

# Option Market Analysis with Evolutionary Game Theory (進化ゲーム理論を用いた オプション市場分析)

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# This Talk (本報告)

- ANALYZEING the financial market (high-frequency )with **Evolutionary game theory**.(金融市場において、進化ゲーム理論を用いて、分析する)
- **PREDICTSING** the next market state with Stability Analysis.(安定性の概念を用いることによって、次期の市場の状態を予測する)
- EXAMINING the **Real Market** (Future Market) to apply this model. (構築したモデルをもとに、実際の市場を分析する)
- MOVIE (avi)



# OUTLINE

1. Introduction (Motivation)
2. Related Literatures and Review
3. Model
4. Apply this model to the Future market (Nikkei 225)
5. Extension (Risk Attitude)
6. Empirical Evidence (Multi-Nominal Logit Model)
7. Option Market (Black-Sholes Eq.)
8. Summary (Future works)



# 1. INTRODUCTION



# Motivation (動機)

- For **Practical Use** (実務への応用を目指して)  
More **Detail** (より具体的で), More **Useful** (より役に立つ)  
→ We construct the market from the **order book**.  
(板情報に着目)
- + Use the “Real Data” (実際のデータを取り扱う)



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# Research Fields (this study)

**Market (市場)**



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Market (市場)



General  
Equilibrium  
(一般均衡理  
論)





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Arrow and  
Debreu (1954),  
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(1973), ...

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# Research Fields (this study)

Invisible Hand (神  
の見えざる  
手) ?

Market (市場)

Arrow and  
Debreu (1954),  
Debreu (1959) ...

Black-Sholes  
(1973), ...

Heat Equation  
(熱方程式)?,  
Micro-  
Foundation ?

General  
Equilibrium  
(一般均衡理  
論)

Mathematic  
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# Research Fields (this study)

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**Game  
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(ゲーム理論)



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Arrow and  
Debreu (1954),  
Debreu (1959) ...

Black-Sholes  
(1973), ...

Dynamic Matching  
and Bargaining  
Game, Strategic  
Market Game,  
Auction

**General  
Equilibrium**  
(一般均衡理  
論)

**Mathematic  
al Finance**  
(数理ファイナ  
ンス)

**Game  
Theory**  
(ゲーム理論)



## **2. RELATED LITERATURES AND PRELIMINARIES**



# Related Literatures(先行研究)

- **Micro Structure**

Roughly speaking, we analysis the agents' behavior from the financial data.(データから市場参加者の行動を探る)

- **Method: Evolutionary Game Theory (進化ゲーム理論)**

→ Esaley and O'hara (1992) [\[HP\]](#)

- **Applied Evolutionary Game Theory**

- 川西 (2008) [\[amazon\]](#)



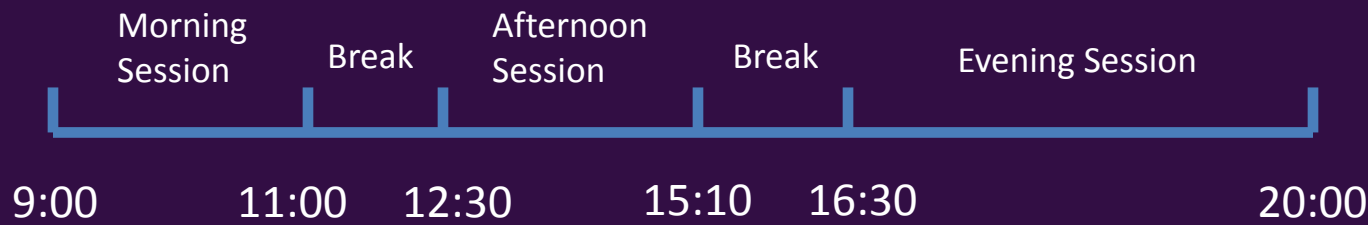


# How are stock prices determined ?

- Stock prices are determined by two methods, the *Itayose*(板寄せ) and *Zaraba*(ザラバ) methods. The *Itayose* method is mainly used to decide opening and closing prices; the *Zaraba* method is used during continuous auction trading for the rest of the trading session.

→ The stock price are determined by Rule.

**[Nikkei 225 Future Market(日経225先物)] [1day]**



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[Nikkei 225 Future Market(日経225先物)] [1day]



# Two Principles (2つの原則)

## 1) Price Priority (価格優先の原則)

means that the lowest sell and highest buy orders take precedence over other orders.

## 2) Time Priority (時間優先の原則)

means that among orders at the same price, the order placed earliest takes precedence.

Offer(sell)	Price	Bid (buy)
-----		
A 3000(5), C 4000(4)	502	early ← → late
-----		
D 10000(3), E 9000(2), F 5000(1)	501	
-----		
	500	H 80000(1), B 1000(2), J 4000(3)
-----		
late ← → early	499	H 1000(4), B 150000(5)



# The Order Book (板情報)

(Offer(sell))	Price	(Bid (buy))
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

In this case, opening price is 500 or 501.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology



# Assume: opening price is 500.

(Offer(sell))	Price	(Bid (buy))
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

- The market orders of 4000 shares to buy and 6000 shares to sell are matched, leaving sell orders of 2000 shares.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology



# Second Step

(Offer(sell))	Price	(Bid (buy))
2000	Market orders	
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

- The market sell orders of 2000 shares and sell orders 6000 shares at limit prices of 499 or less are matched with the buy orders of 8000 shares at limit prices of 501 or more. Thus far, 12000 shares have been matched in total.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology



# Third Step

(Offer(sell))	Price	(Bid (buy))
-----		
Market orders		
-----		
8000	502	
-----		
20000	501	
-----		
4000	500	10000
-----		
	499	8000
-----		
	498	30000

- Finally, the sell orders of 4000 shares at a limit price of 500 are matched with the buy orders of 10000 shares at a limit price of 500. Although this still leaves buy orders of 6000 shares at 500.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology



# Fourth Step

(Offer(sell))	Price	(Bid (buy))
-----		
Market orders		
-----		
8000	502	
-----		
20000	501	
-----		
	500	6000
-----		
	499	8000
-----		
	498	30000

- Thus the opening price is determined at 500 and transactions of 16000 shares are completed at 500.

The stock price and the trade depend on the **order book**. (価格や取引の可否は板情報によって決定する。)





### 3. MODEL



# Model (モデル)

- **Players...** large population : seller and buyer, potentially (大人数の潜在的な売り手と買い手)

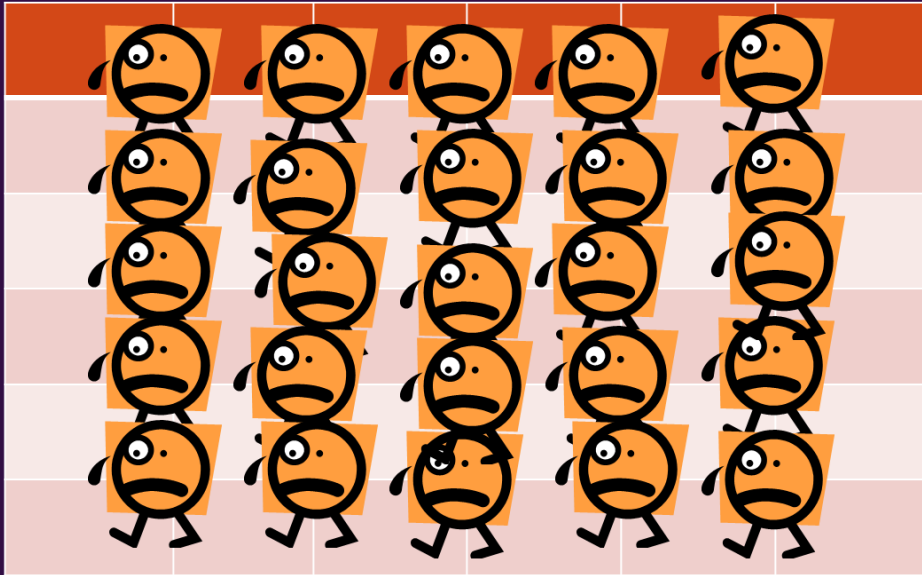
**Seller and Buyer trade an asset.**

- **Goods (財)** ... 1財
- **Strategy (戦略)**...  $n (< \infty)$  個

Here, the strike price : how much do you buy or sell an asset. (ここでは購入、売却価格)

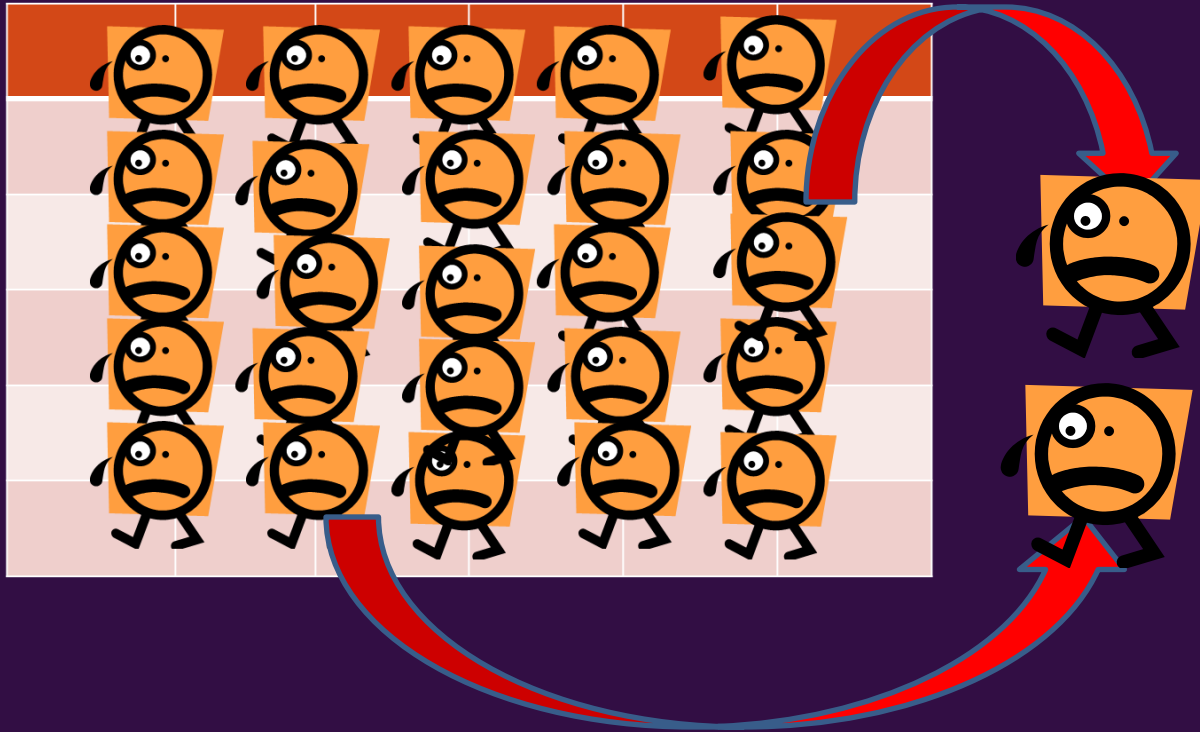


# Situation (Traditional Evolutionary Game Theory)



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At Random (infinitely)



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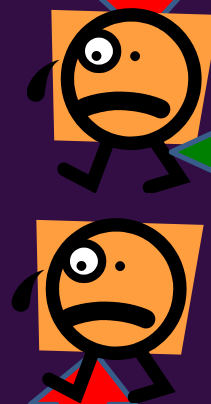
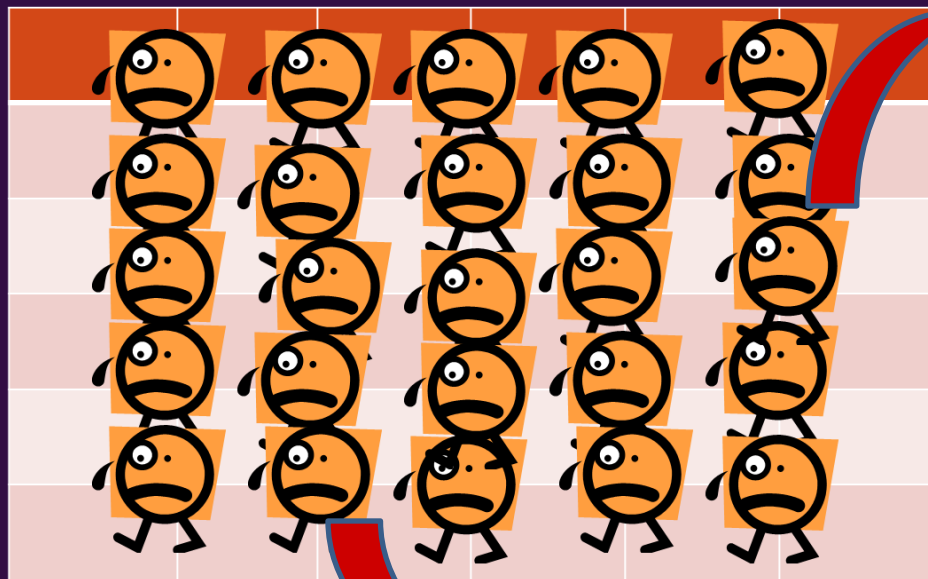


Another players look at the game.



# Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



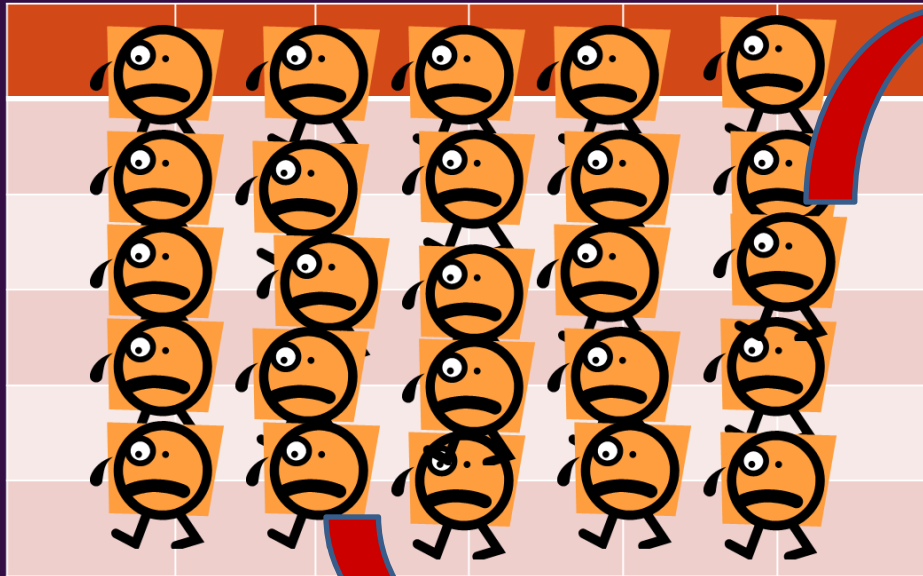
Play a  
game

Another players look at the game.



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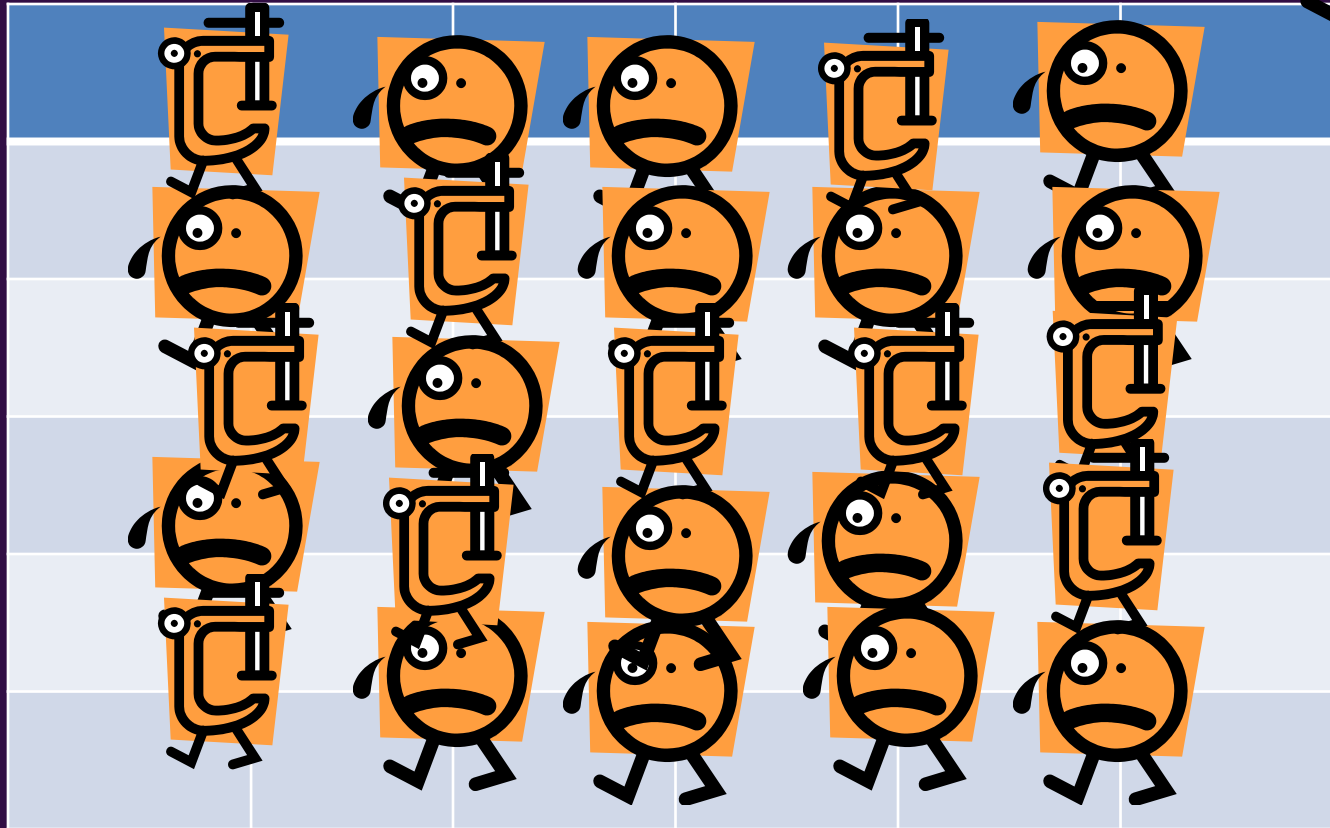
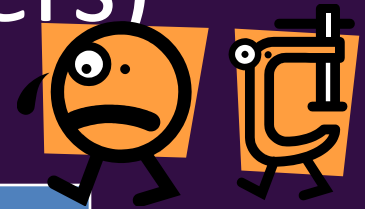
Play a  
game

Another players look at the game.

Replicator Equation



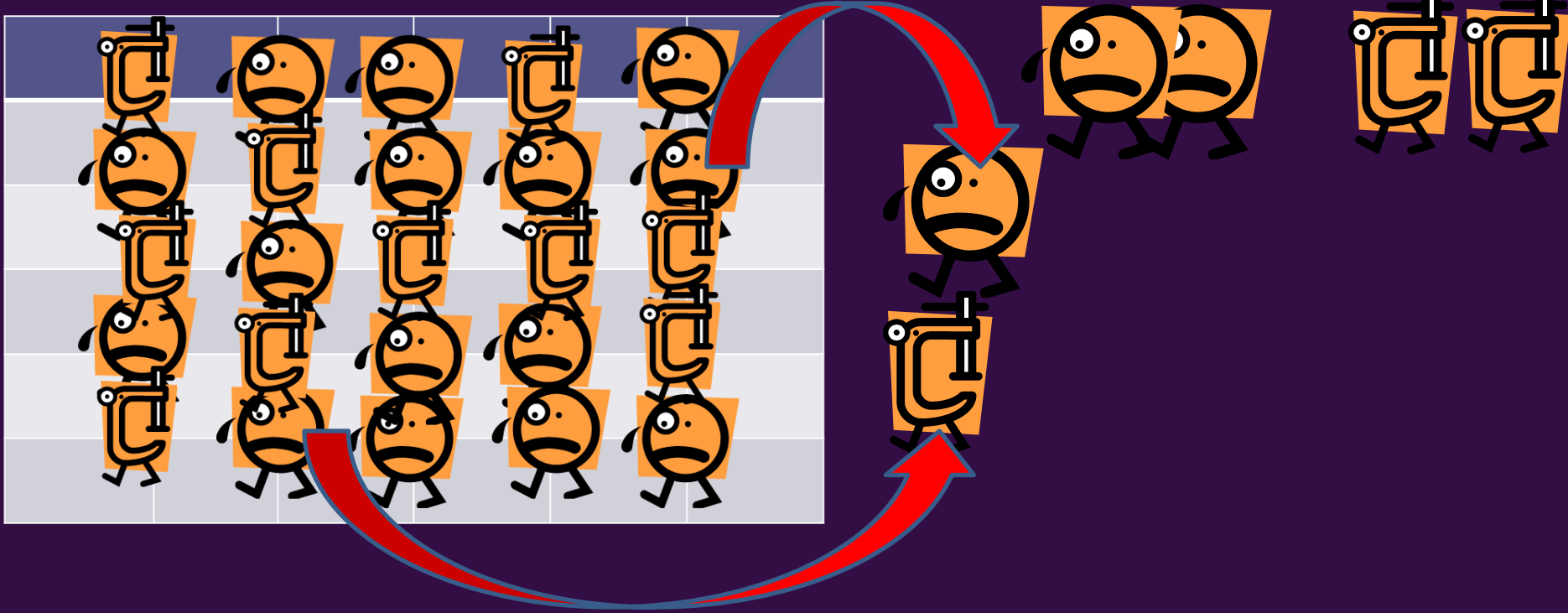
# Situation (two types players)





# Situation

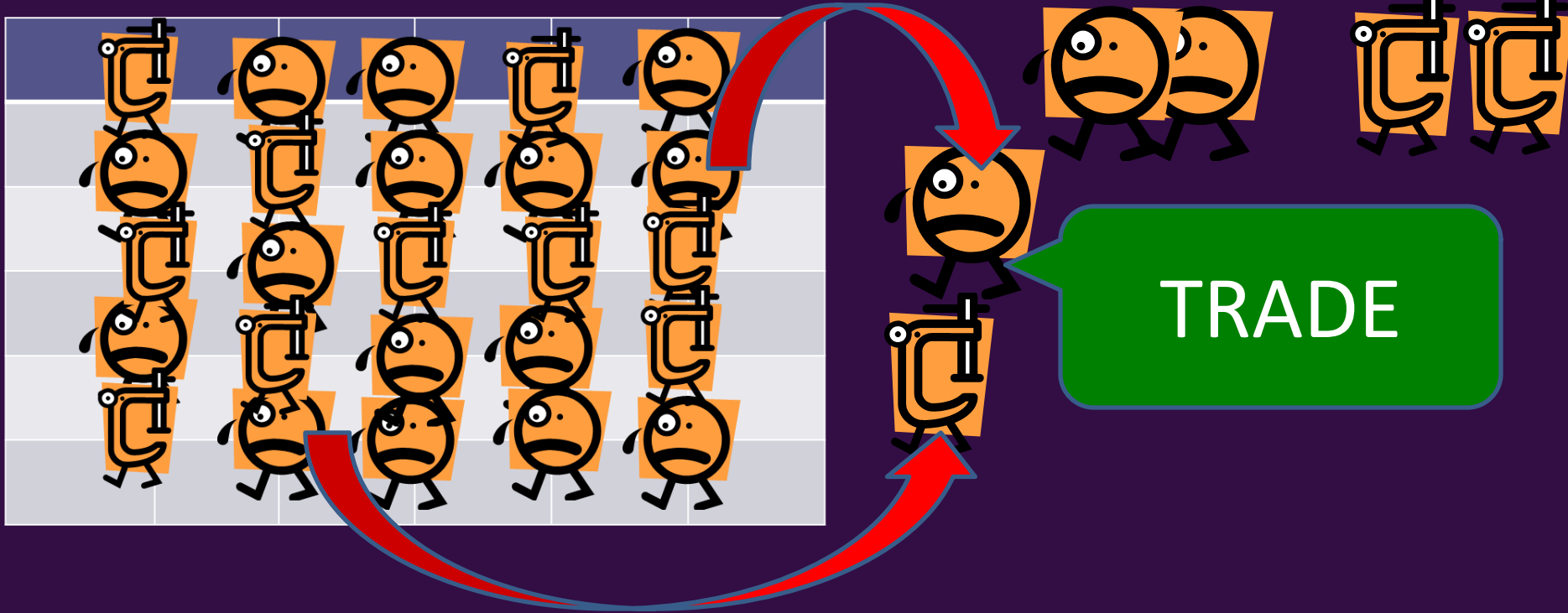
At Random



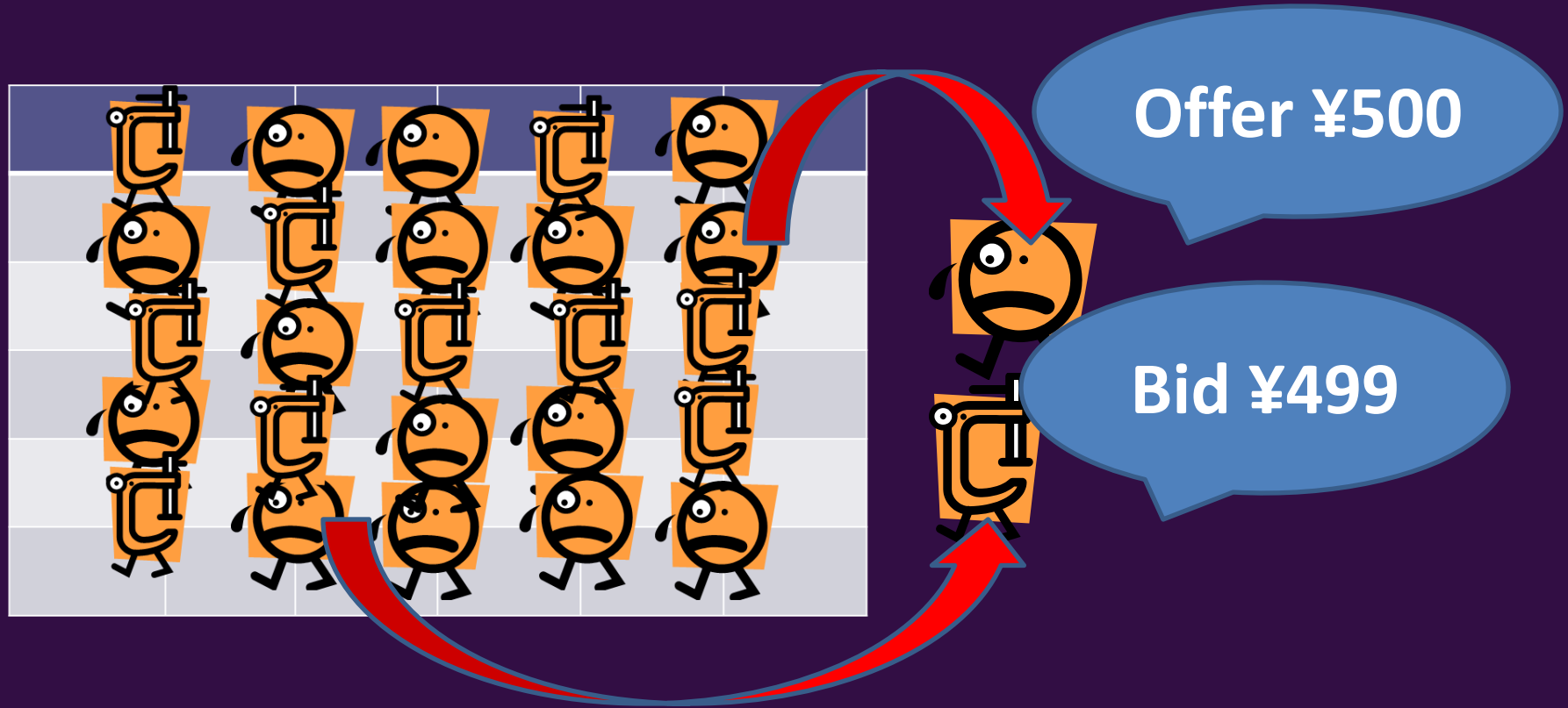
# Situation

No Trade

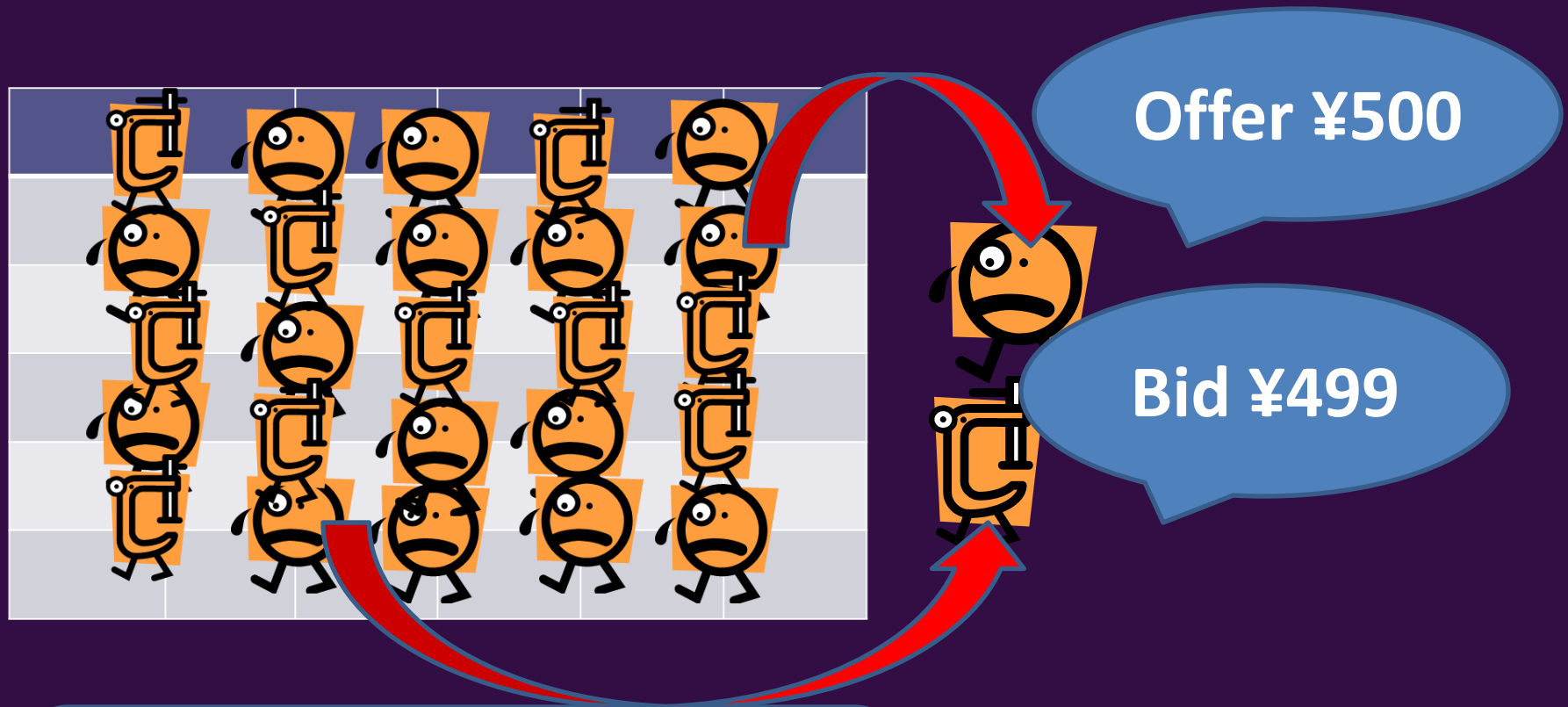
At Random



# Situation



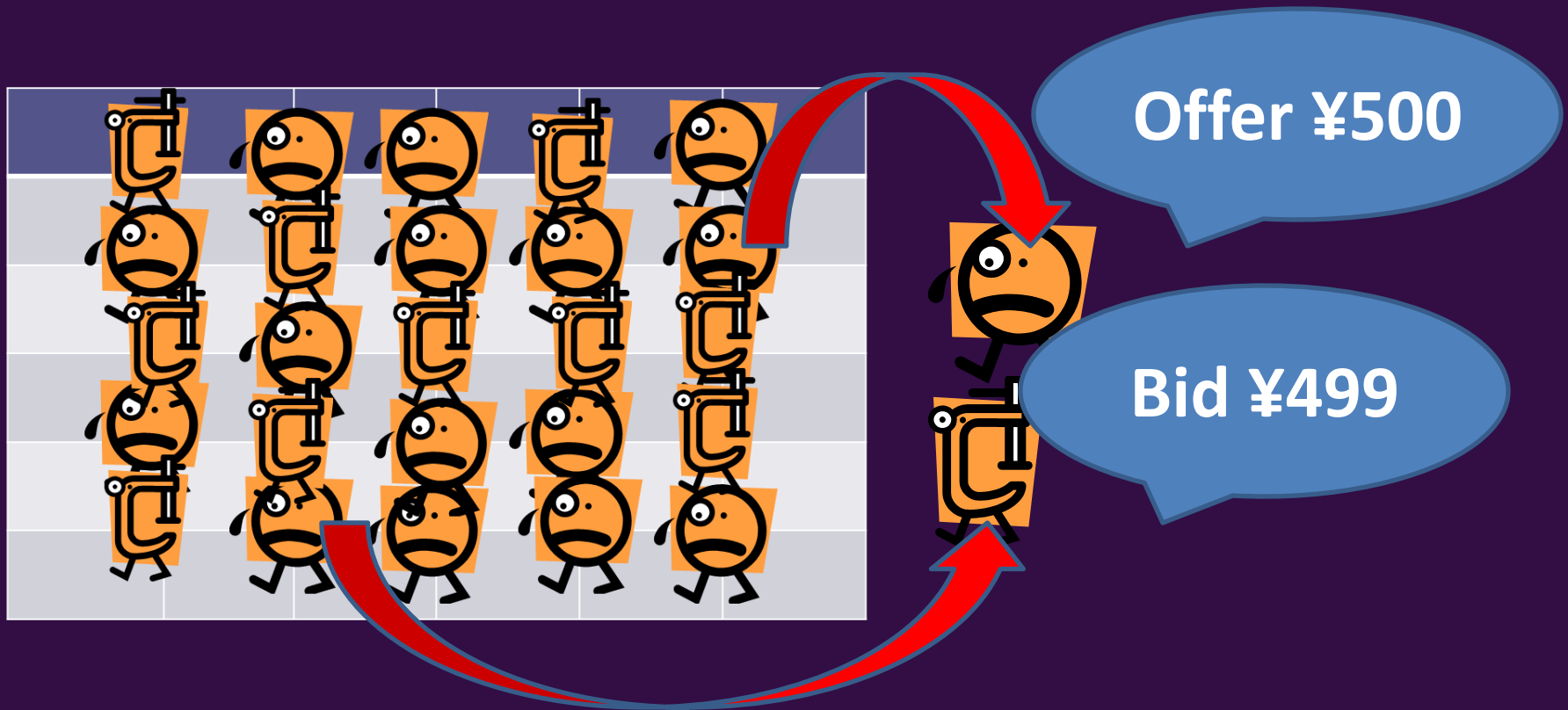
# Situation



Stock Exchange which take account of the order book decides the trade's contract. (取引所が板情報をもとに、売買契約を決定する)



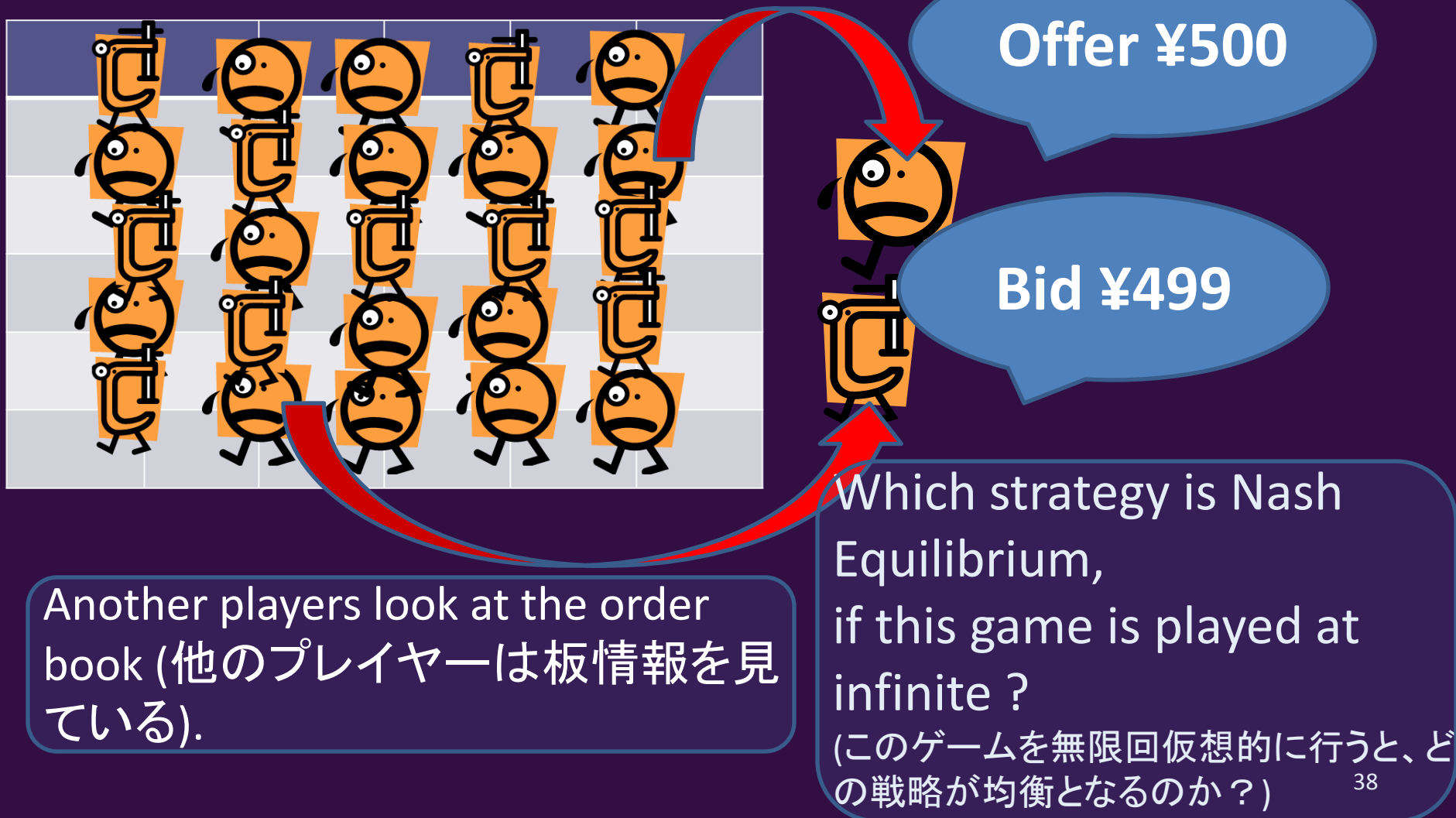
# Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).



# Situation



# Model (モデル)

- **Payoff (利得) ... Buyer :  $S(t)-K$ , Seller :  $K-S(t)$**

where  $S(t)$  : current stock price, Brownian Motion,  $K$ : strike price (行使価格)

- **Replicator Equation**

$$\begin{aligned}\frac{dx_i(t)}{dt} &= x_i(t) \left( g_i(t) - \bar{g}(t) \right) \\ \frac{dy_i(t)}{dt} &= y_i(t) \left( h_i(t) - \bar{h}(t) \right)\end{aligned}$$

where  $x_i, y_i$  : the probability of choosing the strategy 1 for each player.  $g_i, h_i$  : the payoff when each player chooses the strategy 1.



# EVOLUTIONARILY STABLE STRATEGY (ESS)

DEF.: Weibull(1995):  $x \in \Delta$  is an *evolutionarily stable strategy (ESS)* if for every strategy  $y \neq x$  there exists some  $\bar{\varepsilon}_y \in (0,1)$  such that the following inequality holds for all  $\varepsilon \in (0, \bar{\varepsilon}_y)$ .

$$u[x, \varepsilon y + (1 - \varepsilon)x] > u[y, \varepsilon y + (1 - \varepsilon)x].$$





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**INTERPRETATION:** incumbent payoff (fitness) is higher than that of the post-entry strategy

(ESS : ①the solution of the Replicator equation + ② asymptotic stable.)



# PROPOSITION

**PRO.**(Bishop and Cannings (1978)):  $x \in \Delta$  is evolutionary stable strategy if and only if it meets these first-order and second-order best-reply :



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$$(2.4) \quad u(y, x) \leq u(x, x), \quad \forall y,$$

$$(2.5) \quad \begin{aligned} &u(y, x) = u(x, x) \\ &\Rightarrow u(y, y) < u(x, y), \end{aligned} \quad \forall y \neq x,$$



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Nash Eq.

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$$(2.5) \quad \begin{aligned} &u(y, x) = u(x, x) \\ &\Rightarrow u(y, y) < u(x, y), \end{aligned} \quad \forall y \neq x,$$

Asymptotic Stable  
Conditon



# Two Strategies Case (戦略の数が2つ):

- Replicator equation (see next slide)

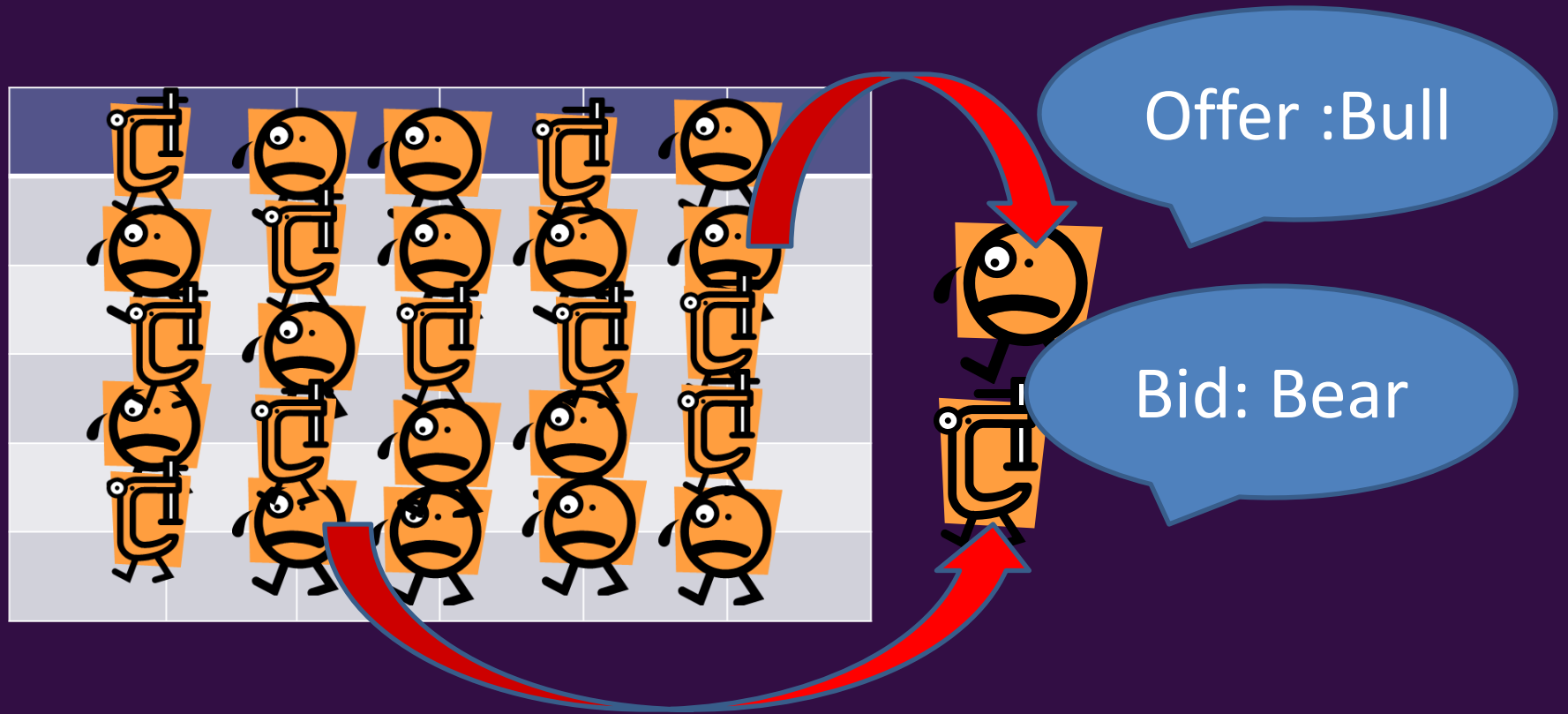
- $\dot{x} = x(1-x)\{-b(t) + (a(t) + b(t))y\},$
- $\dot{y} = y(1-y)\{b(t) - (a(t) + b(t))x\},$

where  $x, y$  is the probability of choosing the strategy 1, 2 for each player.

		Player 2	
		S1	S2
player1	S1	$a(t), -a(t)$	0,0
	S2	0,0	$b(t), -b(t)$



# Situation



# Replicator Equation

REPLICATOR EQ.

$$\dot{x}_i = x_i \left( \left( Ax \right)_i - x \cdot Ax \right), i = 1, \dots, n.$$

If the player's payoff from the outcome  $i$  is greater than the expected utility  $x \cdot Ax$ , the probability of the action  $i$  is higher than before.



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Two Strategies

$$\dot{x} = x(1-x)\{b - (a+b)x\} \dots (*)$$

Classification

(I) **Non-dilemma**:  $a > 0, b < 0$ , ESS : one

(II) **Prisoner's dilemma** :  $a < 0, b > 0$ , ESS : one

(III) **Coordination** :  $a > 0, b > 0$ , ESS two

(IV) **Hawk-Dove** :  $a < 0, b < 0$ , ESS one (mixed strategy)

	2	
	S 1	S 2
1	S 1	a,a
	S 2	0,0

Payoff Matrix

# Replicator Eq. and Payoff Matrix

- Strategy : Two, Player : Two

- Payoff

$$P^1 = \begin{pmatrix} f_1 & f_3 \\ f_2 & f_4 \end{pmatrix}, P^2 = \begin{pmatrix} g_1 & g_3 \\ g_2 & g_4 \end{pmatrix}$$

- Replicator Equation

- $$\dot{y} = y(1-y)\{f_1 - f_2 + x(f_3 - f_4 - f_1 + f_2)\}$$

- $$\dot{x} = x(1-x)\{g_4 - g_2 + y(g_3 - g_4 - g_1 + g_2)\}$$

x is the probability of the type 2 player chooses the strategy 2.  
y is the probability of the type 1 player chooses the strategy 1.



$$f_1 - f_2 := a, f_4 - f_3 := c, g_4 - g_2 := d, g_1 - g_3 := b$$

Derive

$$\dot{y} = y(1-y)\{a - (a+c)x\}, \dot{x} = x(1-x)\{d - (b+d)y\}$$

## Classification

**(I) Non-Dilemma, Prisoner's Dilemma :**

$$ac < 0, bd > 0, \text{ ESS :1}$$

**(II) Coordination :**

$$a > 0, b > 0, c > 0, d > 0, \text{ ESS :2}$$

**(III) Chicken :**

$$a < 0, b < 0, c < 0, d < 0, \text{ ESS :2}$$

**(IV) Matching Pennie:**

$$ab < 0, cd < 0, ac > 0, bd < 0, \text{ ESS: Mixed}$$

	S1	S2
S1	a,b	0,0
S2	0,0	c,d



# Prediction (予測)

- Replicator equation divided by  $xy(1-x)(1-y)$  :

$$\dot{x} = -\frac{b(t)}{y} + \frac{a(t)}{1-y}, \quad \dot{y} = \frac{b(t)}{x} - \frac{a(t)}{1-x}.$$

- Discrete the above equations:

$$x(t + \varepsilon) = x(t) - \left( \frac{b(t)}{y} + \frac{a(t)}{1-y} \right) \varepsilon,$$
$$y(t + \varepsilon) = y(t) + \left( \frac{b(t)}{x} - \frac{a(t)}{1-x} \right) \varepsilon.$$



# Payoff Matrix (利得表)

i)  $\uparrow$  (UP)

N.E.  $(s1,s2),(s2,s2)$

	S 1	S 2
S 1	+, -	0,0
S 2	0,0	0,0

ii)  $\downarrow$  (Down)

N.E.  $(s1,s1),(s1,s2)$

	S 1	S 2
S 1	0,0	0,0
S 2	0,0	-, +

iii)  $\rightarrow$  (No change)

N.E.  $(s1,s2)$

	S 1	S 2
S 1	+, -	0,0
S 2	0,0	-, +

i),ii),iii)  $\rightarrow (s1,s2) (x \rightarrow 1, y \rightarrow 0)$

# Second Step

(Offer(sell))	Price	(Bid (buy))
2000	Market orders	
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

- The market sell orders of 2000 shares and sell orders 6000 shares at limit prices of 499 or less are matched with the buy orders of 8000 shares at limit prices of 501 or more. Thus far, 12000 shares have been matched in total.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology



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N.E.  $(s1,s1),(s1,s2)$

	S 1	S 2
S 1	0,0	<b>0,0</b>
S 2	0,0	-, +

iii)  $\rightarrow$  (No change)

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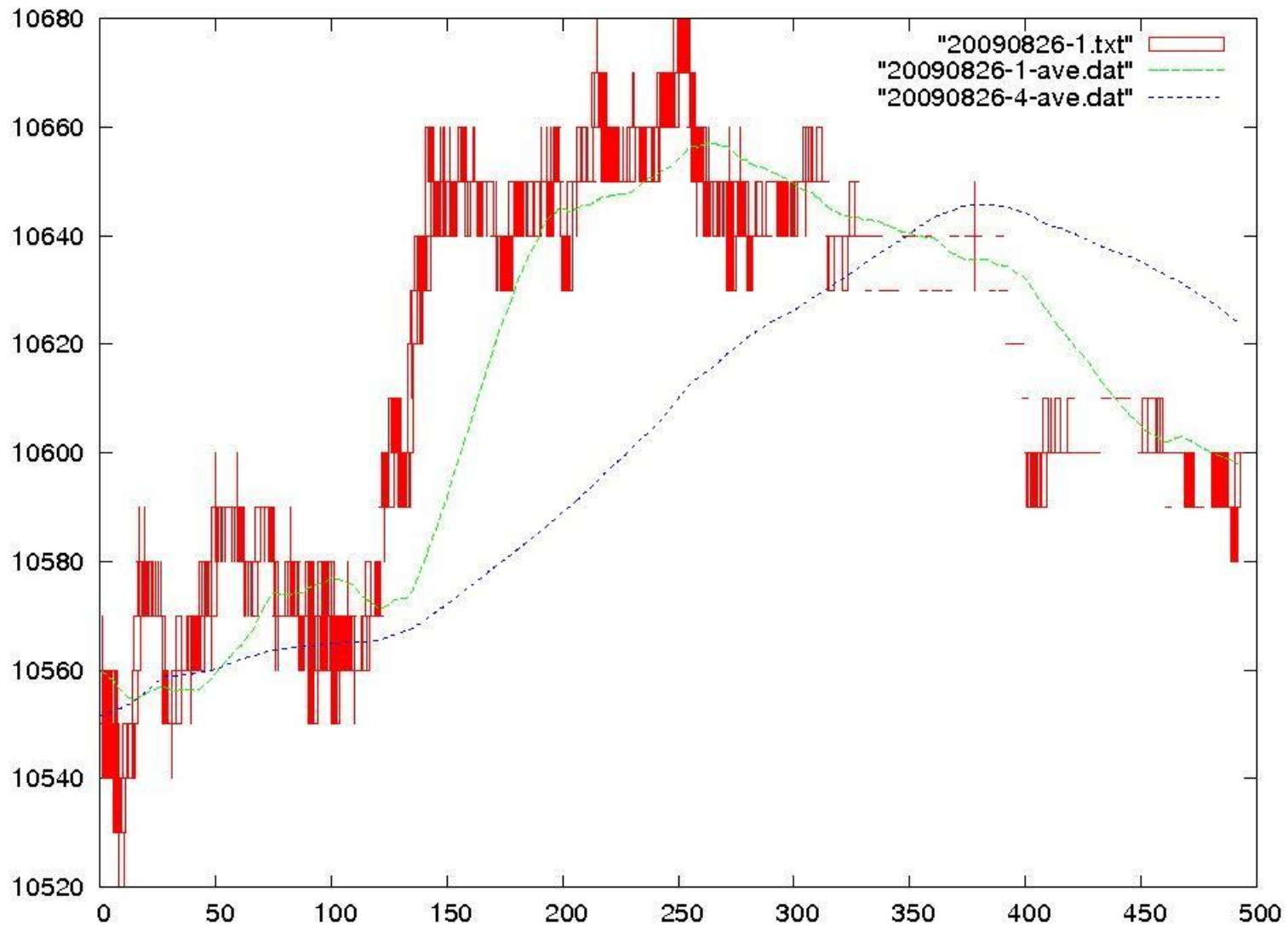
Is This OK?

4. Application:

**NIKKEI 225 FUTURE MARKET**  
**(日経225先物市場)**



# EX: 20090826



# Payoff Matrix (利得表)

i)  $\uparrow$  (UP)

N.E. (s2,s2)

	S 1	S 2
S 1	+, -	0,0
S 2	0,0	<b>+,+</b>

ii)  $\downarrow$  (Down)

N.E. (s1,s1)

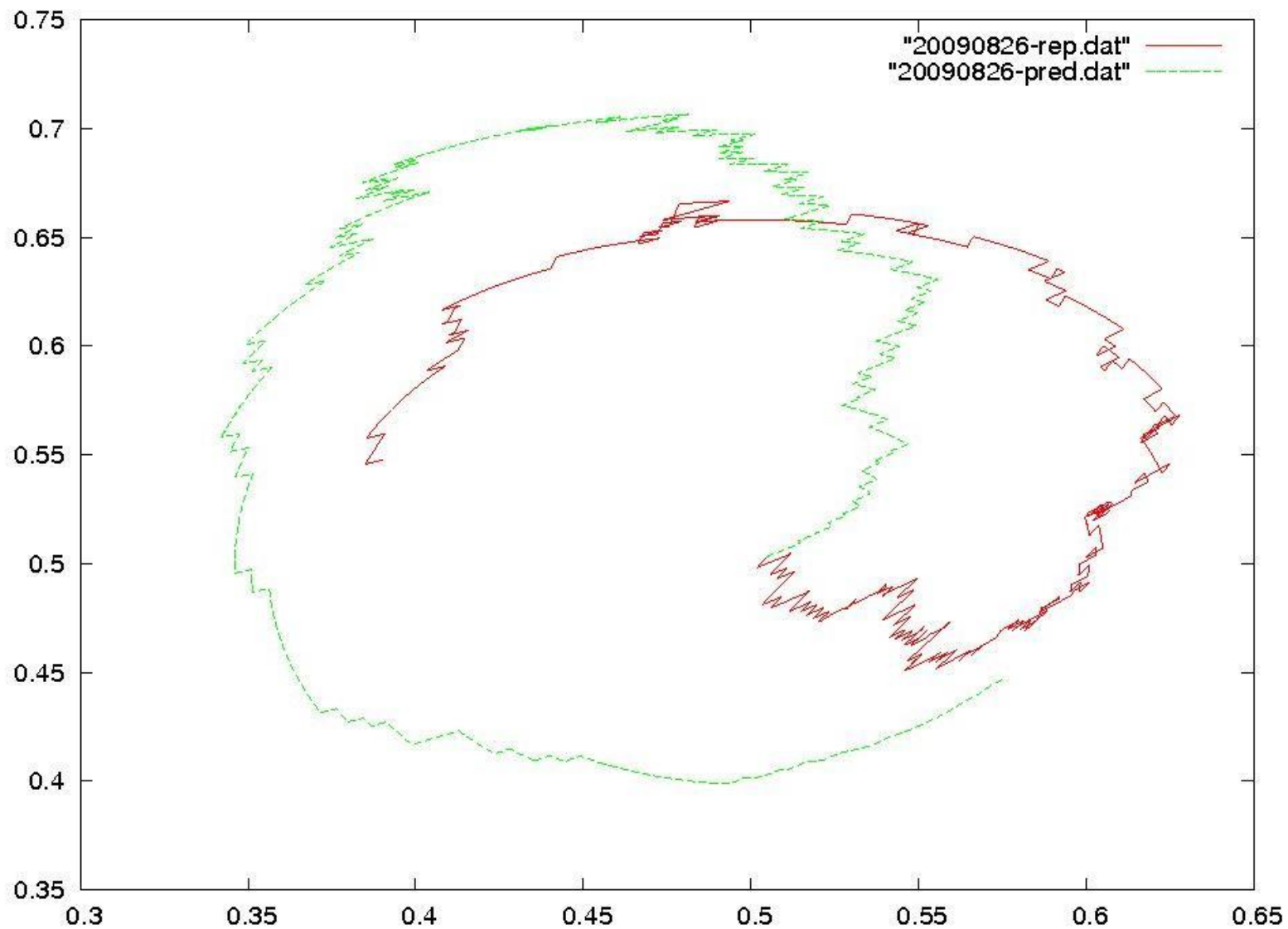
	S 1	S 2
S 1	<b>+,+</b>	0,0
S 2	0,0	-,+

iii)  $\rightarrow$  (No change)

N.E. Mixed Strategy.

	S 1	S 2
S 1	<b>-,+</b>	0,0
S 2	0,0	<b>-,+</b>

# EX: 20090826



## 5. EXTENSION: **RISK ATTITUDE**



# RISK ATTITUDE

- We assume that the own utility is linear function.(今まで主体の効用は線形であると仮定してきた。)
- Each player has the non-linear utility.(そこで非線形の場合をも考慮に入れる。)
- We examine the equilibrium selection with this nonlinear-utility. (そこでこの非線形効用関数を用いて、均衡選択の問題を考察する。)



# EXAMPLE(Allais Paradox)

- Which lotteries do you like ?
- There are three possible monetary prizes.
- First Prize       Second Prize       Third Prize
- 2500 000 USD   500 000 USD       0 USD
- The decision maker is subjected to two choice tests. The first consists of a choice between the lotteries  $L_1$  and  $L'_1$ :
- $L_1=(0,1,0)$     $L'_1=(0.10,0.89,0.01)$ .
- The second consists of a choice between the lotteries  $L_2$  and  $L'_2$ :
- $L_2=(0,0.11,0.89)$     $L'_2=(0.10,0,0.90)$ .
- $\Rightarrow L_1 > L'_1$  and  $L'_2 > L_2$ .





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- $\Rightarrow L_1 \succ L'_1$  and  $L'_2 \succ L_2$ .



- Utility function :  $g(x)$  ,  $z$  : payoff
- Taylor Expansion:
- $g(x+z)-g(x)=g'(x)z+0.5g''(x)z^2+O(z^3) \dots (*)$

**Def.** Given a (twice-differentiable) Bernoulli utility function  $u(.)$  for money, the ***Arrow-Pratt coefficient of absolute risk aversion*** at  $x$  is defined as  $r_A(x)=-u''(x)/u'(x)$ .

- (\*)  $g(x+z)-g(x) = zg'(x)(1-0.5zr_A(x))$
- (In economics, we assume  $g'(x)>0$ ,  $g''(x)<0$ )



# EQUILIBRIUM SELECTION (均衡選択)

- Under  $g'(x) > 0$ ,  $g''(x) < 0$
- $\frac{d}{dx} \left( \frac{g'(x)}{g''(x)} \right) > \alpha$

	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

## i) Non-Dilemma ( $a > 0, b < 0$ )

- $zr_A(x) < 2$  and  $a > 0, b < 0$

## ii) Dilemma ( $a < 0, b > 0$ ) : $zr_A(x) < 2$ and $a < 0, b > 0$

## iii) Coordination ( $a, b > 0$ ) : $zr_A(x) < 2$ and $a > 0, b > 0$

## iv) Hawk-Dove ( $a, b < 0$ )

- $zr_A(x) < 2$  and  $a < 0, b < 0$      $zr_A(x) > 2$  and  $a > 0, b < 0$

$zr_A(x) > 2$  and  $a < 0, b > 0$      $zr_A(x) > 2$  and  $a > 0, b > 0$



# Payoff Matrix (利得表)

i)  $\uparrow$  (UP)

N.E. (s2,s2)

	S 1	S 2
S 1	$-,-$	0,0
S 2	0,0	$+,+$

ii)  $\downarrow$  (Down)

N.E. (s1,s1)

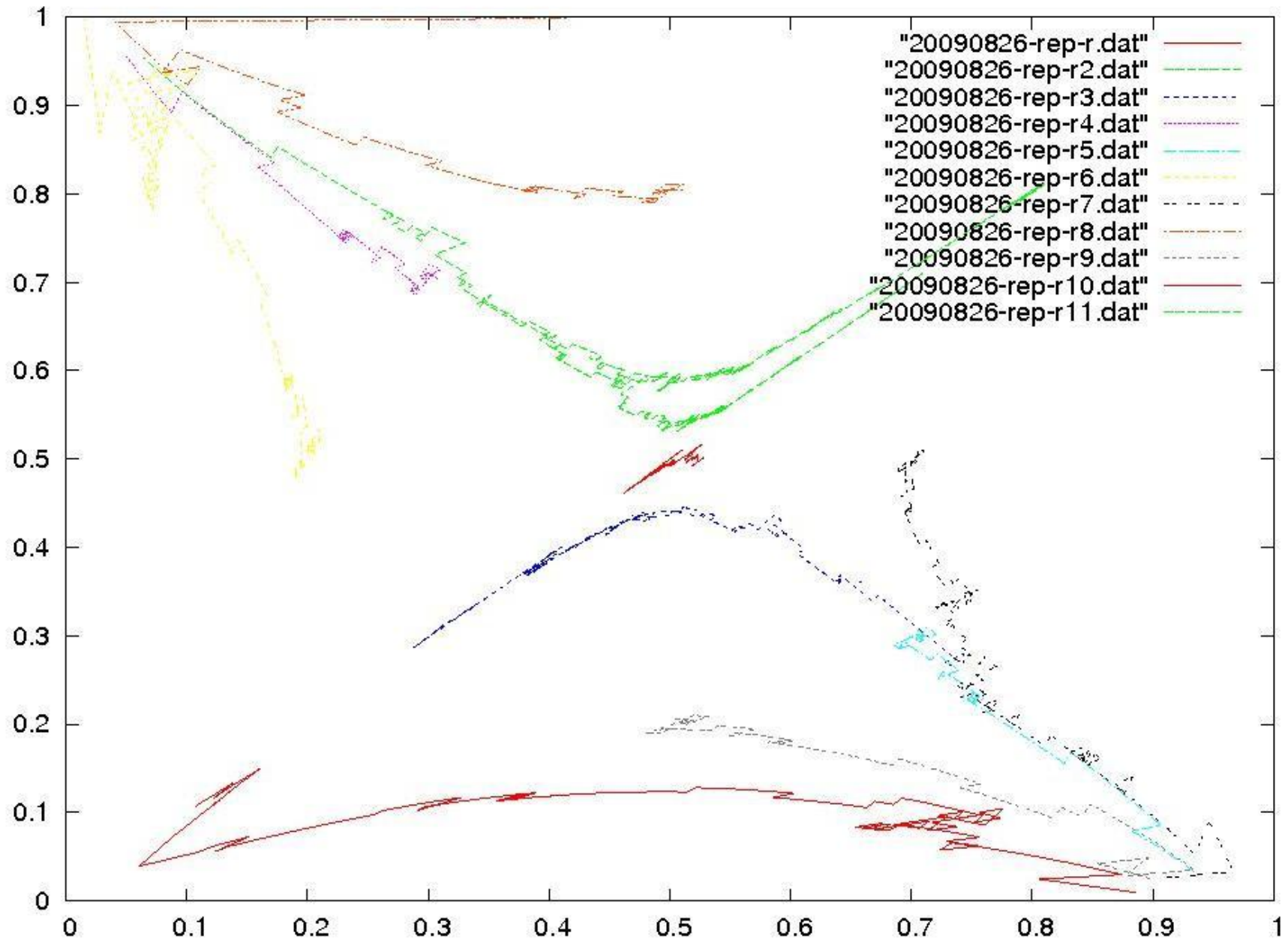
	S 1	S 2
S 1	$+,+$	0,0
S 2	0,0	$-,-$

iii)  $\rightarrow$  (No change)

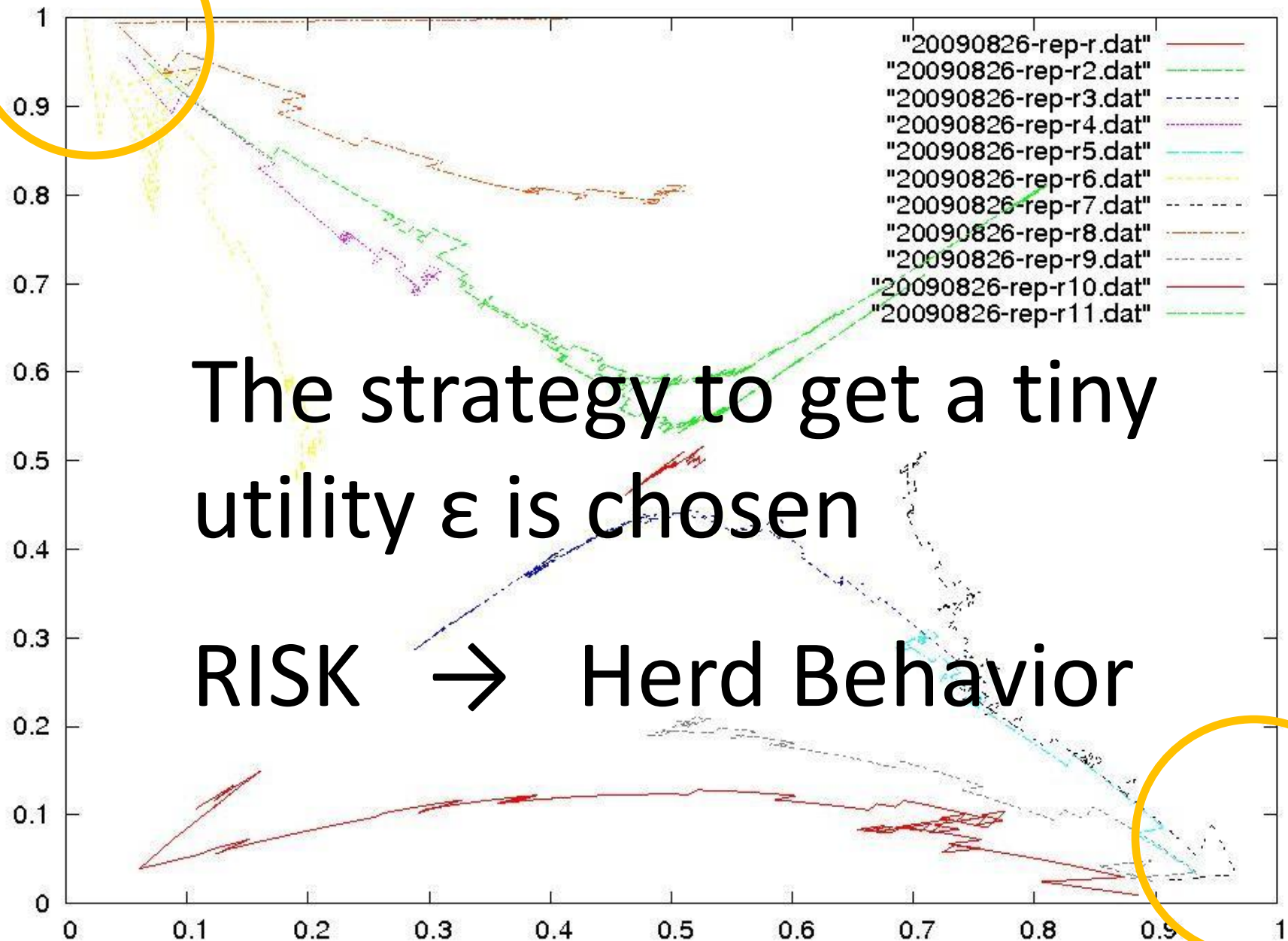
N.E. Mixed Strategy.

	S 1	S 2
S 1	$-, -$	0,0
S 2	0,0	$-, -$

# EX: 20090826 (RISK)



# EX: 20090826 (RISK)



# 6. Empirical Evidence:

## **MICRO ECONOMETRICS**



# Multinomial Logit Model (多項選択モデル)

- Order Book (板情報) is expressing that the players choose the strategy/price at that time.(ある時間におけるどの価格(戦略)を採用しているのかを表している.)
- Strategy/Price :  $J (< \infty)$  個,
- $x_i$ という属性を持った個人 $i$ が選択肢 $j$ を選ぶ確率:  $\pi_{ij}$ , (  $P(y_i=j \mid x_i) = \pi_{ij}$  )

$$\pi_{ij} = \frac{\exp(x'_i \beta_j)}{\sum_{r=1}^J \exp(x'_i \beta_r)} \quad j = 1, \dots, J.$$





# PANEL ANALYSIS(パネル分析):

- TIME SERIES(時系列): **PARTICLE FILTER**(粒子フィルタ) (Kikkawa[4]の動学に対応)
- State Space Model (状態空間モデル)
- System model: UTILITY (ex.  $Y_i = \alpha + \beta X_i + u_i$ )
- Observation model: 
$$\pi_{ij} = \frac{\exp(x'_i \beta_j)}{\sum_{r=1}^J \exp(x'_i \beta_r)} \quad j = 1, \dots, J.$$
- We examine empirical evidence : why the player chooses this strategy. (なぜこの戦略を採用したのか？を時系列に実証分析)



## 7. Application:

# OPTION MARKET

(オプション市場)



# Black-Sholes Formula in this setting

- When we derive the Black-Sholes formula, the strike price is influence by the boundary condition.(行使価格の影響があるのは、境界条件を使用するとき)
- $K := \bar{K}$

$$f(S, t) = S \cdot N\left(\frac{u}{\sigma\sqrt{\tau}} + \sigma\sqrt{\tau}\right) - \bar{K} \cdot e^{-r\tau} \cdot N\left(\frac{u}{\sigma\sqrt{\tau}}\right).$$

where  $\bar{K} =$  the strategy 1's strike price in equilibrium (平衡時の戦略1における行使価格)  $\cdot s_1^*$  + the strategy 2's the strike price in equilibrium.  $\cdot (1 - s_1^*)$   
 $s_1^*$  is the probability of choosing the mixed strategy(混合戦略を採用する場合の確率)



## 8. SUMMARY AND FUTURE WORKS



# Summary

- **MODELING** the Financial Market.
- **DERIVING** the payoff matrix for each player.
- **APPLYING** the Real Market.
- **DERIVING** the Optimal Behavior for each player.
- **CONSIDERING** the each player's Risk Attitude.



# Future Works

- **EMPIRICAL EVIDENCE (実証研究) : Particle Filter (粒子フィルタ)**
- **GET** the Online Financial Data, **CALCULATE** and **DISPLAY**. (オンラインでデータを手し、計算し、それを表示する)
- **MAKE** the software like a PUCK based on the Evolutionary Game Theory. (PUCKの進化ゲーム理論版の構築)



# Thank You For Your Attention

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<http://kikkawa.cyber-ninja.jp/>



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