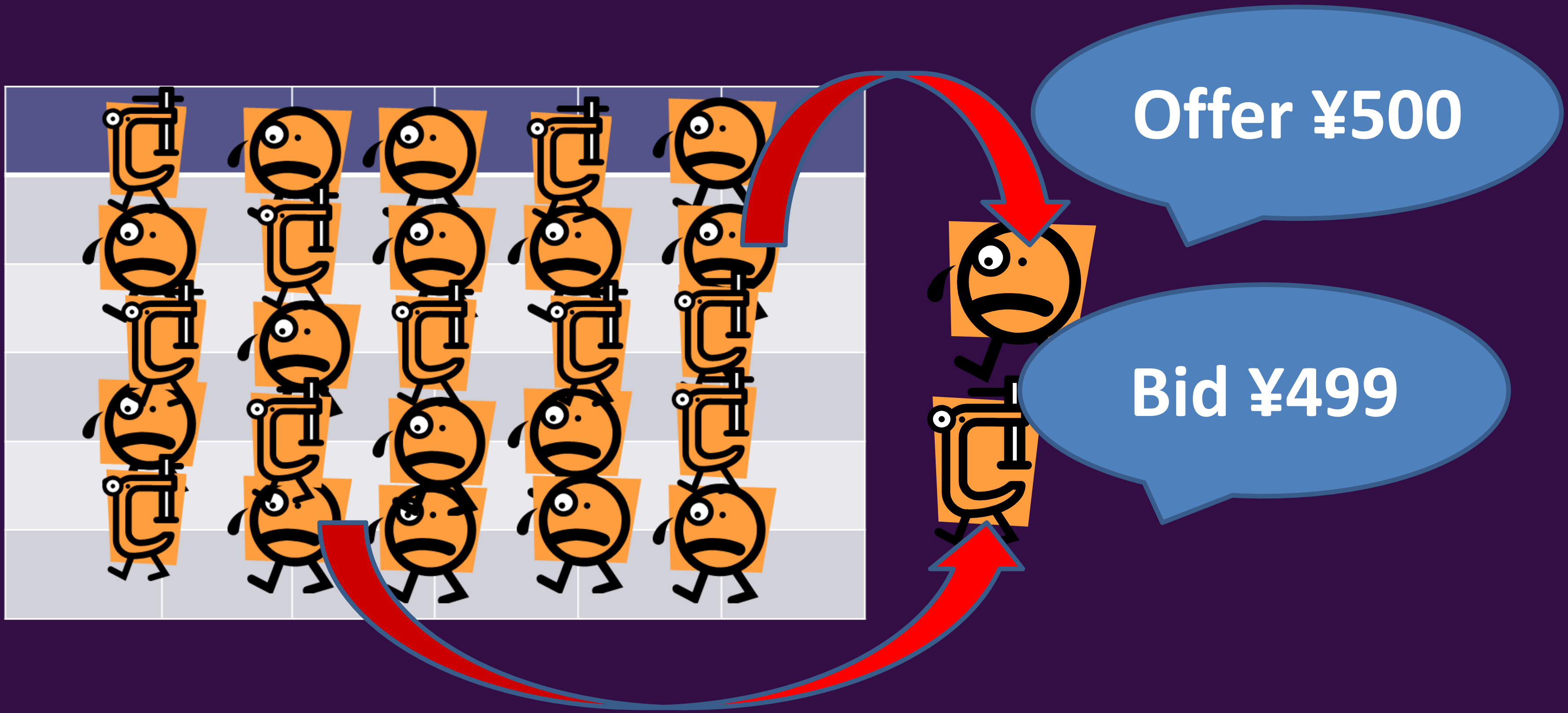


Situation



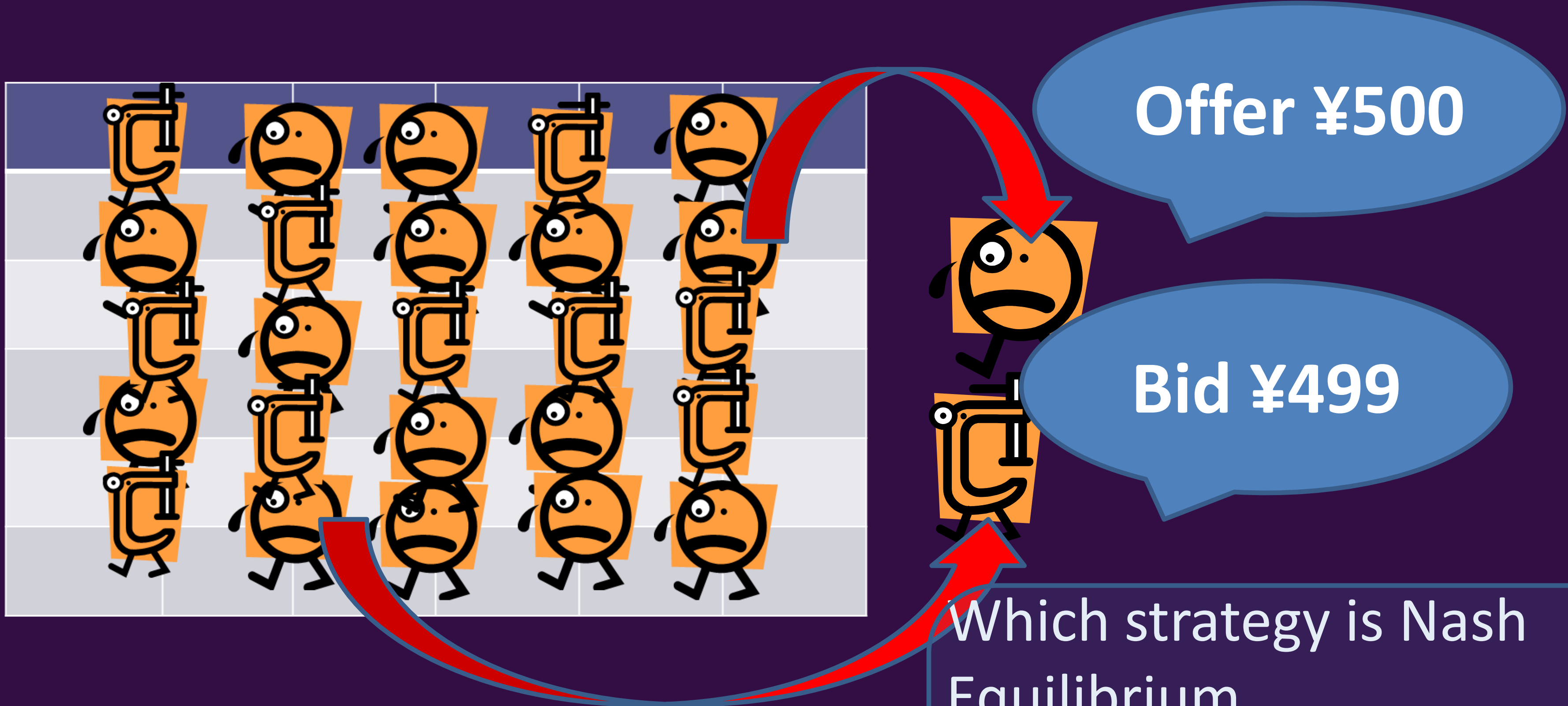
Stock Exchange which take account of the order book decides the trade’s contract. (取引所が板情報をもとに、売買契約を決定する)

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Which strategy is Nash Equilibrium,
if this game is played at infinite ?
(このゲームを無限回仮想的に行うと、どの戦略が均衡となるのか？)

Model (モデル)

- **Payoff (利得) … Buyer : S(t)-K, Seller : K-S(t)**

where S(t) : current stock price, Brownian Motion. K: strike price (行使価格)

- **Replicator Equation**

$$\frac{dx_i(t)}{dt} = x_i(t)\left(g_i(t) - \bar{g}(t)\right), g_i(t) = g_i + \zeta(t)$$

$$\frac{dy_i(t)}{dt} = y_i(t)\left(h_i(t) - \bar{h}(t)\right), h_i(t) = h_i + \zeta'(t)$$

where x_i, y_i : the probability of choosing the strategy 1 for each player. g_i, h_i : the payoff when each player chooses the strategy 1.

EVOLUTIONARILY STABLE STRATEGY（ESS)

DEF.: Weibull(1995): $x \in \Delta$ is an ***evolutionarily stable strategy (ESS)*** if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$.

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

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INTERPRETATION：incumbent payoff (fitness) is higher than that of the post-entry strategy

（ESS：①the solution of the Replicator equation + ② asymptotic stable.）

PROPOSITION

PRO.(Bishop and Cannings (1978)): $x \in \Delta$ is evolutionary stable strategy if and only if it meets these first-order and second-order best-reply :

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$$(2.4) \quad u(y,x) \leq u(x,x), \quad \forall y,$$

$$(2.5) \quad \begin{array}{l} u(y,x) = u(x,x) \\ \Rightarrow u(y,y) < u(x,y), \end{array} \quad \forall y \neq x,$$

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Nash Eq.

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$$(2.5) \quad \begin{array}{l} u(y,x) = u(x,x) \\ \Rightarrow u(y,y) < u(x,y), \end{array} \quad \forall y \neq x,$$

Asymptotic Stable Conditon

Two Strategies Case
(戦略の数が2つ) :

- Replicator equation (see next slide)

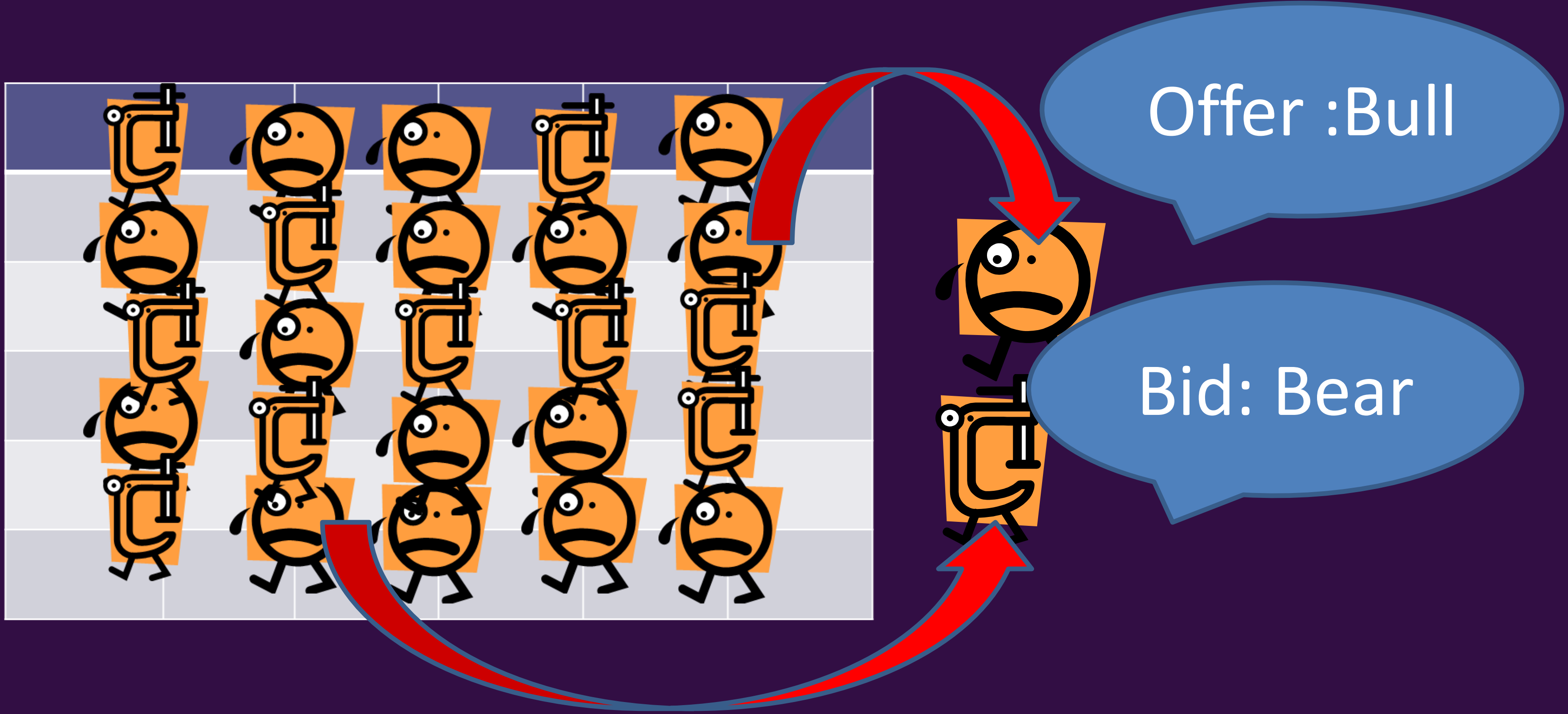
$$\dot{x} = x(1-x)\left\{-b(t)+\big(a(t)+b(t)\big)y\right\},$$

$$\dot{y} = y(1-y)\left\{b(t)-\big(a(t)+b(t)\big)x\right\},$$

where x, y is the probability of choosing the strategy 1, 2 for each player.

		Player 2	
		S1	S2
player1	S1	a(t),-a(t)	0,0
	S2	0,0	b(t),-b(t)

Situation



Replicator Equation

REPLICATOR EQ.

$$\dot{x}_i = x_i\left(\underline{(Ax)_i - x \cdot Ax}\right), i = 1, \cdots, n.$$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before.

Replicator Equation

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If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before. And this equation shows that the probability of the action *i* chosen by another players is also higher than before (**externality**).

Replicator Equation

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Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \quad \cdots(*)$$

Classification

(I) **Non-dilemma**: a > 0, b < 0, ESS : one

(II) **Prisoner's dilemma :** a < 0, b > 0, ESS :one

(III) **Coordination** : a>0,b>0, ESS two

(IV) **Hawk-Dove** : a<0,b < 0, ESS one (mixed strategy)

2

1

	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

Replicator Eq. and Payoff Matrix

- Strategy : Two, Player : Two
- Payoff

$$P^1 = \begin{pmatrix} f_1 & f_3 \\ f_2 & f_4 \end{pmatrix}, P^2 = \begin{pmatrix} g_1 & g_3 \\ g_2 & g_4 \end{pmatrix}$$

- Replicator Equation

$$\dot{y} = y(1-y)\{f_1 - f_2 + x(f_3 - f_4 - f_1 + f_2)\}$$

$$\dot{x} = x(1-x)\{g_4 - g_2 + y(g_3 - g_4 - g_1 + g_2)\}$$

x is the probability of the type 2 player chooses the strategy 2.
y is the probability of the type 1 player chooses the strategy 1.

$$f_1 - f_2 := a, f_4 - f_3 := c, g_4 - g_2 := d, g_1 - g_3 := b$$

Derive

$$\dot{y} = y(1-y)\{a-(a+c)x\}, \dot{x} = x(1-x)\{d-(b+d)y\}$$

Classification

(I) Non-Dilemma, Prisoner's Dilemma :

ac < 0, bd >0,   ESS :1

(II) Coordination :

a>0,b>0,c>0,d>0, ESS :2

(III) Chicken :

a<0,b<0,c<0,d<0, ESS :2

(IV) Matching Pennie:

ab<0,cd<0, ac>0,bd<0, ESS:Mixed

	S1	S2
S1	a,b	0,0
S2	0,0	c,d

Prediction (予測)

- Replicator equation divided by xy(1-x)(1-y) :

$$\dot{x} = -\frac{b(t)}{y} + \frac{a(t)}{1-y}, \dot{y} = \frac{b(t)}{x} - \frac{a(t)}{1-x}.$$

- Discrete the above equations:

$$x(t+\varepsilon) = x(t) - \left(\frac{b(t)}{y} + \frac{a(t)}{1-y} \right) \varepsilon,$$

$$y(t+\varepsilon) = y(t) + \left(\frac{b(t)}{x} - \frac{a(t)}{1-x} \right) \varepsilon.$$

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s1,s2),(s2,s2)

	S 1	S 2
S 1	+, -	0,0
S 2	0,0	0,0

ii) ↓ (Down)

N.E. (s1,s1),(s1,s2)

	S 1	S 2
S 1	0,0	0,0
S 2	0,0	-,+

iii) → (No change)

N.E. (s1,s2)

	S 1	S 2
S 1	+,-	0,0
S 2	0,0	-,+

i),ii),iii) → (s1,s2) (x→1, y→0)

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S 1	0,0	**0,0**
S 2	0,0	-,+

iii) → (No change)

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ii) ↓ (Down)

N.E. (s1,s1),(s1,s2)

	S 1	S 2
S 1	0,0	0,0
S 2	0,0	-,+

Is This OK?

iii) → (No change)

N.E. (s1,s2)

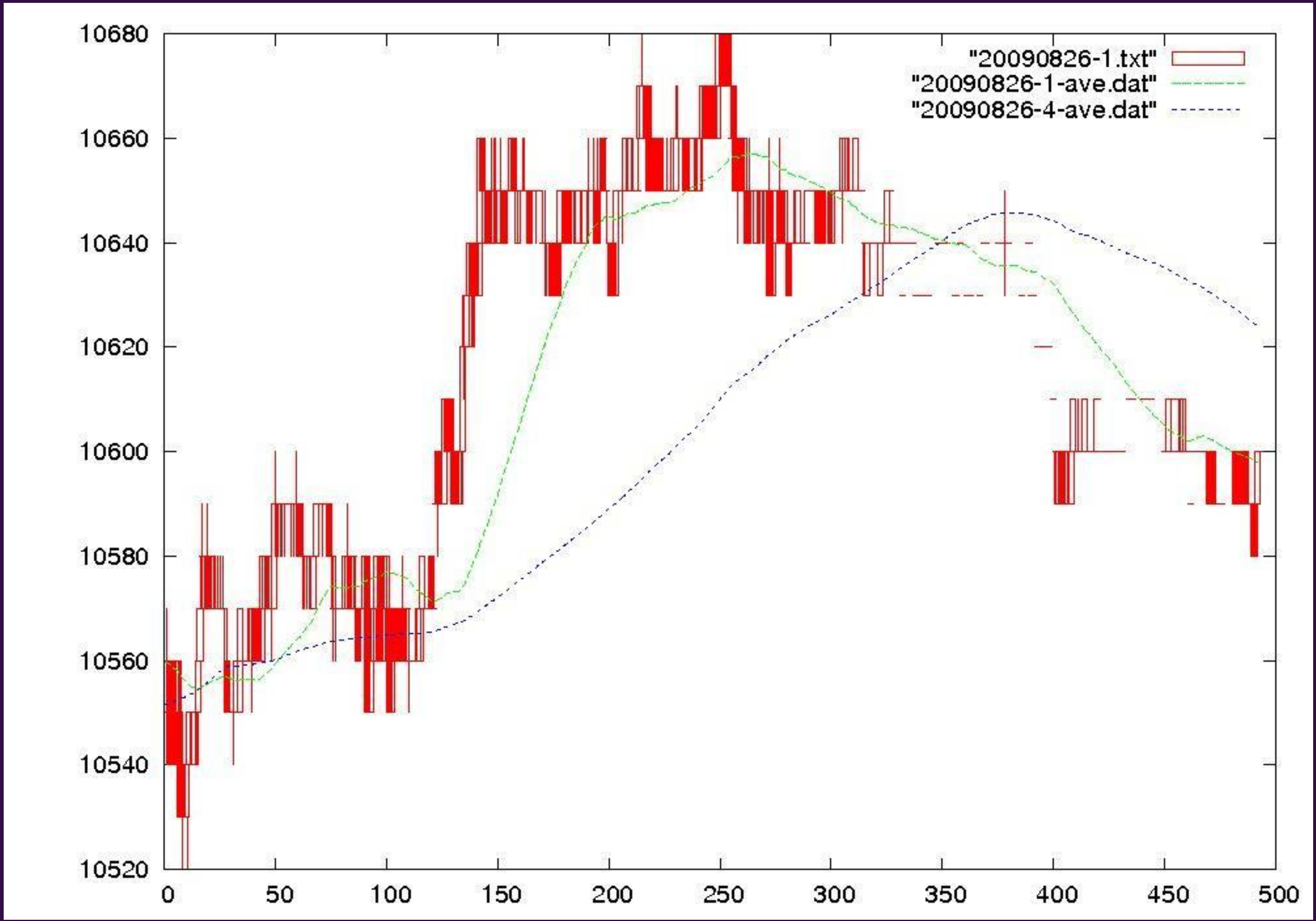
	S 1	S 2
S 1	+,-	0,0
S 2	0,0	-,+

i),ii),iii) → (s1,s2) (x→1, y→0)

4. Application:

NIKKEI 225 FUTURE MARKET
(日経225先物市場)

EX: 20090826


"20090826-1.txt"
"20090826-1-ave.dat"
"20090826-4-ave.dat"
10680
10660
10640
10620
10600
10580
10560
10540
10520
0
50
100
150
200
250
300
350
400
450
500

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

	S 1	S 2
S 1	+, -	0,0
S 2	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

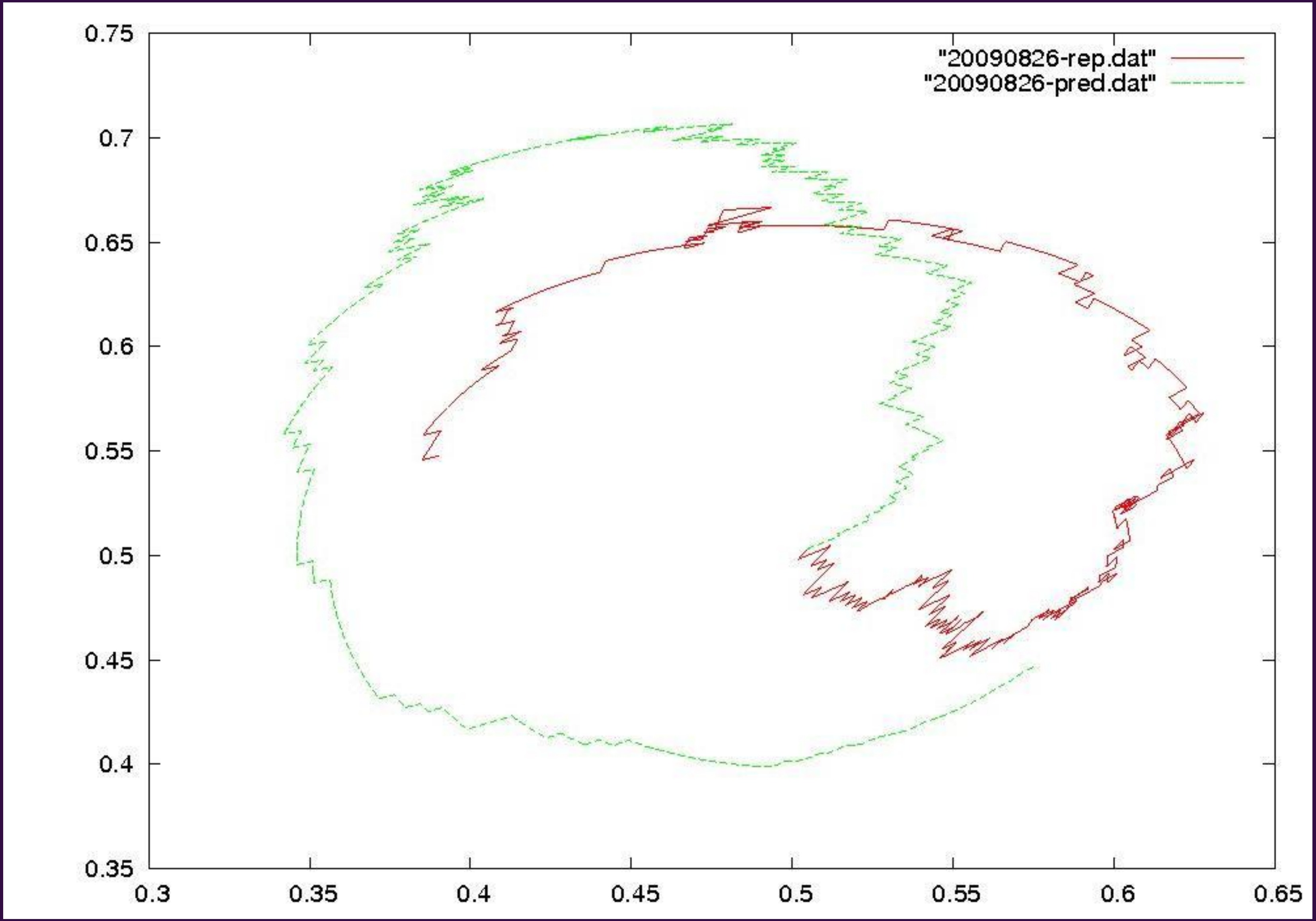
	S 1	S 2
S 1	+,+	0,0
S 2	0,0	-,+

iii) → (No change)

N.E. Mixed Strategy.

	S 1	S 2
S 1	-,+	0,0
S 2	0,0	-,+

EX: 20090826


"20090826-rep.dat"
"20090826-pred.dat"
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65

5. Application:

OPTION MARKET
(オプション市場)

OPTION

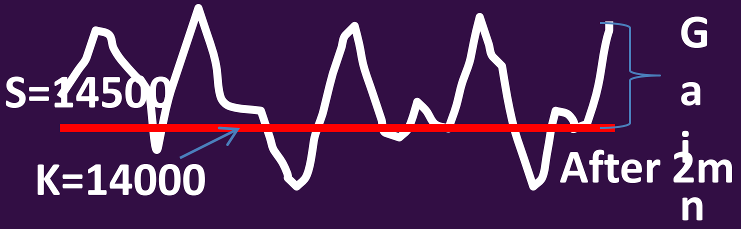
- Option gives the buyer the right, but not the obligation, to buy or sell an asset at a set price on or before a given date.(オプション とは売買を行う権利)　Call(put) is the right which the buy(sell) an asset (買い付ける権利を**コール(call)**、売り付ける権利を**プット(put)**と言います)
- Stock Index Options are traded in Japan are **Nikkei 225 option**、**Nikkei 300 option**(Osaka Securities Exchange)、**TOPIX option**(Tokyo Stock Exchange)

[DATA] Osaka Securities Exchange

http://www.nippo.ose.or.jp/pdf.html

- **Black-Sholes's Formula**･･･value the price of European Option

PROBLEM

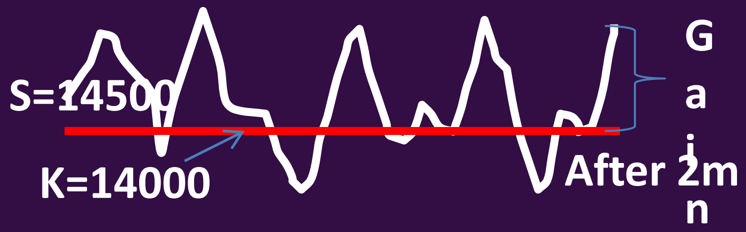


- Value the European Call Option under the following. (次のヨーロピアン･コールオプションの価格を求めよ)

  Current Stock Price(現在の株価) S:=¥14500, Strike Price(権利行使価格) K:=¥14000, Maturity(満期):=2 month, Volatility σ :=38%, Riskless interest rate (非危険利子率)r:=6%

→ How much do you value this right under the above conditions？(あなたは上記の条件でいくら支払い購入する権利をするのか)

PROBLEM



- Value the European Call Option under the following. (次のヨーロピアン･コールオプションの価格を求めよ)

  Current Stock Price(現在の株価) S:=¥14500, Strike Price(権利行使価格) K:=¥14000, Maturity(満期):=2 month, Volatility σ :=38%, Riskless interest rate (非危険利子率)r:=6%

→ How much do you value this right under the above conditions ? (あなたは上記の条件でいくら支払い購入する権利をするのか)

THEORETICAL ANSWER

- **Black-Sholes Formula**

$$f(S,t)=S\cdot N\left(\frac{u}{\sigma\sqrt{x}}+\sigma\sqrt{x}\right)-K\cdot\exp(-rx)\cdot N\left(\frac{u}{\sigma\sqrt{x}}\right).$$

- Maturity (オプションの期間) T-t=2/12=0.1667
- $u=\log(S/K)+(r-\sigma^2/2)(T-t)=0.0331$
- u/σ√x+σ√x=0.3685, u/σ√x=0.2133.
- From Standard Normal distribution Table(標準正規分布の数表から)
- N(u/σ√x+σ√x)=0.6437, N(u/σ√x)=0.5845.

We derive the European Call Option price from the above.
f(S,t)=14500×0.6437-14000×exp(-0.06×0.1667)×0.5845
= **¥1232.0884**

⇒Stock Price after two month(2ヶ月後の株価)

＞ **¥15232**「GAIN」

＜ 「LOSS」

Black-Sholes Formula in this setting

- When we derive the Black-Sholes formula, the strike price is influence by the boundary condition.(行使価
- 格の影響があるのは、**境界条件**を使用するとき)
- K := $\overline{K}$

$$f(S,t) = S \cdot N\left(\frac{u}{\sigma\sqrt{\tau}} + \sigma\sqrt{\tau}\right) - \boxed{\overline{K}} \cdot e^{-r\tau} \cdot N\left(\frac{u}{\sigma\sqrt{\tau}}\right).$$

where $\overline{K} =$ the strategy 1's strike price in equilibrium (平衡時の戦略1における行使価格) $\cdot s_1^*$ + the strategy 2's the strike price in equilibrium. $\cdot (1 - s_1^*)$

s_1^* is the probability of choosing the mixed strategy(混合戦略を採用する場合の確率)

6. SUMMARY AND FUTURE WORKS

Summary and Future Works

Summary

- **MODELING** the Financial Market.
- **DERIVE** the payoff matrix for each player.
- **APPLY** the Real Market.
- **DERIVE** the Optimal Behavior for each player.

Future Works

- **GET** the Online Financial Data, **CALCULATE** and **DISPLAY**. (オンラインでデータを入手し、計算し、それを表示する)
- **MAKE** the software like a PUCK based on the Evolutionary Game Theory. (PUCKの進化ゲーム理論版の構築)

Summary and Future Works

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etc

Thank You For Your Attention

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This File is available at

http://kikkawa.cyber-ninja.jp/

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