明治大学グローバルＣＯＥプログラム【現象数理学の形成と発展】
第３回現象数理若手ミニシンポジウム

２０１０年１１月２２日（月）１０：３０−１７：４５
明治大学 駿河台校舎 大学会館８階 第４会議室

「ファイナンスと現象数理学」

10:30ー11:30
吉川　満（明治大学）
「板情報に着目した市場モデル：進化ゲーム理論」

13:00ー14:00
宇野　淳（早稲田大学）
「取引スピードと流動性：東証アローヘッドのケース」

14:15ー15:15
古幡征史（北陸先端技術科学大学院大学）
「東京証券取引所の現物市場における売買制度の特性に関する研究」

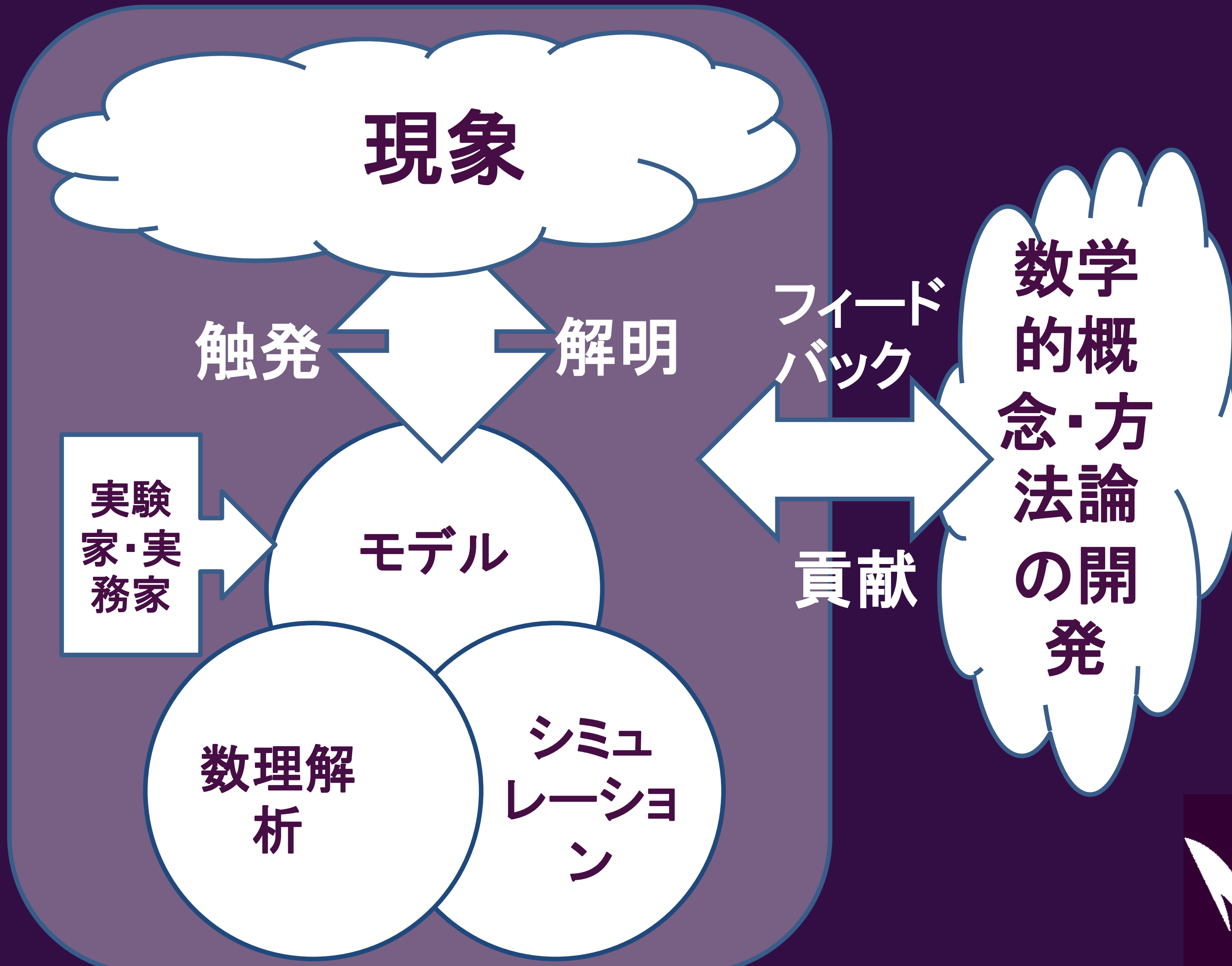
15:30ー16:30
刈屋武昭（明治大学）
「倒産確率の期間構造と回収率を導出するための社債価格付けモデルとその応用」

16:45ー17:45
八丁地園子（明治大学）
「情報の非対称性と意思決定における上司と部下の性格の違いによる企業の戦略」

MEIJI UNIVERSITY GCOE PROGRAM

現象数理学

(Mathematical Sciences Based on Modeling and Analysis)



Market Model Focused On the Order Book

: Evolutionary Game Theory

(板情報に着目した市場モデル
:進化ゲーム理論)

Mitsuru KIKKAWA (吉川満)

（**Department of Science and Technology, Meiji University**)

THIS FILE IS AVAILABLE AT

http://kikkawa.cyber-ninja.jp/

Today's Talk

Modeling

- **Order Book**
- **Double Auction**

Short-term

- **Kikkawa (2009)**
- Micro-Econometrics (**Multinominal Logit model**)

Long-term

- Dynamical system (**Replicator equation**)
- Similarity between of the markets (**SOM**)

Practical Use (Excel)

Case Studies

１）Trading Strategy (Algorithm Trading)

２）Detection of the fraud (不正の発見):

EX.) Insider Trading, Manipulating Quotations (相場操縦)

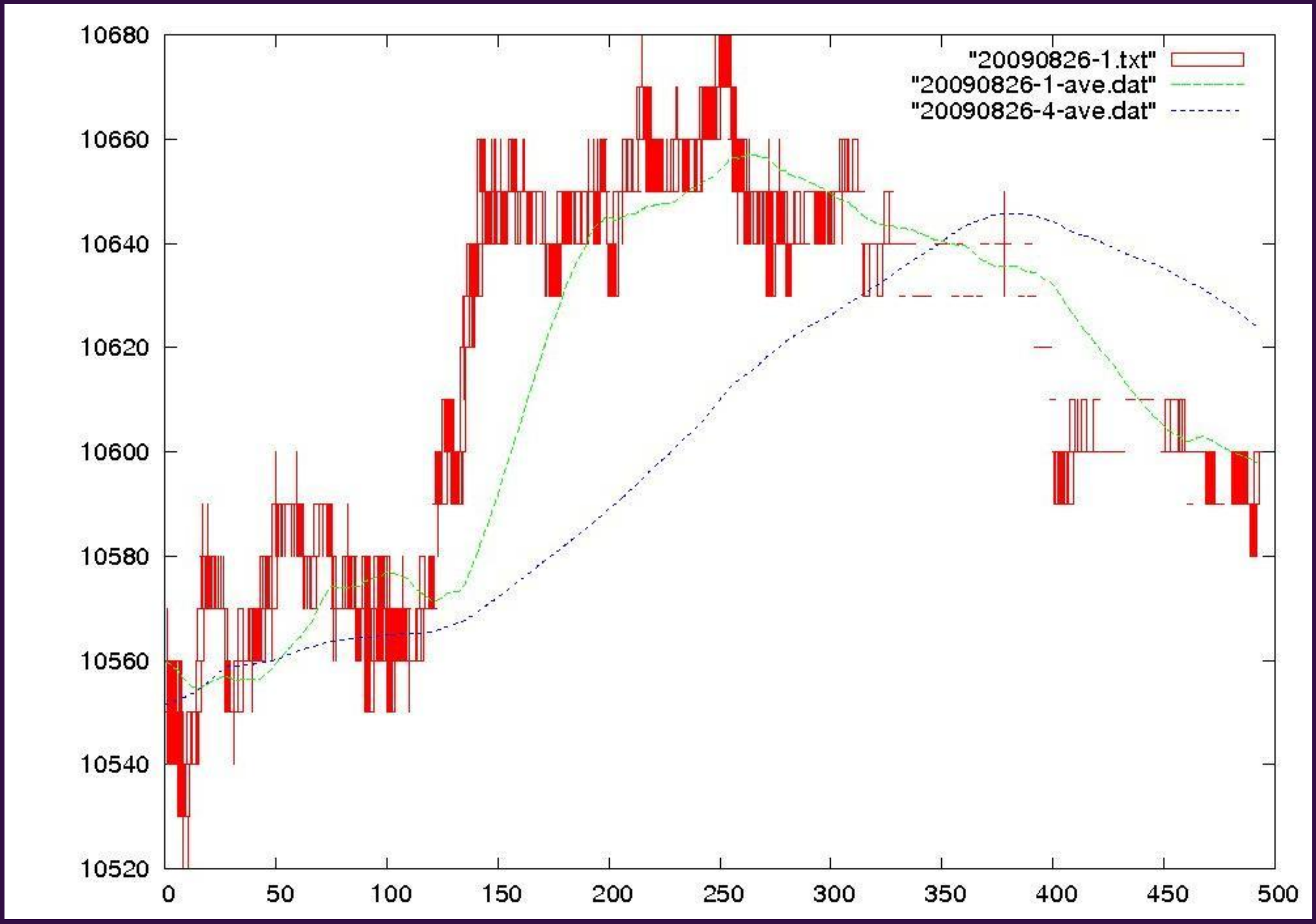
３）Detection of bungled trade(誤発注), Circuit Breaker：

EX.) Black Monday (19^{th} , Oct., 1987)

４）Constructing new trading institutions

1. INTRODUCTION

EX: 20090826



10680
10660
10640
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10600
10580
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10520
0
50
100
150
200
250
300
350
400
450
500
"20090826-1.txt"
"20090826-1-ave.dat"
"20090826-4-ave.dat"

The Order Book (板情報)

（ Bid (sell)）	Price	（Ask (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

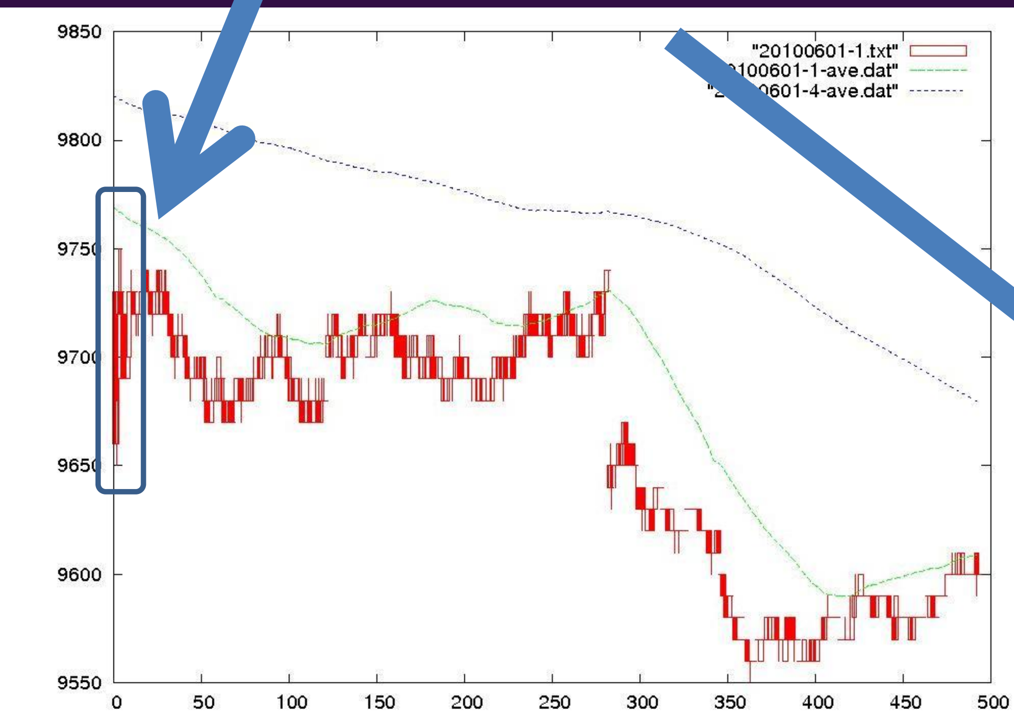
The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

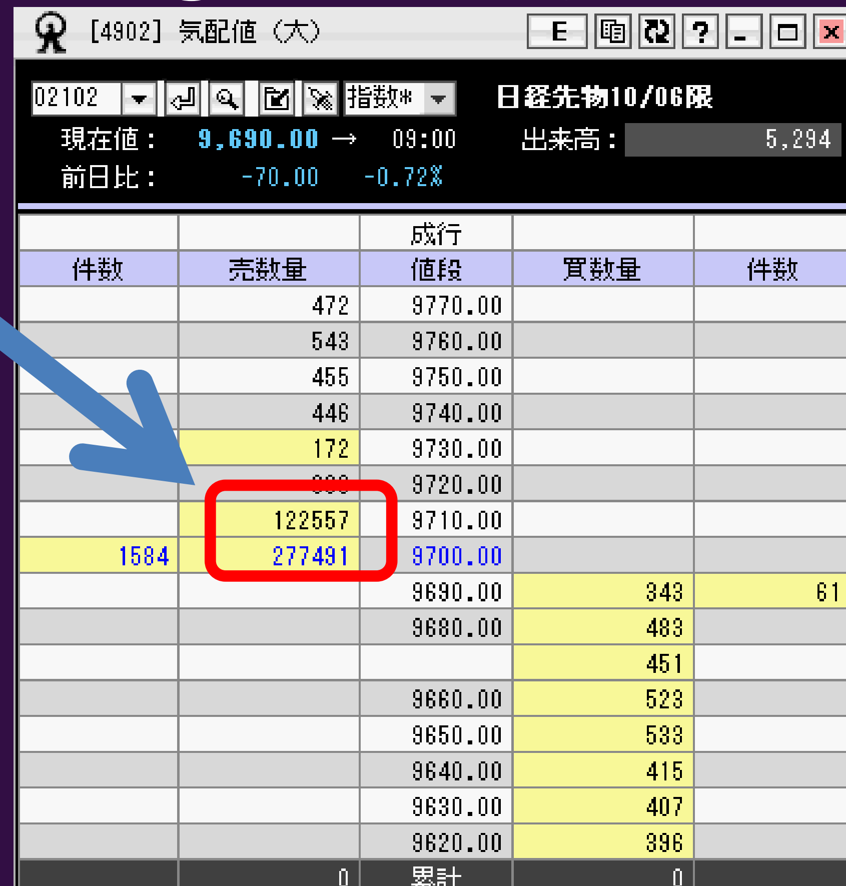
In this case, opening price is 500 or 501.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

【MOVIE】

Bungled trade (Deutsche Bank, 2010/06/01, Opening session)





		成行		
件数	売数量	値段	買数量	件数
	472	9770.00		
	543	9760.00		
	455	9750.00		
	446	9740.00		
	172	9730.00		
	[illegible]	9720.00		
	122557	9710.00		
1584	277491	9700.00		
		9690.00	343	61
		9680.00	483	
			451	
		9660.00	523	
		9650.00	533	
		9640.00	415	
		9630.00	407	
		9620.00	396	
	0	累計	0	

【MOVIE】

この板には、9710円に、約12万、9700円に、約28万、合計40万、約4兆円分の注文がある。

WHAT IS THE "GAME" ? (Non-cooperative Game)

There are two interacting players (Player 1, Player 2).

If player 1 chooses strategy 1 and player 2 chooses strategy 1, player 1's payoff is *a* , player 2's payoff is *b*.

In this situation, which strategy does each player choose ?

(The game is played only once.)

→This game's solution is **Nash Equilibrium**.

player 2

player1		S1	S2
	S1	a,b	0,0
	S2	0,0	c,d

Nash equilibrium depends on the signs: *a,b,c,d*.

Background: Can we compare own utility with an another player's utility ?

【No】In microeconomics, we use an **ordinal** (序数的, not measurable, only order) utility function.

＋

【Yes】In game theory, we use a **cardinal** (基数的, measurable) utility function.

→

Estimate the utility function which can't be observed with "**data**". (**Structural Econometrics (構造推定)**)

Double Auction

- Buyers and sellers each have private information about their valuations trade with the good.

- 次の3つの条件を同時に満たすような取引制度は存在しない。

  i) Individual Rationality (IR, 個人合理性)

  ii) Pareto Efficient (PE, パレート効率性)

  iii) Incentive Compatible (IC, 誘因両立性)

Example) McAfee(1992, JET), IR〇, IC 〇, PE×

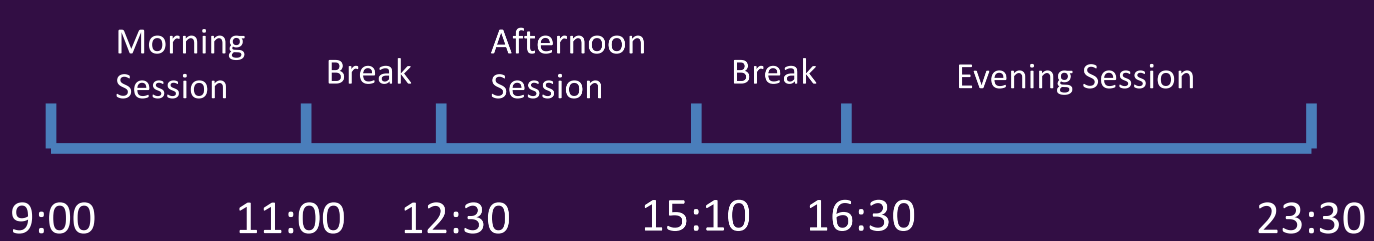
2. RELATED LITERATURES AND PRELIMINARIES

How are stock prices determined ?

- Stock prices are determined by two methods, the *Itayose(板寄せ)* and *Zaraba(ザラバ)* methods. The *Itayose* method is mainly used to decide opening and closing prices; the *Zaraba* method is used during continuous auction trading for the rest of the trading session.

→ The stock price are determined by **Rule**.

[Nikkei 225 Future Market(日経225先物)] [1day]



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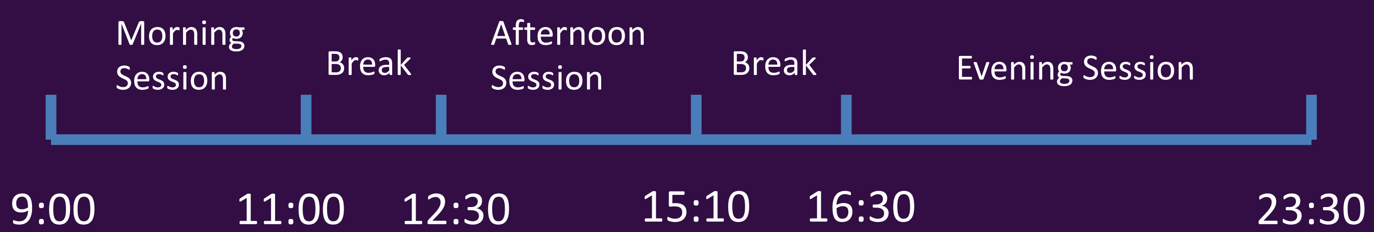

Two Principles (2つの原則)

1) **Price Priority** (価格優先の原則)

means that the lowest sell and highest buy orders take precedence over other orders.

2) **Time Priority** (時間優先の原則)

means that among orders at the same price, the order placed earliest takes precedence.

Bid (sell)	Price	Ask (buy)
A 3000(5), C 4000(4)	502	early ← → late
D 10000(3), E 9000(2), F 5000(1)	501	
	500	H 80000(1), B 1000(2), J 4000(3)
late← → early	499	H 1000(4), B 150000(5)

The Order Book (板情報)

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In this case, opening price is 500 or 501.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

【MOVIE】

Assume: opening price is 500.

（Bid (sell)）	Price	（Ask (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- The market orders of 4000 shares to buy and 6000 shares to sell are matched, leaving sell orders of 2000 shares.

Second Step

（Bid (sell)）	Price	（Ask (buy)）
2000	Market orders	
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- The market sell orders of 2000 shares and sell orders 6000 shares at limit prices of 499 or less are matched with the buy orders of 8000 shares at limit prices of 501 or more. Thus far, 12000 shares have been matched in total.

Third Step

（Bid (sell)）	Price	（Ask (buy)）
Market orders		
8000	502	
20000	501	
4000	500	10000
	499	8000
	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- Finally, the sell orders of 4000 shares at a limit price of 500 are matched with the buy orders of 10000 shares at a limit price of 500. Although this still leaves buy orders of 6000 shares at 500.

Fourth Step

（ Bid (sell)）	Price	（Ask (buy)）
	Market orders	
8000	502	
20000	501	
	500	6000
	499	8000
	498	30000

- Thus the opening price is determined at 500 and transactions of 16000 shares are completed at 500.

The stock price and the trade depend on the **order book**. (価格や取引の可否は板情報によって決定する。)

3. MODEL

Model (Ref. Chatterjee and Samuelson (1983))

- **Players**… large population : seller and buyer, potentially (大人数の潜在的な売り手と買い手)

  Seller and Buyer trade an asset.

- **Goods** (財) … 1財
- **Strategy** (戦略)… **n** (<∞) 個、p_s, p_b

Here, the strike price : how much do you buy or sell an asset. (ここでは購入、売却価格)

- **Payoff (利得) …**

  Buyer : $\max[v_b - p_b]$ Prob(OB),

  Seller : $\max[p_s - v_s]$ Prob(OB).

where Prob(OB) ∝ Prob ($p_b \geqq p_s(v_s)$) × Prob(OA)

(取引が起こる条件)　(取引への積極度)

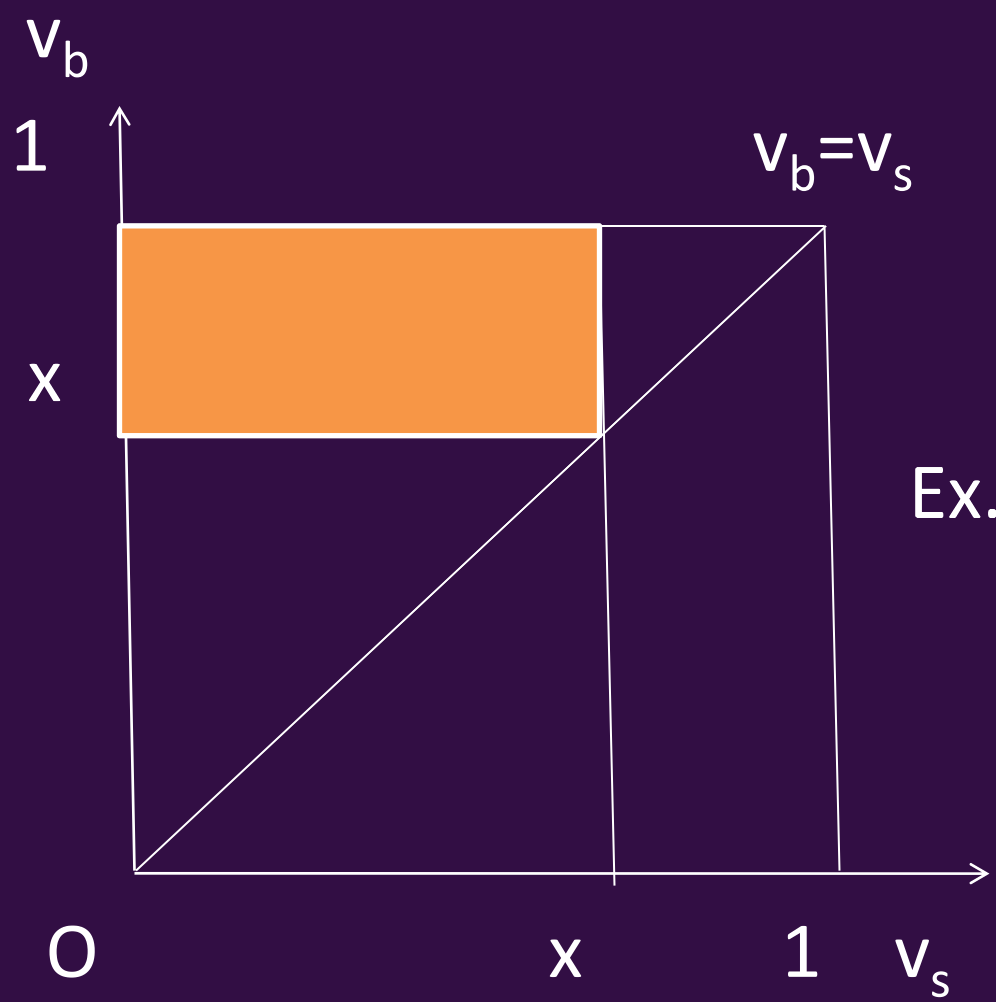
EXAMPLE: The Order Book (板情報)

（ Bid (sell)）	Price	（Ask (buy)）
0	Market orders	0
492	9840	-----
506	9830	-----
444	9820	-----
530	9810	----
784	9800	-----
-----	9790	197
-----	9780	734
-----	9770	640
------	9760	643
-----	9750	598

This order book is on Nikkei Future Market(9:03, 5^{th}, November, 2009.)

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

One-Price Equilibrium



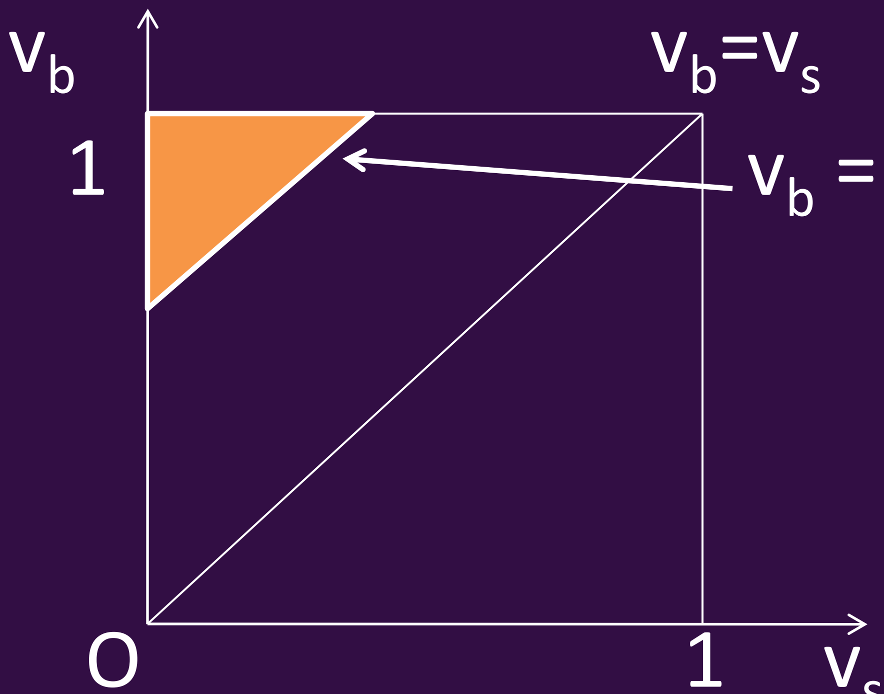
Ex. Itayose Method

- This square is a turnover(出来高)

Linear Equilibrium

- Seller's Strategy：$p_s(v_s)=a_s+c_sv_s$、p_s : uniform distribution on $[a_s,a_s+c_s]$ → $p_s=(a_b+c_b+v_s)/2$
- Buyer's Strategy：$p_b(v_b)=a_b+c_bv_b$、p_b : uniform distribution on $[a_b,a_b+c_b]$ → $p_b=(v_b+a_s)/2$

⇒ $p_s(v_s)=2/3+v_s/2$, $p_b(v_b)=1/3 + v_b/2$.



Myerson and Satterthwaite (1983)
→線形均衡の方が高い利得を得られる。

Kikkawa (2009) (related : Logit model)

- **Many players** play the game simultaneously.

- Kikkawa (2009) formulates this situation with statistical mechanics (統計力学).

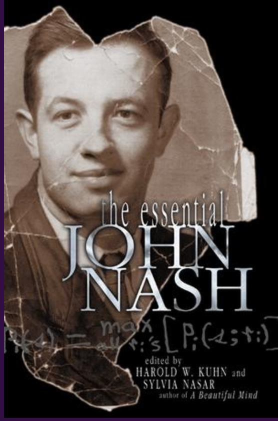
Prop. We obtain the probability distribution of actions, $\{S_i\}$, $i=1,\dots,N$, and the player's payoff from the outcome is f,

$$P(\{S_i\})=Z^{-1}\exp(\gamma f),$$

$\{S_i\}$:a player i's action, γ: non-negative constant, f: the player's payoff from outcome $\{S_i\}$, Z: normalization parameter.

- Kikkawa (2009) is similar to **Quantal Response Equilibrium**. (Mckelvey and Palfrey (1995, 1996))

Interpretation of Nash Equilibrium (J.F.Nash's Ph D. Thesis)



- 1. "**Rationality**" ･･･ the players are perceived as rational and they have complete information about the structure of the game, including all of the players' preferences regarding possible outcomes, where this information about each other's strategic alternatives and preferences, they can also compute each other's optimal choice of strategy for each set of expectations. If all of the players expect the same Nash equilibrium, then there are no incentives for anyone to change his strategy.
- 2. "**Statistical Populations**" ･･･ is useful in so-called evolutionary games. This type of game has also been developed in biology in order to understand how the principles of natural selection operate in strategic interaction within among species.(→ **Mass Action**)

WHAT IS THE “GAME” ? (Non-cooperative Game)

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In this situation, which strategy does each player choose ?

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player 2

player1		S1	S2
	S1	a,b	0,0
	S2	0,0	c,d

Nash equilibrium depends on the signs: *a,b,c,d*.

PROPOSITION (Kikkawa, 2009)

PRO.: $x \in \Delta$ is an evolutionary stable strategy in an evolutionary game with statistical mechanics, if there exists some *m* such that the inequality (*)

holds for all m^*.

$$u(y,x) \leqq u(x,x), \quad \forall y,$$

(*) $$| m - m^* | < \varepsilon$$ ← Lyapunov Stable Condition

where m^* stands for the index of the strategy

Multinominal Logit Model

- From Kikkawa (2009), we can know the probability of choosing the strategy for each player.

＋

- Data (the probability of choosing the strategy for each player)

- Regression analysis(回帰分析)

$$Y_i=\alpha + \gamma f + u, \qquad u : \text{logistic distribution.}$$

- We can estimate optimal parameters in this model with **Least Squares Method** (最小2乗法)

How to analyze the order book

Step 1) Logit Model (Derive the probability of choosing the strategy and transform this into log function.)

Step 2) Regression analysis.

OA: $Y=-0.65307+94079.26X_1-9.59255X_2$,

$Y=-0.66468+74928.44X_1-7.6642X_2$.

Step 3) Derive v_s, v_b. $v_s=9776$, $v_b=9807.53$.

~~OA~~:

Step 3') Derive the Demand and Supply function.

$Y=583.93-146.27X$, $Y=-237.14+59.57X$

Step 4') In equilibrium, we know that the quantity demanded is equal to the quantity supplied.

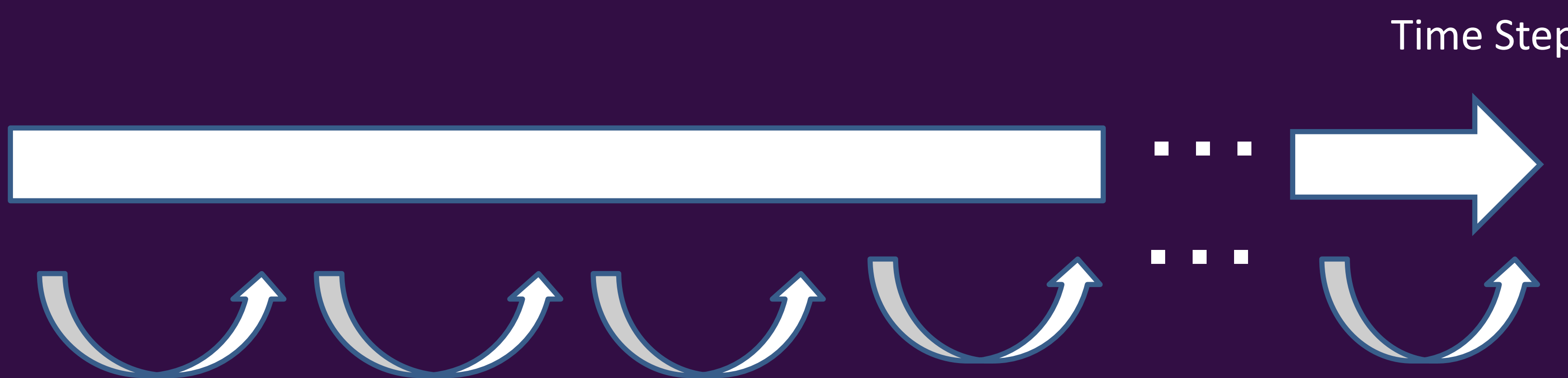
Step 5') Derive the Nash equilibrium.

$X^*=9740$.

4. DYNAMICAL SYSTEM
- REPLICATOR EQUATION-

Dynamical Logit Model

Estimate the parameter with Dynamical Logit model (Rust, 1987) and predict the future states using the constructed model in each period.



Estimate, update the parameters and predict the order book

Merit: easy
Demerit: robustness

[Another method] : Particle Filter

Dynamical QRE (Weibull (1995))

$$\dot{x}_i = \left(\frac{\exp\left[\sigma u(e^i, x)\right]}{\sum_h x_h \exp\left[\sigma u(e^i, x)\right]} - 1 \right) x_i$$

→If we transform and rescale this equation, then we can obtain the following equation.

$$\dot{x}'_i = \left(\exp\left[\sigma u(e^i, x)\right] - \bar{u} \right) x'_i, \; \bar{u} := \sum_h x_h \exp\left[\sigma u(e^i, x)\right]$$

Replicator equation

$$\frac{dx_i(t)}{dt} = x_i\left(Ax_i - x \cdot Ax\right), i = 1, \cdots, n$$

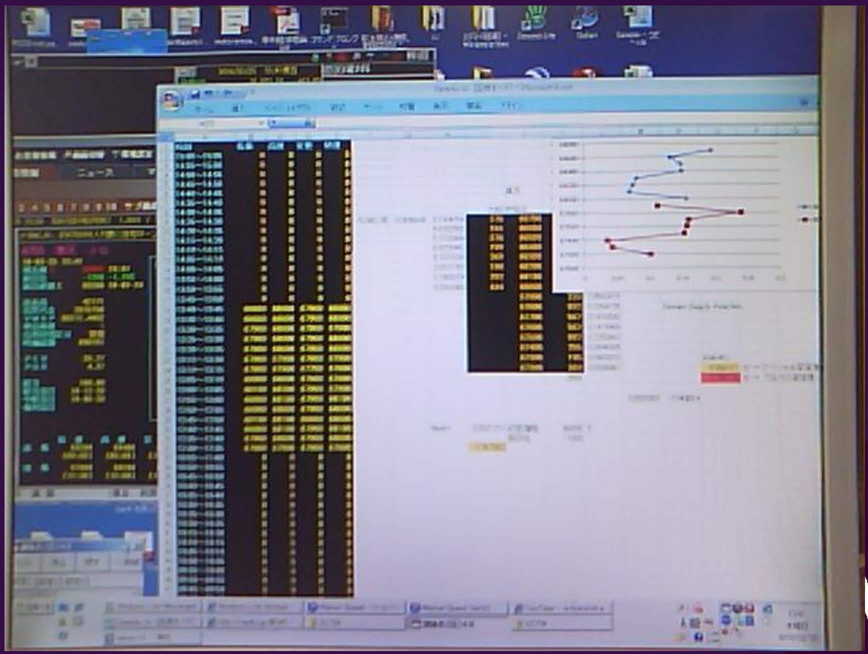
- x_i : the probability of choosing strategy *i*, Ax_i : the payoff of choosing the strategy *i*, *x Ax* : expected utility.

Practice Use

- **Excel**
- **Realtime Spread Sheet** (provided by Rakuten Securities, Inc. (楽天証券))

- **[MOVIE] (YouTube)**

  2010/05/10 09:42-



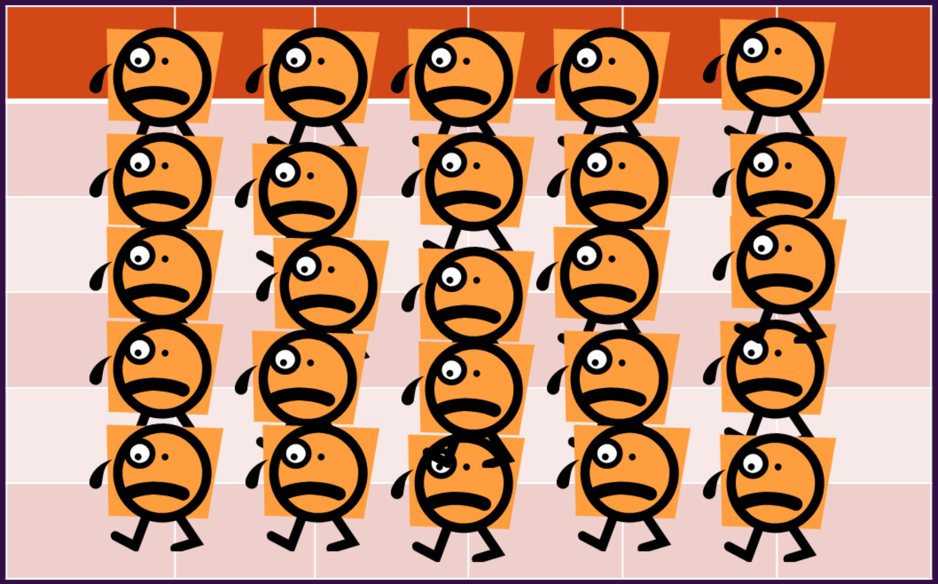
Kikkawa (2010a, Submitted)

- “An information is very important, but …”

- Kikkawa (2010a) shows that Bayesian learning leads to Nash equilibrium in a game under some signal structures due to **payoff monotone**.

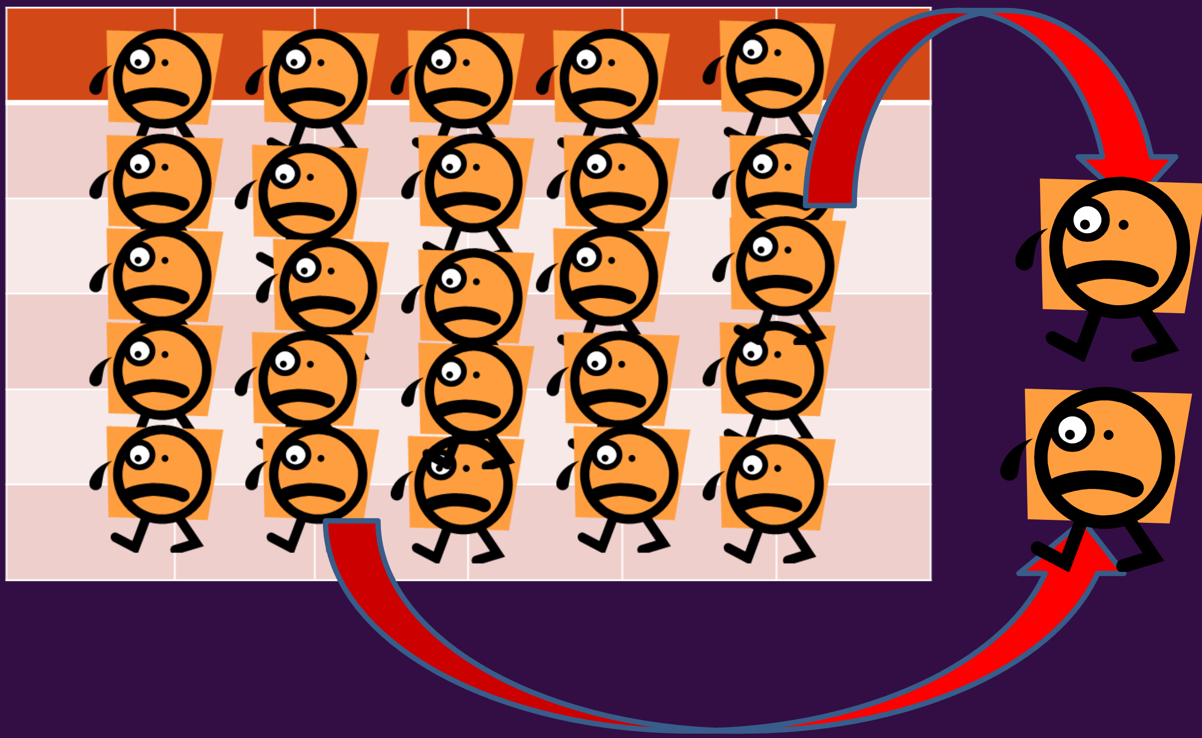
- We can interpret that each player’s behavior is formulated with Replicator equation regardless of with or without a signal structure under some assumptions.

Situation (Traditional Evolutionary Game Theory)



Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)

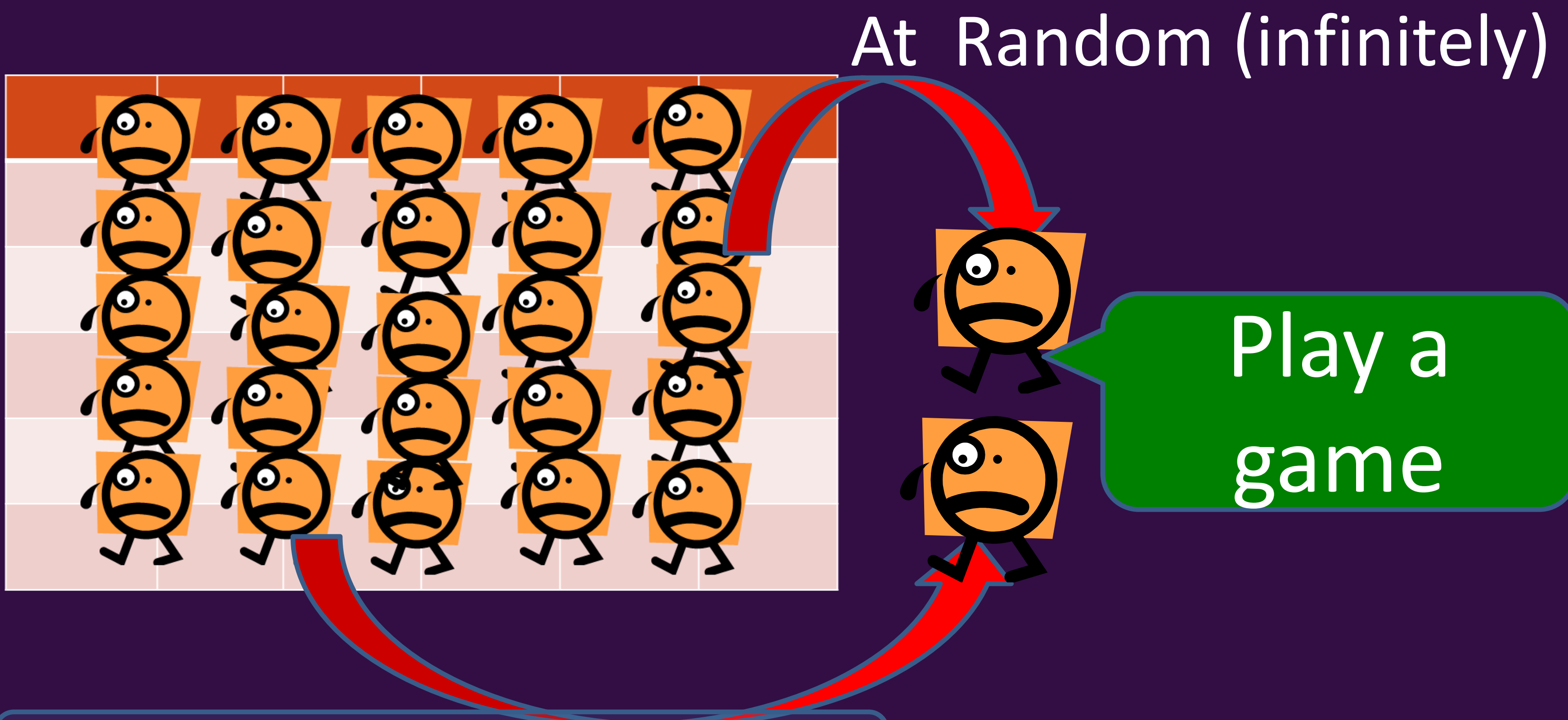


Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)

Another players look at the game.

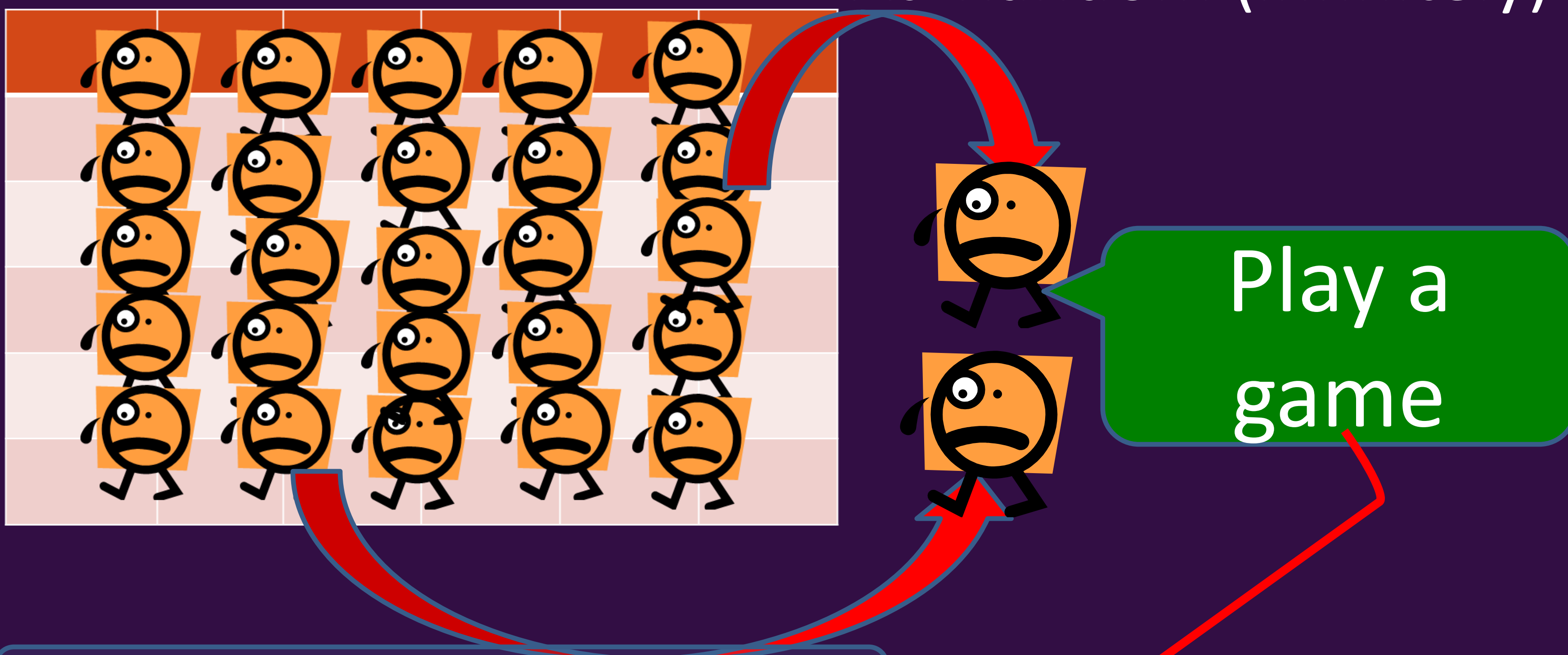
Situation (Traditional Evolutionary Game Theory)



Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

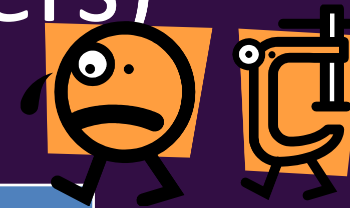
At Random (infinitely)

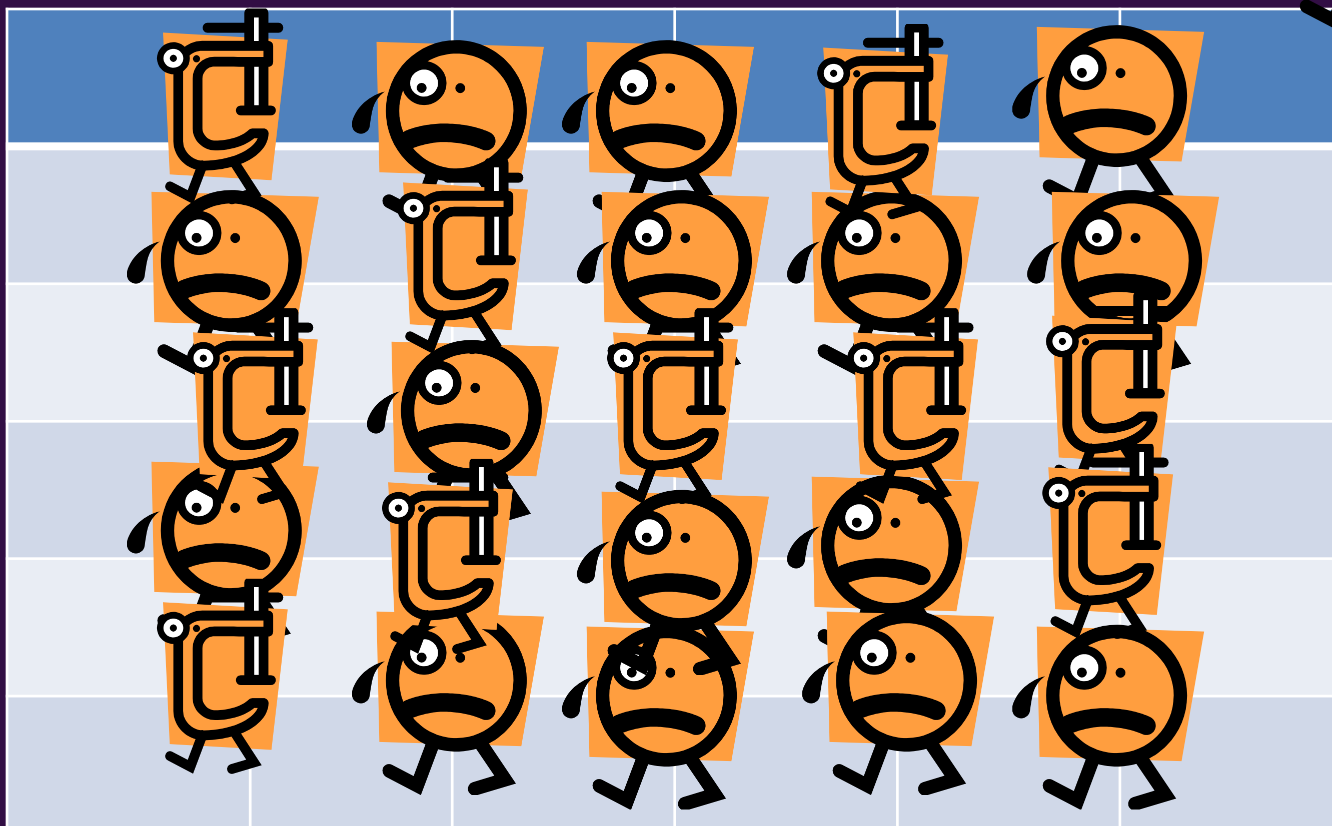


Another players look at the game.

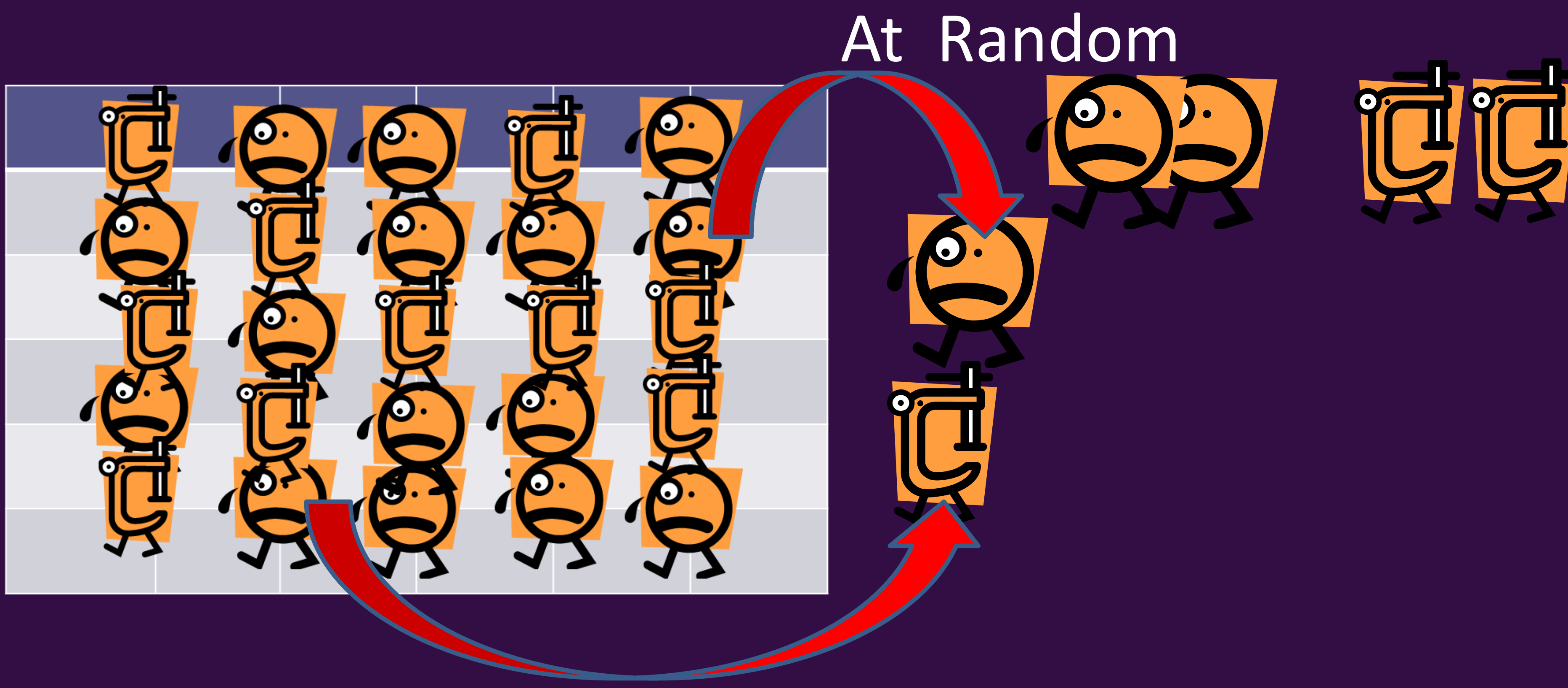
Replicator Equation

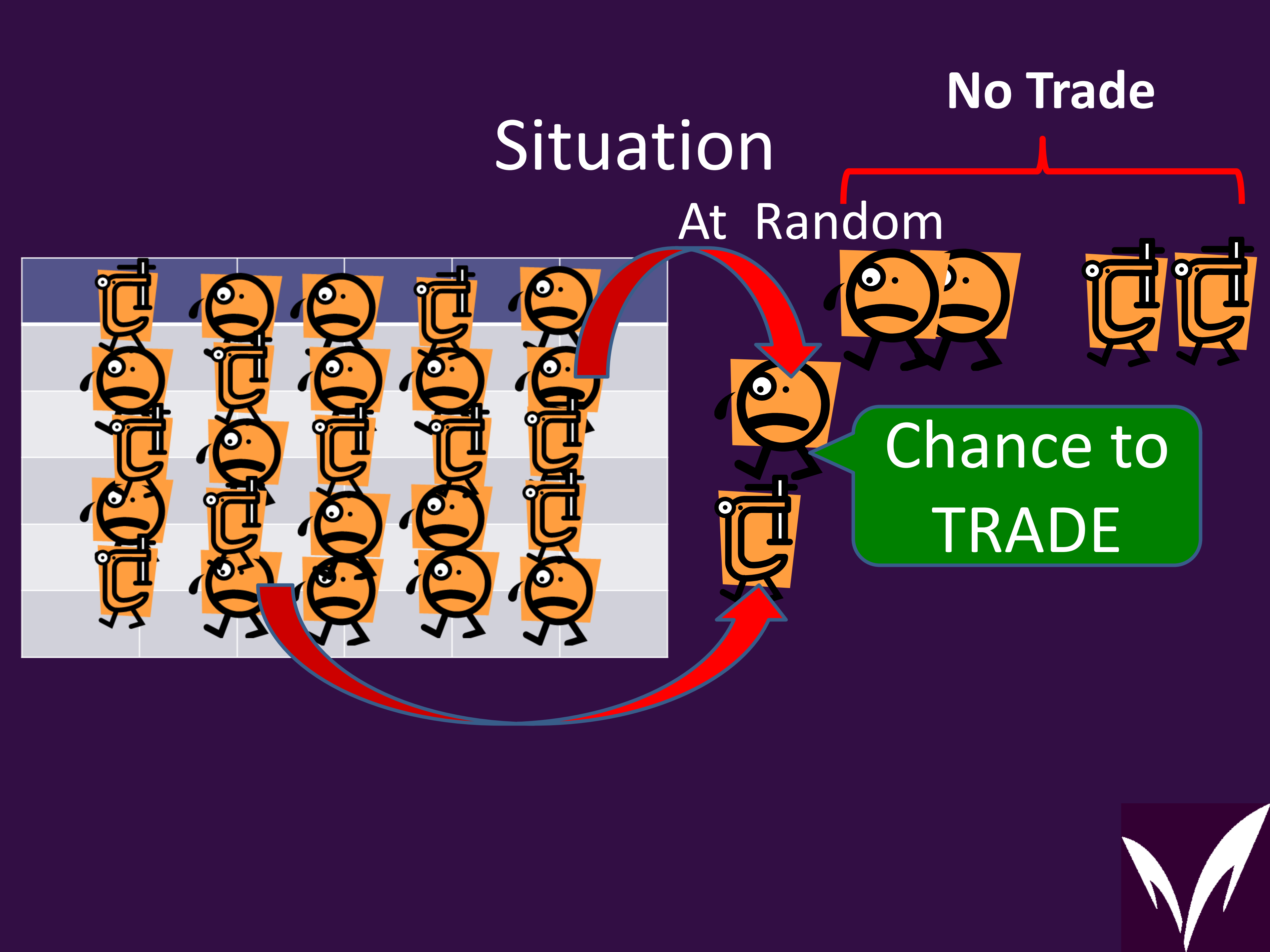
Situation (two types players)



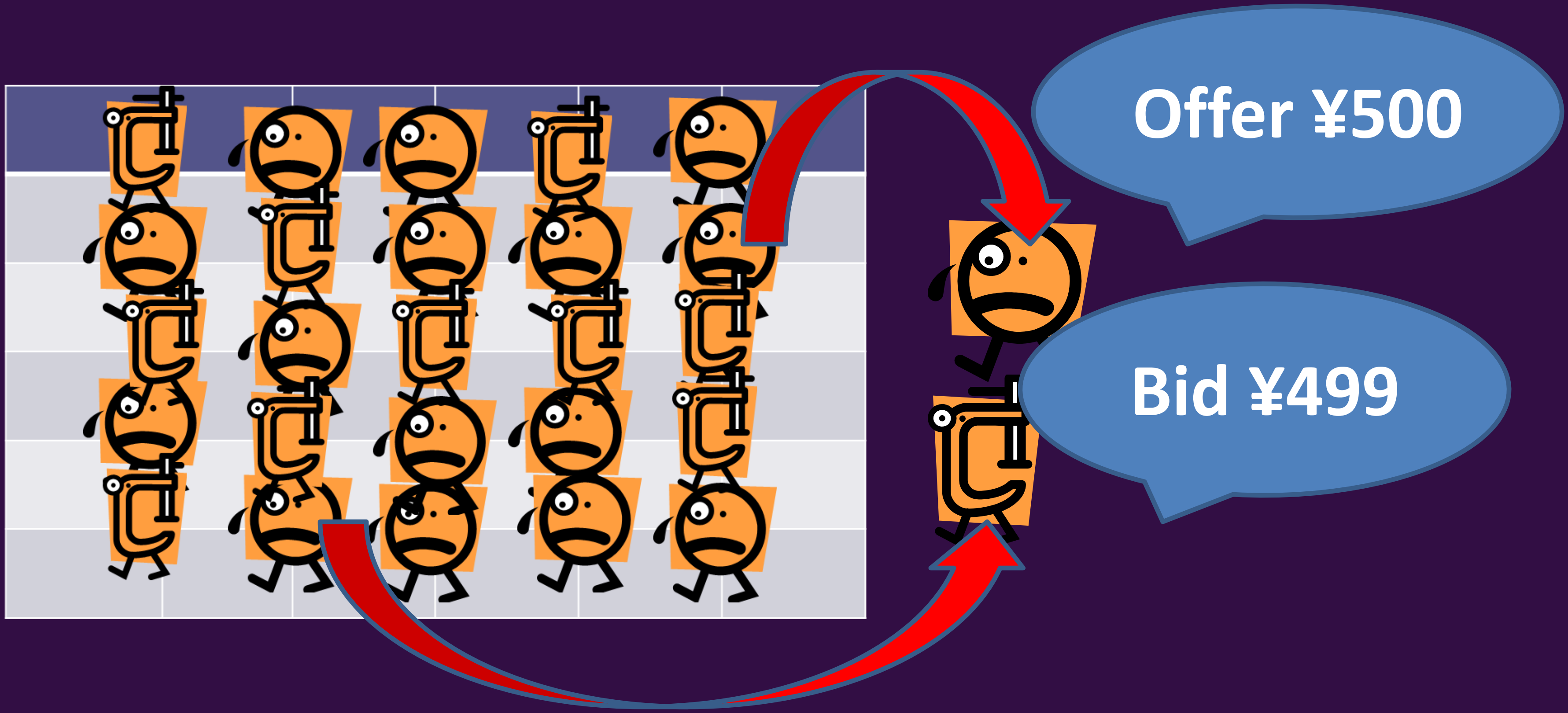


Situation

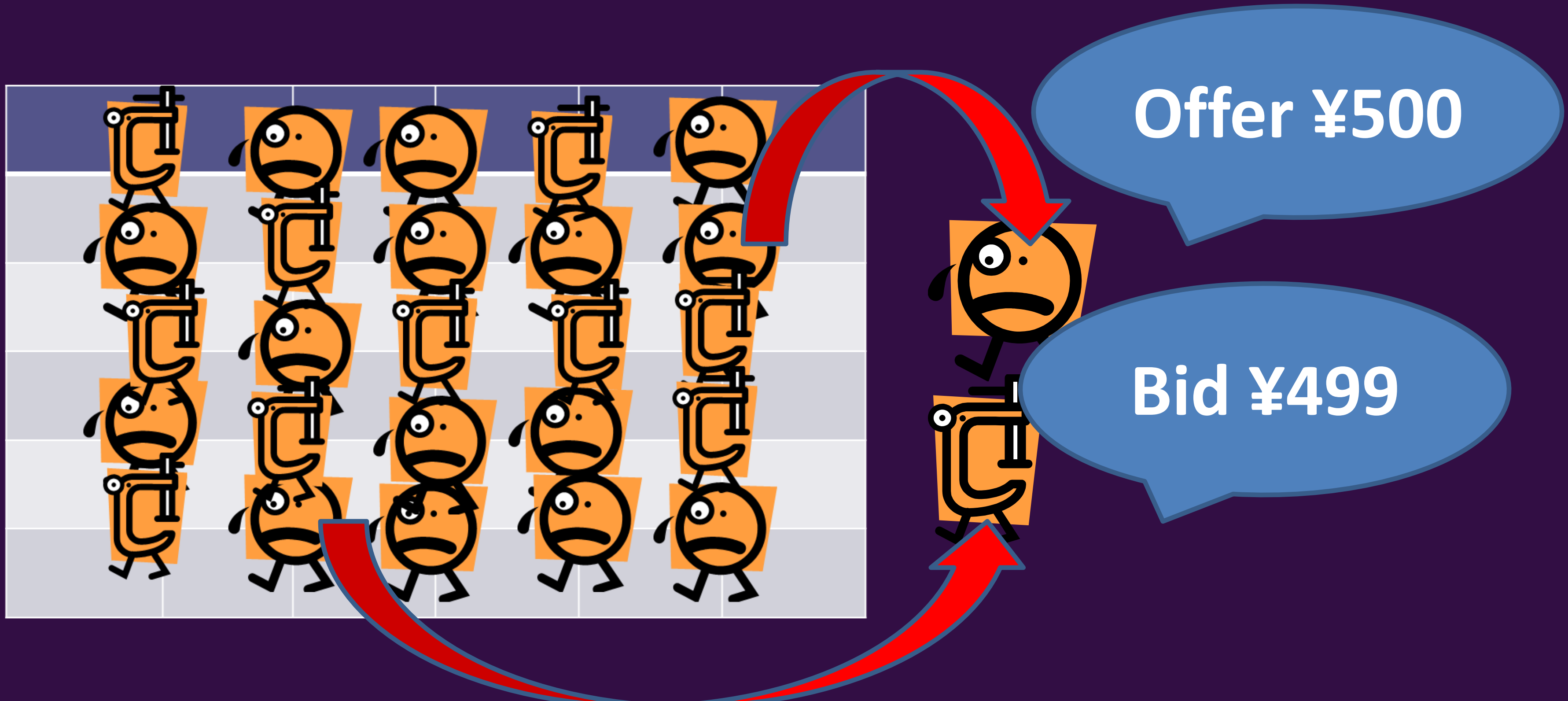



No Trade
Situation
At Random
Chance to TRADE

Situation

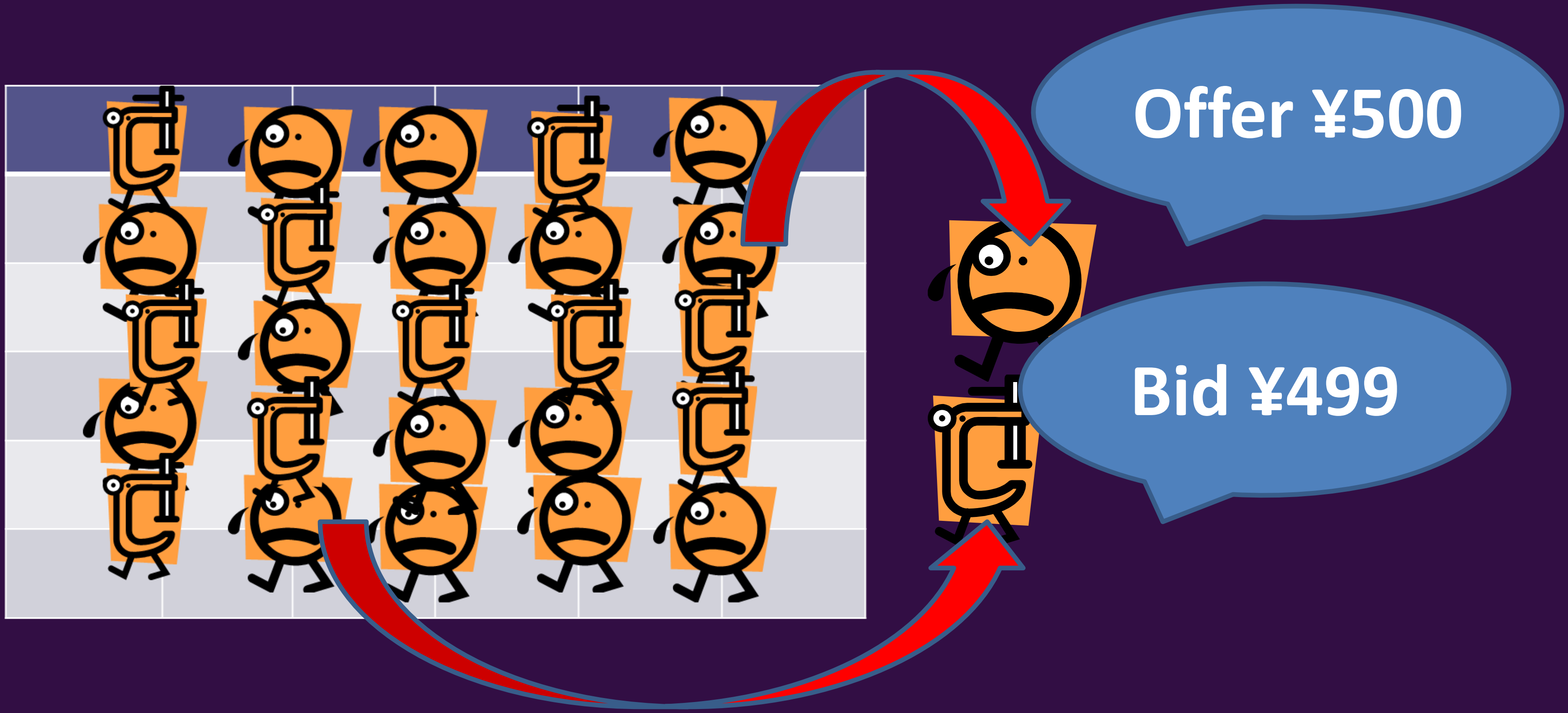


Situation



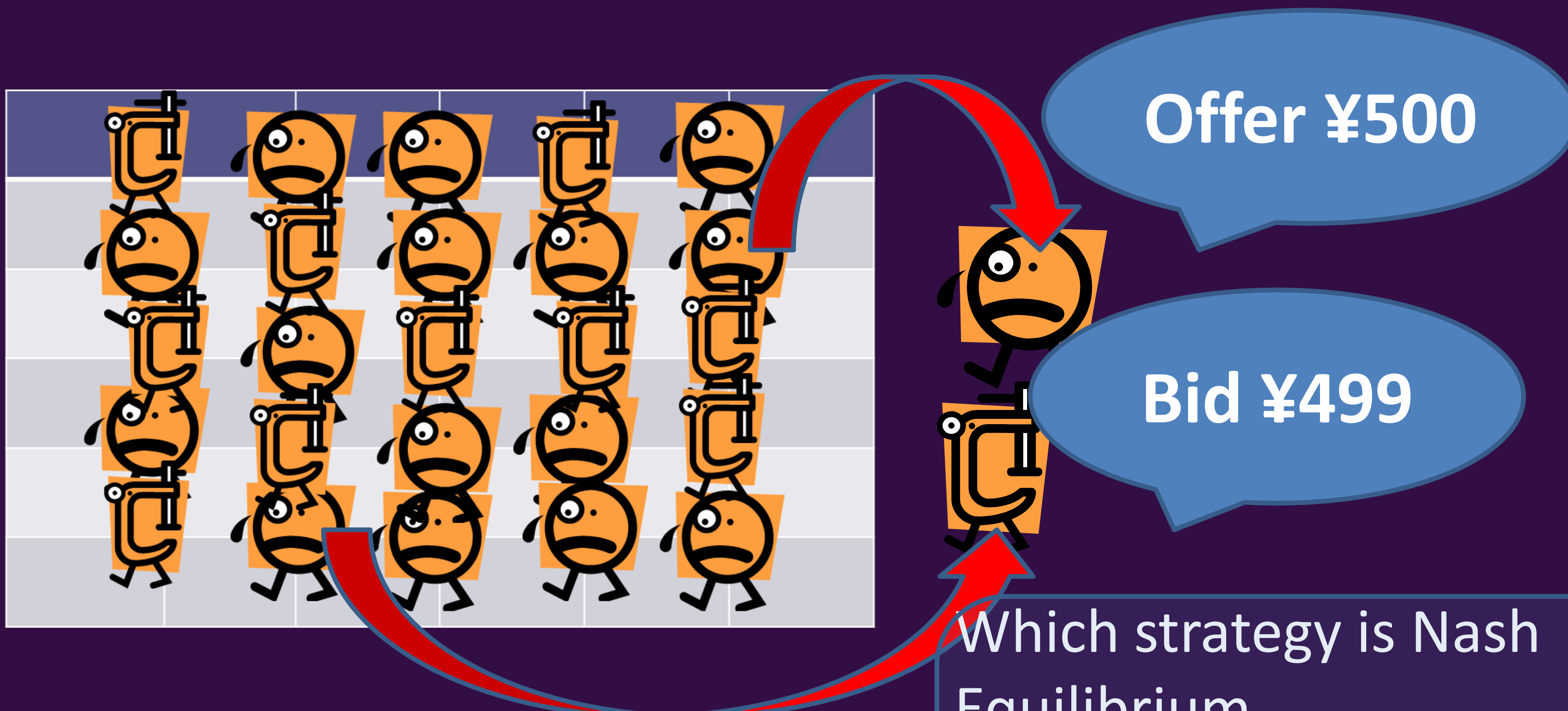
Stock Exchange which take account of the order book decides the trade’s contract. (取引所が板情報をもとに、売買契約を決定する)

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Which strategy is Nash Equilibrium,
if this game is played at infinite ?
(このゲームを無限回仮想的に行うと、どの戦略が均衡となるのか？)

Model (モデル)

- **Replicator Equation**

$$\frac{dx_i(t)}{dt} = x_i(t)\left(g_i(t) - \bar{g}(t)\right)$$

$$\frac{dy_i(t)}{dt} = y_i(t)\left(h_i(t) - \bar{h}(t)\right)$$

where x_i, y_i : the probability of choosing the strategy 1 for each player. g_i, h_i : the payoff when each player chooses the strategy 1.

Two Strategies Case (戦略の数が2つ) :

- Replicator equation (see next slide)

$$\dot{x} = x(1-x)\left\{-b(t) + \left(a(t)+b(t)\right)y\right\},$$

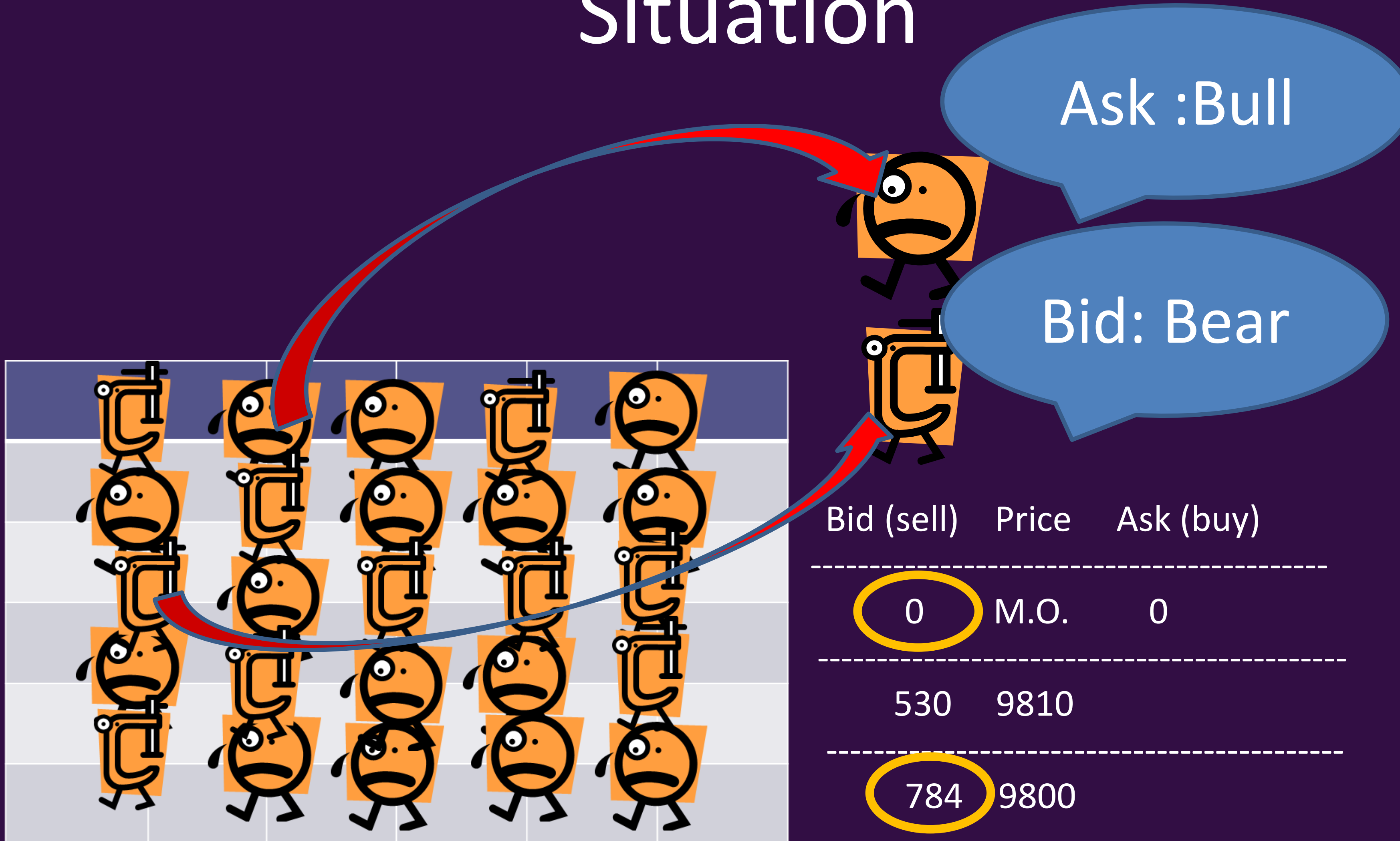
$$\dot{y} = y(1-y)\left\{b(t) - \left(a(t)+b(t)\right)x\right\},$$

where x, y is the probability of choosing the strategy 1, 2 for each player.

Player 2

player1		S1	S2
	S1	a(t),-a(t)	0,0
	S2	0,0	b(t),-b(t)

Situation



Bid (sell)	Price	Ask (buy)
0	M.O.	0
530	9810	
784	9800	
	9790	197
	9780	734

Price ↓

Prediction (予測)

- Replicator equation divided by xy(1-x)(1-y) :

$$\dot{x} = -\frac{b(t)}{y} + \frac{a(t)}{1-y}, \dot{y} = \frac{b(t)}{x} - \frac{a(t)}{1-x}.$$

- Discrete the above equations:

$$x(t+\varepsilon) = x(t) - \left(\frac{b(t)}{y} + \frac{a(t)}{1-y}\right)\varepsilon,$$

$$y(t+\varepsilon) = y(t) + \left(\frac{b(t)}{x} - \frac{a(t)}{1-x}\right)\varepsilon.$$

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

Buyer

Seller

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	+, -	0,0
S 2(BEAR)	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	+,+	0,0
S 2(BEAR)	0,0	-,+

iii) → (No change)

N.E. Mixed Strategy.

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	-,+	0,0
S 2(BEAR)	0,0	-,+

Payoff Matrix (利

i) ↑ (UP)

N.E. (s2,s2)

Seller

		LL)
S 1(BULL)	+, -	0,0
S 2(BEAR)	0,0	+,+

価格上昇時、売り手は約定価格よりも**強気に**高い売り、買い手は**弱気**で高い価格で購入

ii) ↓ (Down)

N.E. (s1,s1)

	S 1(B	
S 1(BULL)	+,+	
S 2(BEAR)	0,0	-,+

価格下落時、売り手は約定価格よりも**弱気に**安く売り、買い手は**強気**で安い価格で購入

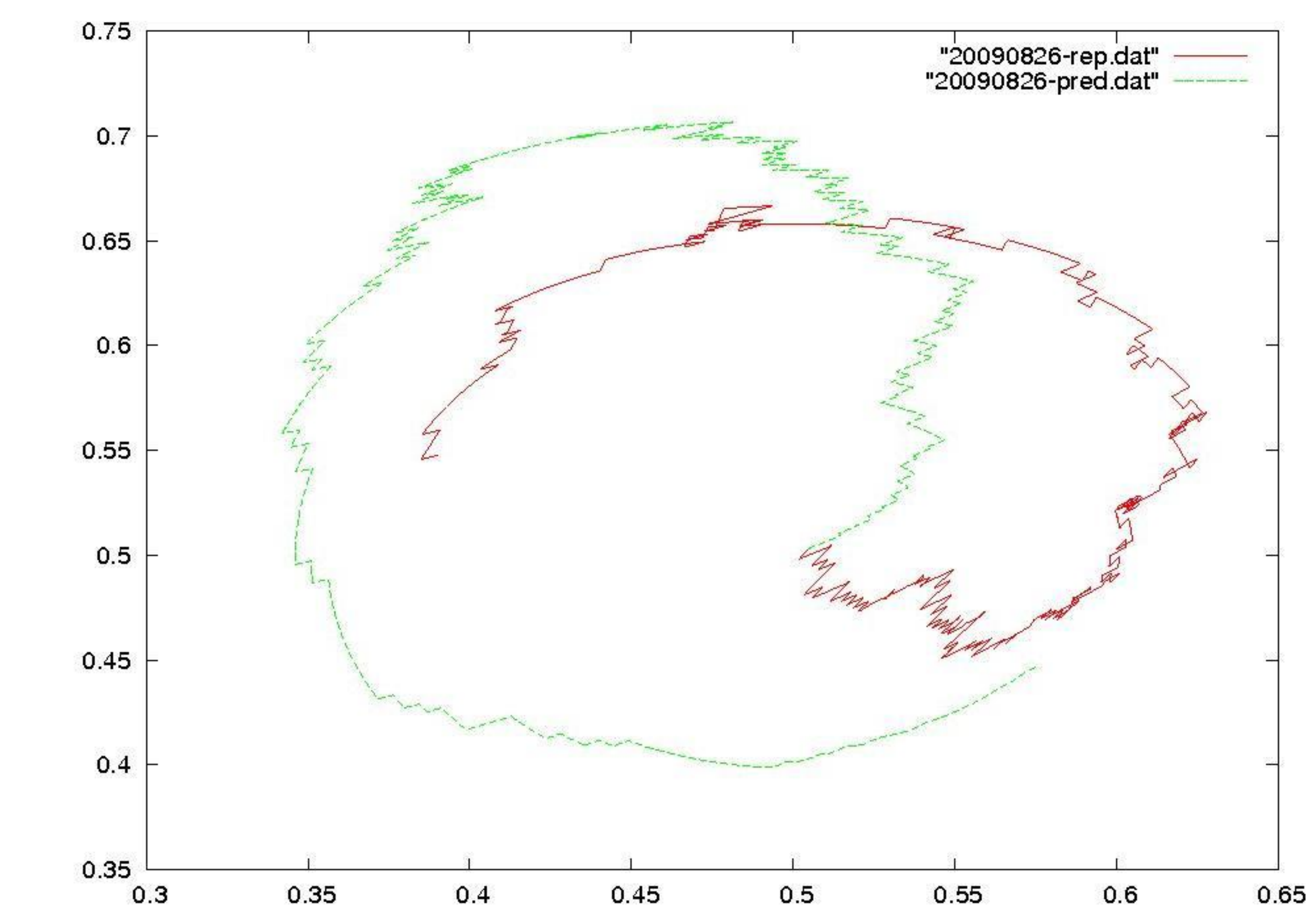
iii) → (No change)

N.E. Mixed Strategy.

	S 1(B	
S 1(BULL)	-,+	
S 2(BEAR)	0,0	-,+

価格変化しない時、売り手は約定価格よりも**弱気に**安く売り、買い手は**強気**で安い価格で購入

EX: 20090826



"20090826-rep.dat"
"20090826-pred.dat"
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65

Indirect Evolutionary Approach
: RISK ATTITUDE (Kikkawa(2010b) submitted)

- Indirect evolutionary approach (Samuelson (2001):

  preference ≠ material payoff

- We assumed that the own utility is linear function. The preference was equal to the material payoff. (今まで主体の効用は線形であると仮定してきた。選好は利得に等しかった。)

- Each player has the non-linear utility.(そこで非線形の場合をも考慮に入れる。)

- Utility function : $g(x)$, z : material payoff
- Taylor Expansion:
- $g(x+z)-g(x)=g'(x)z+0.5g''(x)z^2+O(z^3)$ .... (*)

Def. Given a twice-differentiable Bernoulli utility function $u(.)$ for money, the ***Arrow-Pratt coefficient of absolute risk aversion*** at x is defined as $r_A(x)=-u''(x)/u'(x)$.

- (*) $g(x+z)-g(x) = zg'(x)(1-0.5zr_A(x))$
- (In economics, we assume $g'(x)>0$, $g''(x)<0$)

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

	S 1	S 2
S 1	-, -	0,0
S 2	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

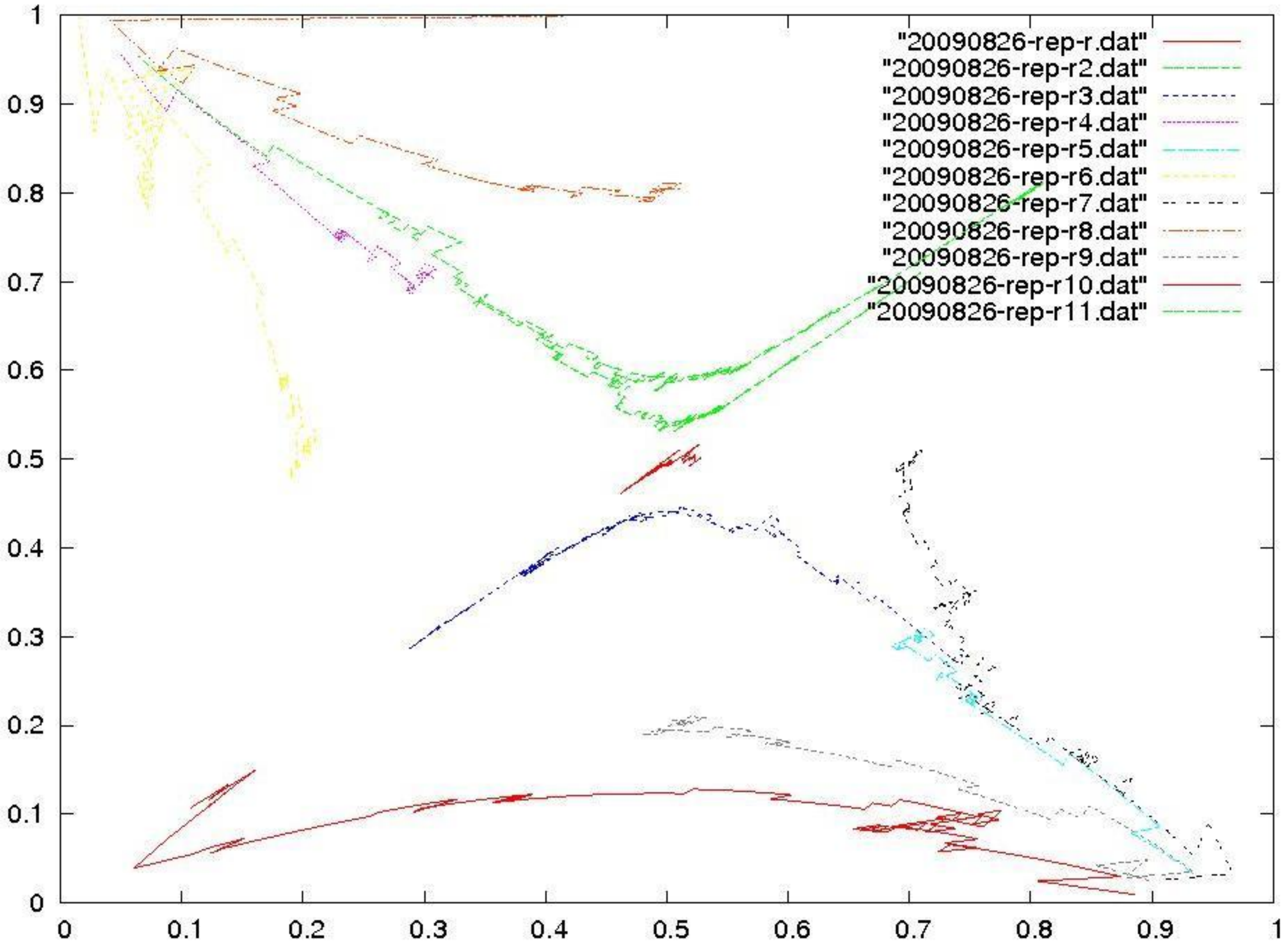
	S 1	S 2
S 1	+,+	0,0
S 2	0,0	-,-

iii) → (No change)

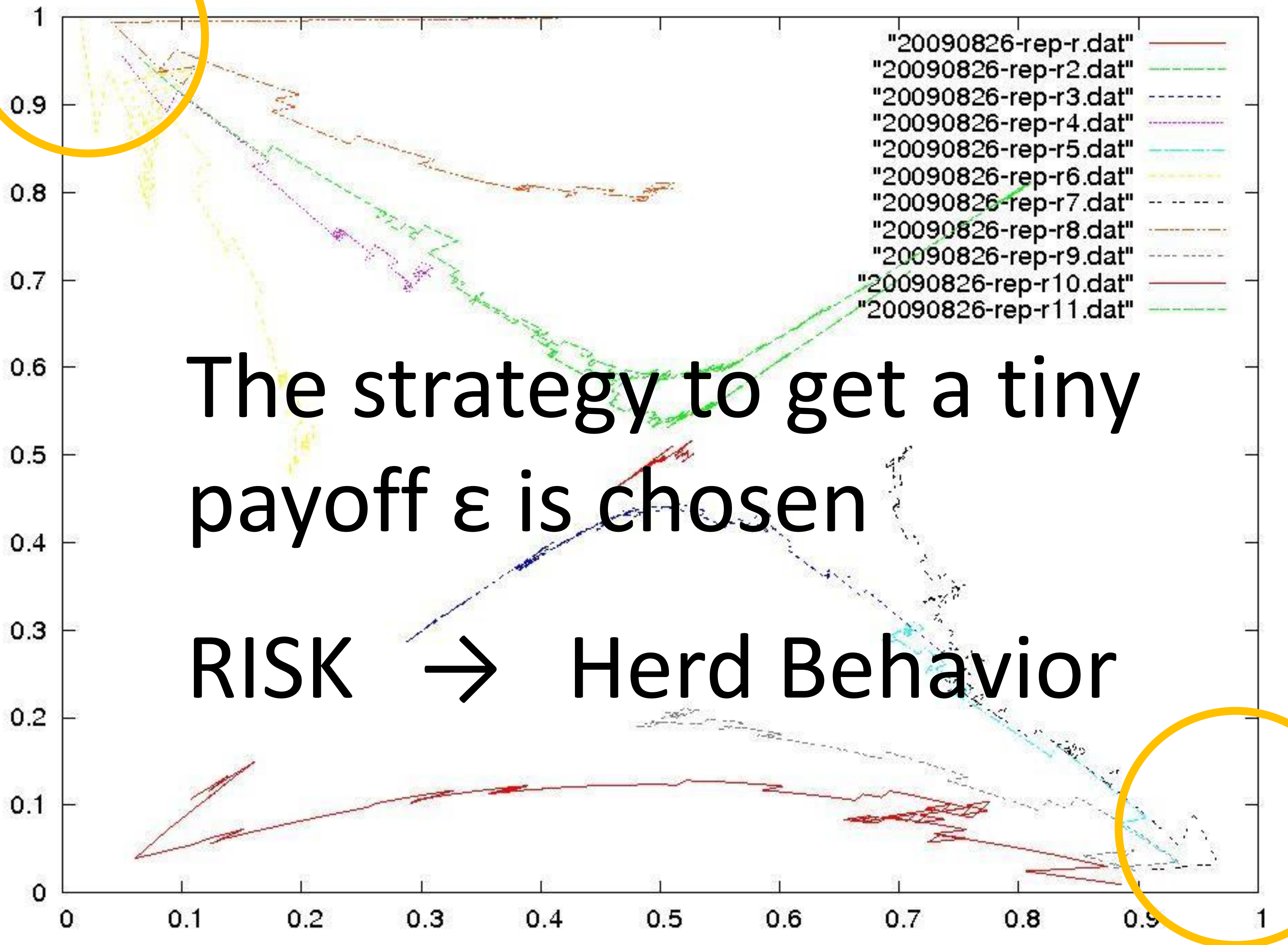
N.E. Mixed Strategy.

	S 1	S 2
S 1	-,-	0,0
S 2	0,0	-,-

EX: 20090826 (RISK)



EX: 20090826 (RISK)



The strategy to get a tiny payoff ε is chosen

RISK → Herd Behavior

5. SIMILARITY BETWEEN BEHAVIORS OF THE MARKETS (SELF-ORGANIZING MAP)

Comparing the Markets

We can not compare the markets for different economic situations.

Similar News

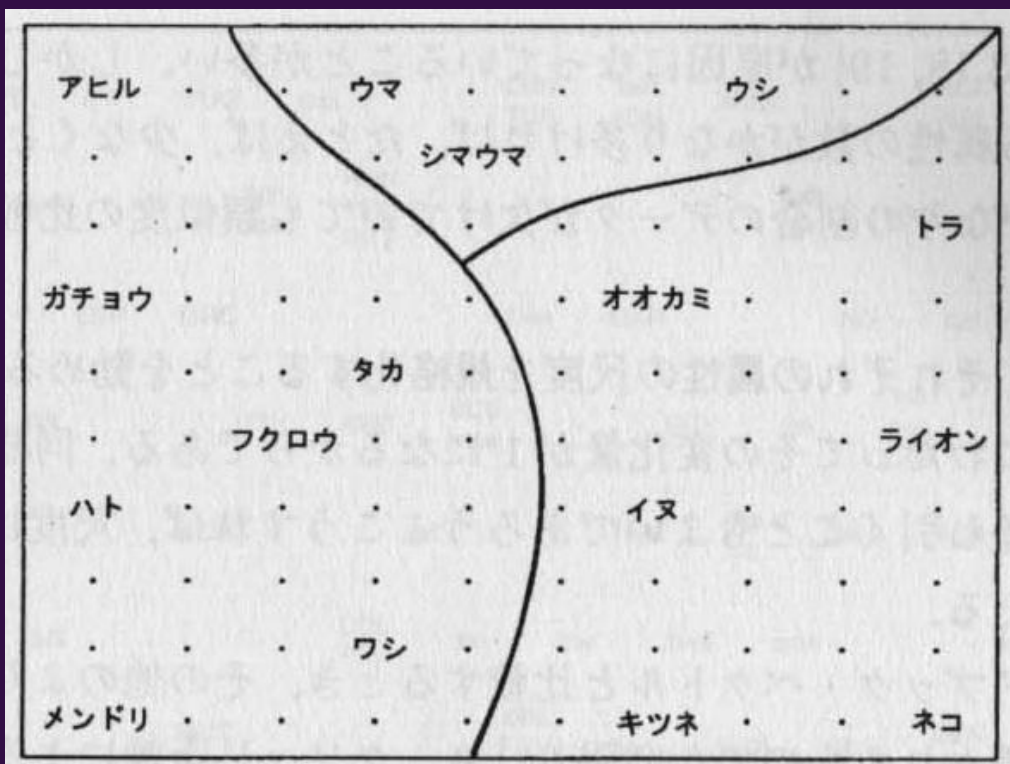
data

Efficient Market Hypothesis (Fama 1970)

We can compare the markets by using data regardless of economic situations.

Self-Organizing Map (SOM, 自己組織化マップ)

classifies the vector data and is a data **visualization technique** which reduce the dimensions of data through the use of self-organizing neural networks.



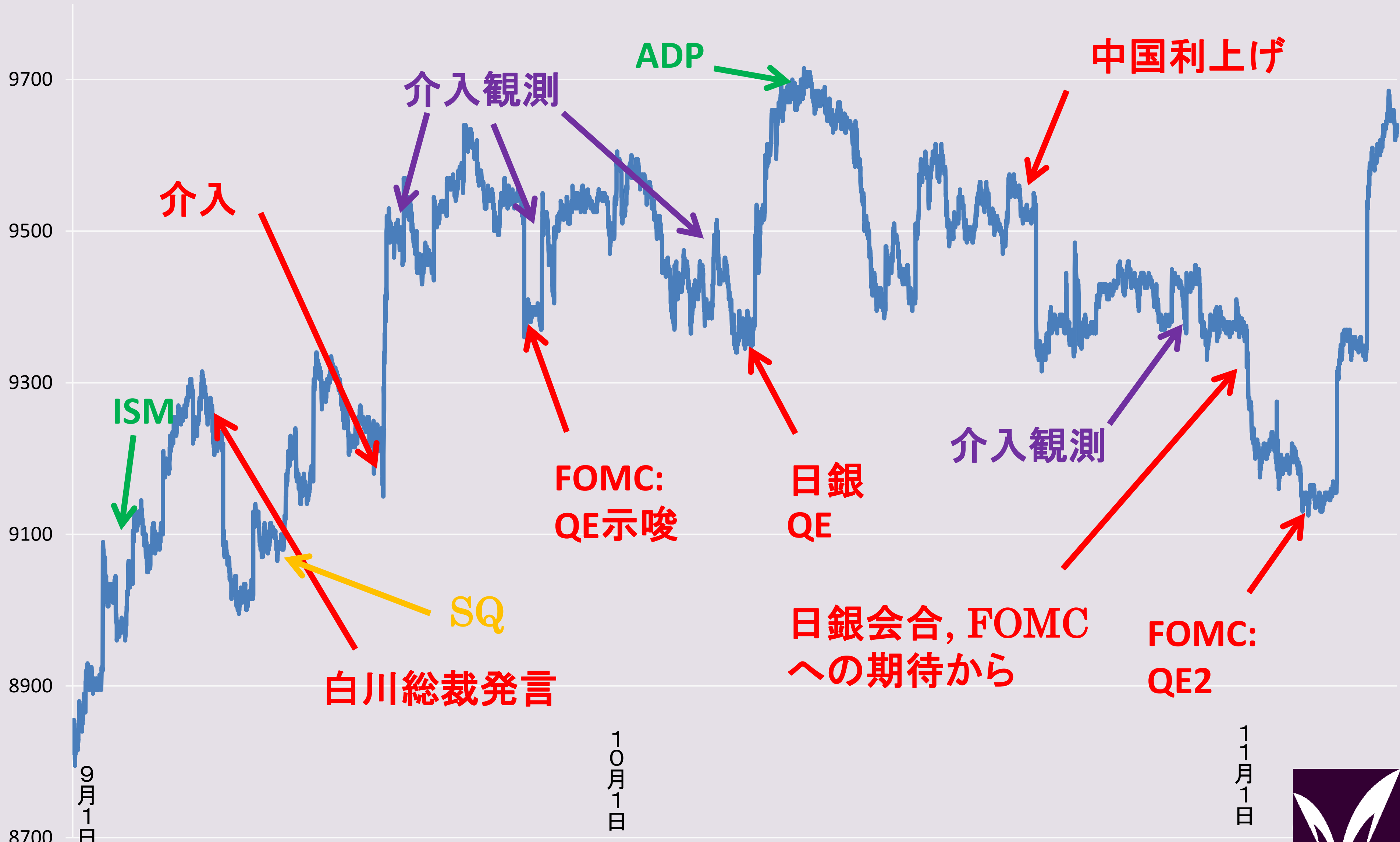
T.Kohonen「自己組織化マップ」

Algorithm : Hebbian rule (Spin-Glass)

similar samples are mapped **close** together and dissimilar **apart**.

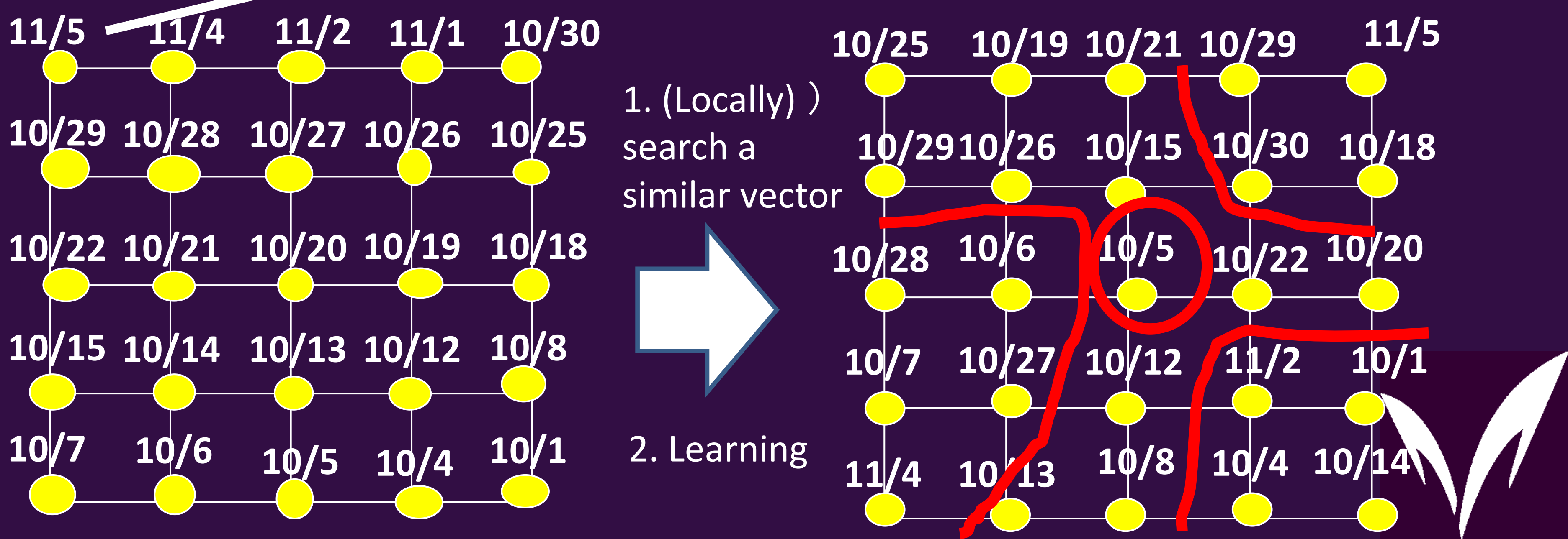
Nikkei 225 Future, 1分足 (2010/9/1-11/05, 44日間)

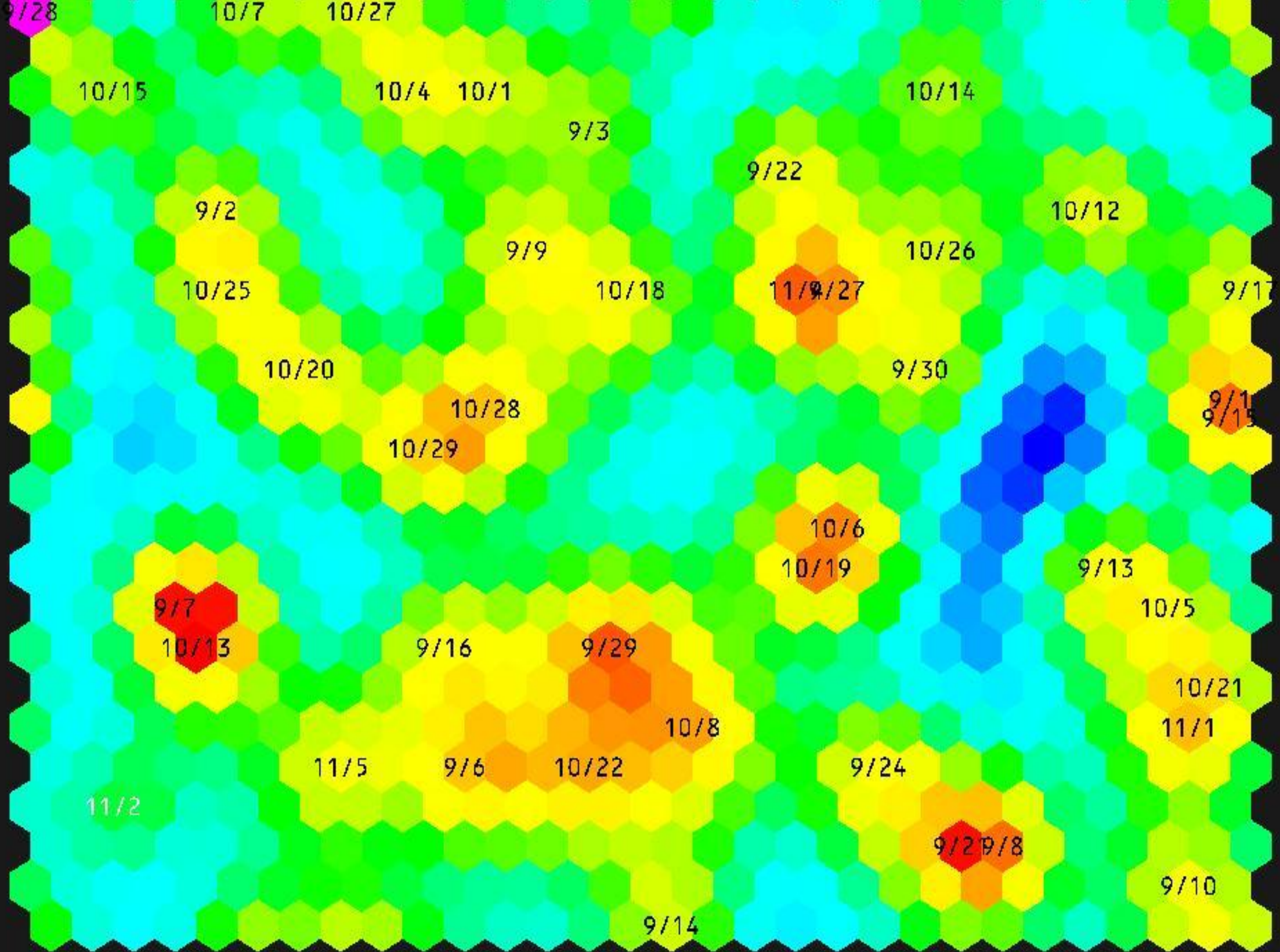
Data



In Detail

- Nikkei 225 Mini Futures (1 month) : 2010/9/1-2010/11/5 (44 days)
- Data : price dynamics every one minute in a day (states: ↑,↓,→)
- Example : 11/5 {0 1 0 0.5 0.5 , …, 0.5 1 1 0} (281)

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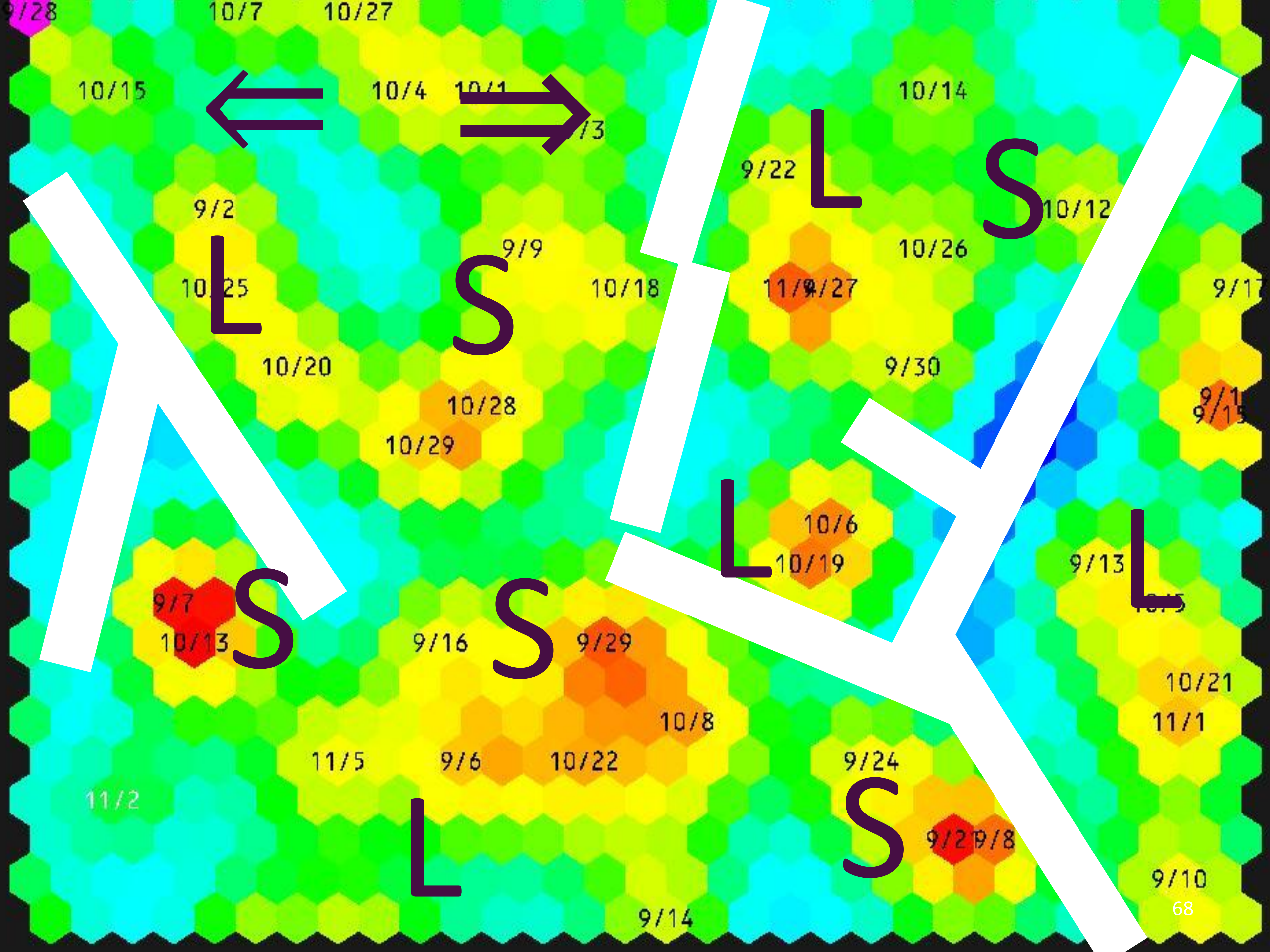
9/14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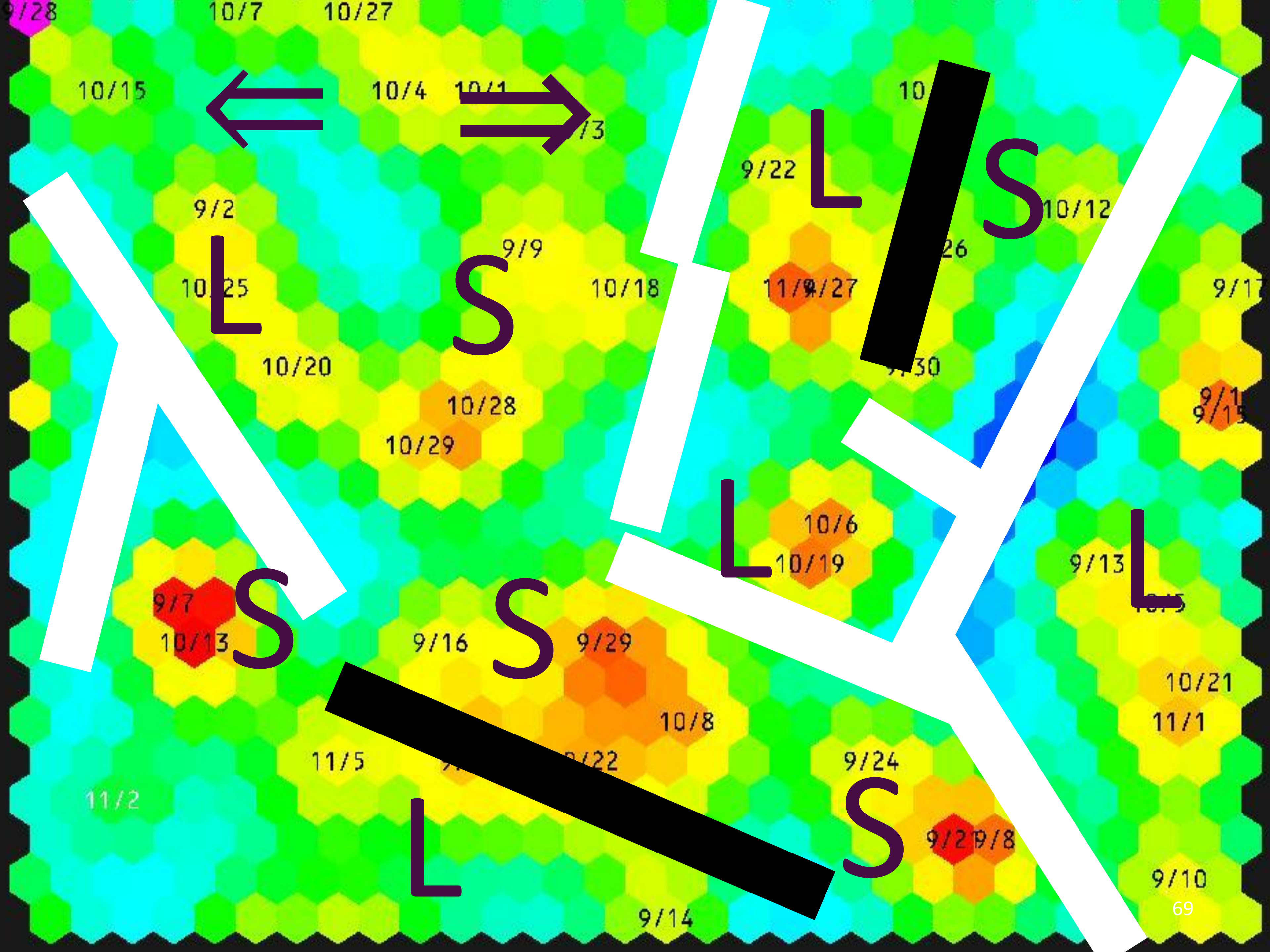

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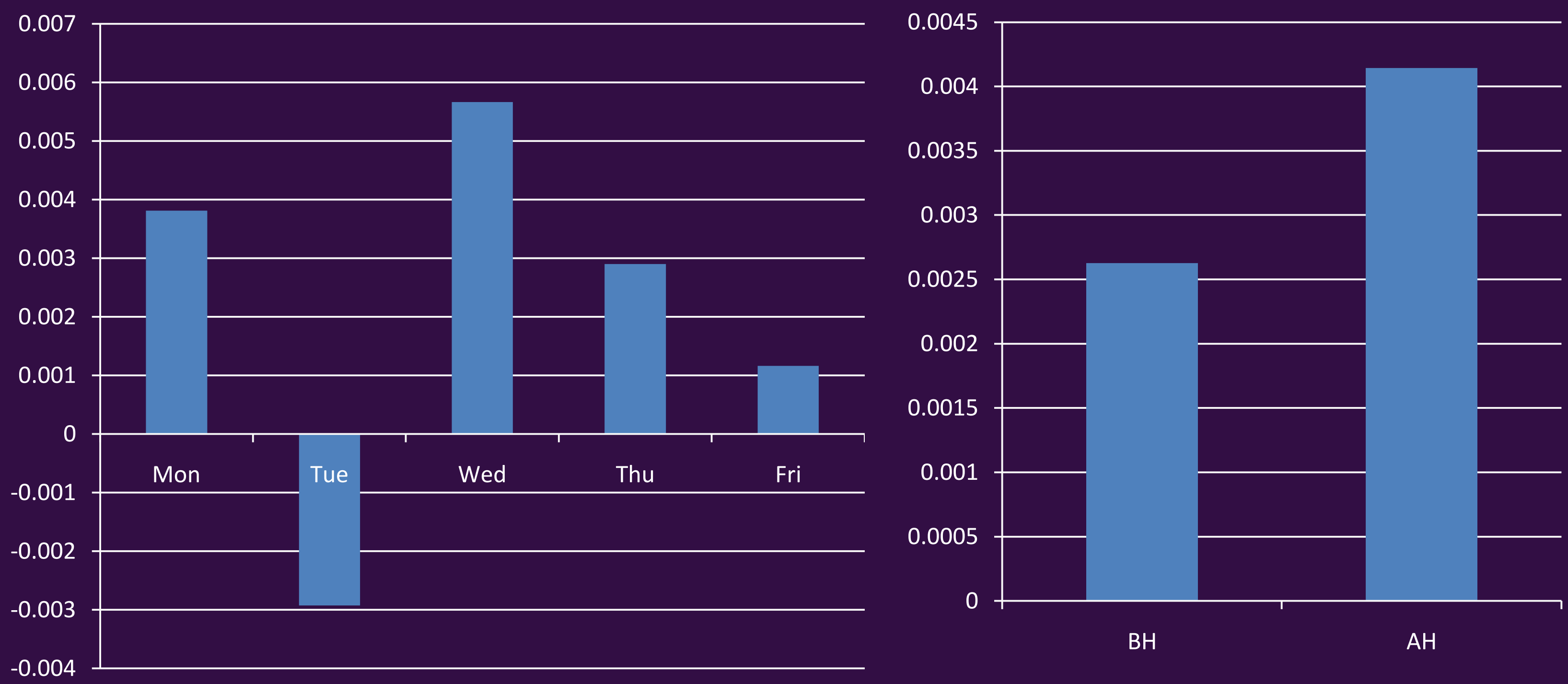
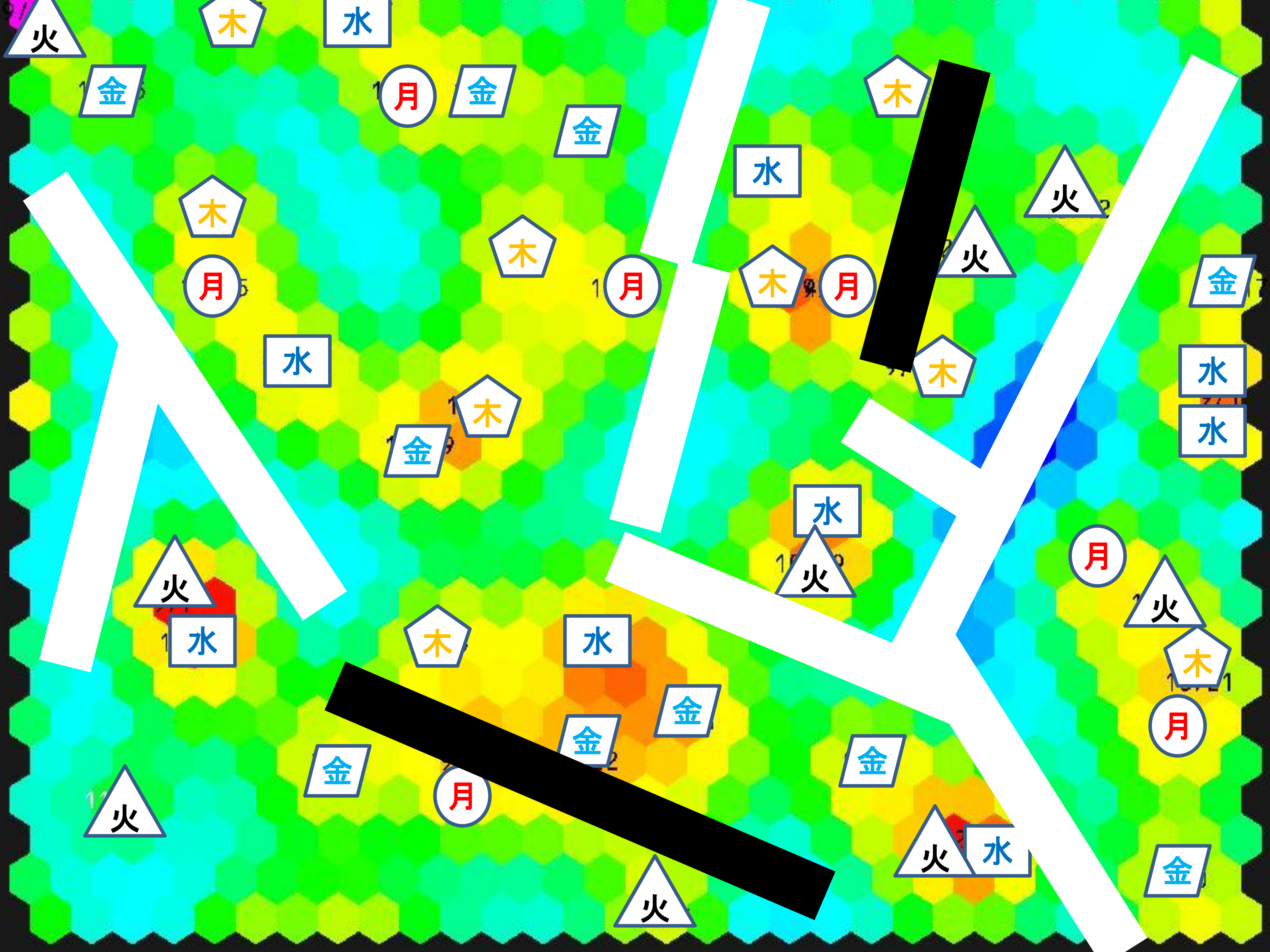
Day of the week effect (French (1980))

- 一般には、週末の曜日における収益率(R(t)=(S(t)-S(t-1))/S(t-1))は他の曜日に比べて高く，月曜日の収益率は他の曜日に比べて低い。また休日前の取引日の収益率が高く，休日明けは平均して低い。

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6. SUMMARY AND FUTURE WORKS

Summary

Modeling

- FORMULATING the financial market focused on the **order book**.
- This market is treated as a **double auction**.

Short-term

- PREDICTING the future execution price with **Kikkawa (2009)**.
- Micro-Econometrics (**Multinominal Logit model**)

Long-term

- **Replicator equation**: Risk attitudes
- PREDICTING the market behavior with the market's similarity. (**SOM**)

Practical Use (Excel)

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[7] Myerson, Roger B. and Satterthwaite, Mark A. : ‘’Efficient Mechanisms for Bilateral Trading,” Journal of Economic Theory, Vol.29 (1983), pp. 265-281. **[HP]**

[You Tube] mitsurukikkawa’s Channel : http://www.youtube.com/mitsurukikkawa

Thank You For Your Attention

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http://kikkawa.cyber-ninja.jp/