第3回 ファイナンスにおける人工知能応用研究会（SIG-FIN） 9/12 11:25-11:50

# Option Market Analysis with Evolutionary Game Theory (進化ゲーム理論を用いたオプション市場分析)

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# This Talk (本報告)

- ANALYSIS the financial market with **Evolutionary game theory**.(金融市場において、進化ゲーム理論を用いて、分析する)
- **PREDICT** the next market state with Stability Analysis.(安定性の概念を用いることによって、次期の市場の状態を予測する)
- EXAMINE the **Real Market** (Future Market) to apply this model. (構築したモデルをもとに、実際の市場を分析する)
- MOVIE (avi)

# OUTLINE

1. Introduction (Motivation)
2. Related Literatures and Review
3. Model
4. Apply this model to the Future market (Nikkei 225)
5. Option Market (Black-Sholes Eq.)
6. Summary (Future works)

# 1. INTRODUCTION

# Motivation　(動機)

- For **Practical Use** (実務への応用を目指して)

More **Detail** (より具体的で), More **Useful** (より役に立つ)

→We construct the market from the **market depth**. (板情報に着目)

- ＋Use the "Real Data" (実際のデータを取り扱う)

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# Self-Introduction : Research

- Research Field : Evolutionary Game Theory : Theory and its Applications.
- Master Course: Game Theory(Bargaining Game) :an approach to General Equilibrium (一般均衡理論をゲーム理論の立場で考える) → Dynamic Matching and Bargaining Game (Rubinstein (1985), Gale (1986a,b), Gale (2000) …)

- **DESCRIBE** the Market with Evolutionary Game Theory.

# 2. RELATED LITERATURES

# Related Literatures(先行研究)

- **Micro Structure**

- **Applied Evolutionary Game Theory**
- **川西 (2008) [amazon]**

- **PUCK (Econophysics)** PUCK 動画 (real player)
- Takayasu, et al. (2006) **[HP]**, Yamada, et al. (2008) **[HP]**, Yamada, et al. (2009) **[HP]**

# Market Micro Structure (市場のマイクロストラクチャー)

- Roughly Speaking, We analysis the agents’ behavior from the financial data.(データから市場参加者の行動を探る)
- Method: Evolutionary Game Theory (進化ゲーム理論)

→ Esaley and O’hara (1992)  **[HP]**

- ＋　In this talk,

we use the “Real Data.” (実際のデータを扱う)

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# 3. MODEL

# Model (モデル)

- **Players**… large population : seller and buyer, potentially (大人数の潜在的な売り手と買い手)

**Seller and Buyer trade an asset.**

- **Goods** (財) … 1財
- **Strategy** (戦略)… **n** (<∞) 個

Here, the price : how much do you buy or sell a goods. (ここでは購入、売却価格)

# Market Depth (板情報)

| （売 呼 値） | 銘柄（値段） | （買 呼 値） |
|---|---|---|
| 2 4<br>H I | 成行呼値 | 1 3<br>K M |
| ○○○ | 503円 | |
| ○○○ | 502円 | 1<br>T |
| ○○ | 501円 | 5 2<br>P N |
| 1 1 1<br>G F E | 500円 | 4 3 2 1<br>A B C D |
| 2<br>S | 499円 | ○○○ |
| 4<br>R | 498円 | ○○○ |
| | 497円 | ○○○ |

東京証券取引所のホームページより

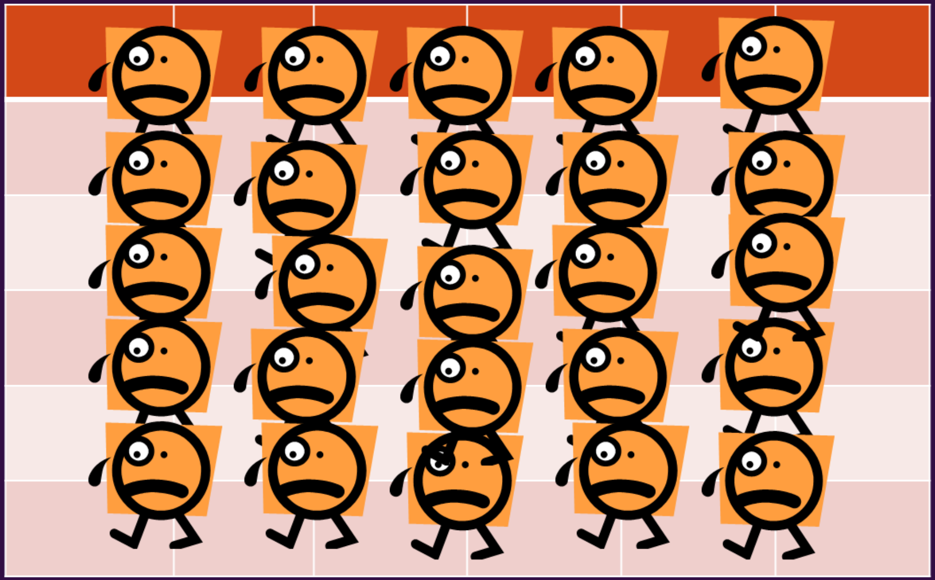
a.まず、成行の売呼値 6,000株(H2,000株、I4,000株)と、成行の買呼値 4,000株(K1,000株、M3,000株)を対当させます。この時点では、成行の売呼値が2,000株残ります。

b.次に、始値を500円と仮定して、成行の売呼値の残りの2,000株及び499円以下の売呼値 6,000株(S2,000株、R4,000株)と、501円以上の買呼値8,000 株(P5,000株、N2,000株、T1,000株)を対当させます。

以上の結果、売呼値が12,000株、買呼値が12,000株で、株数が合致します。

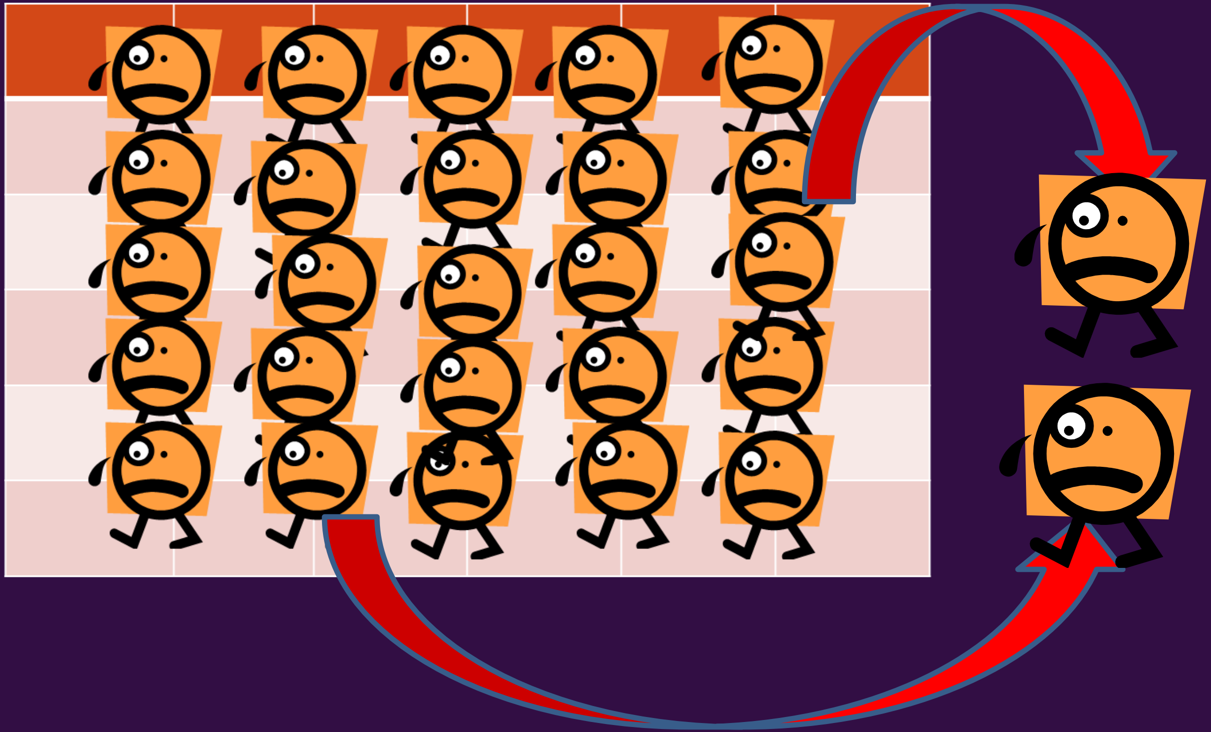
略

# Situation (Traditional Evolutionary Game Theory)



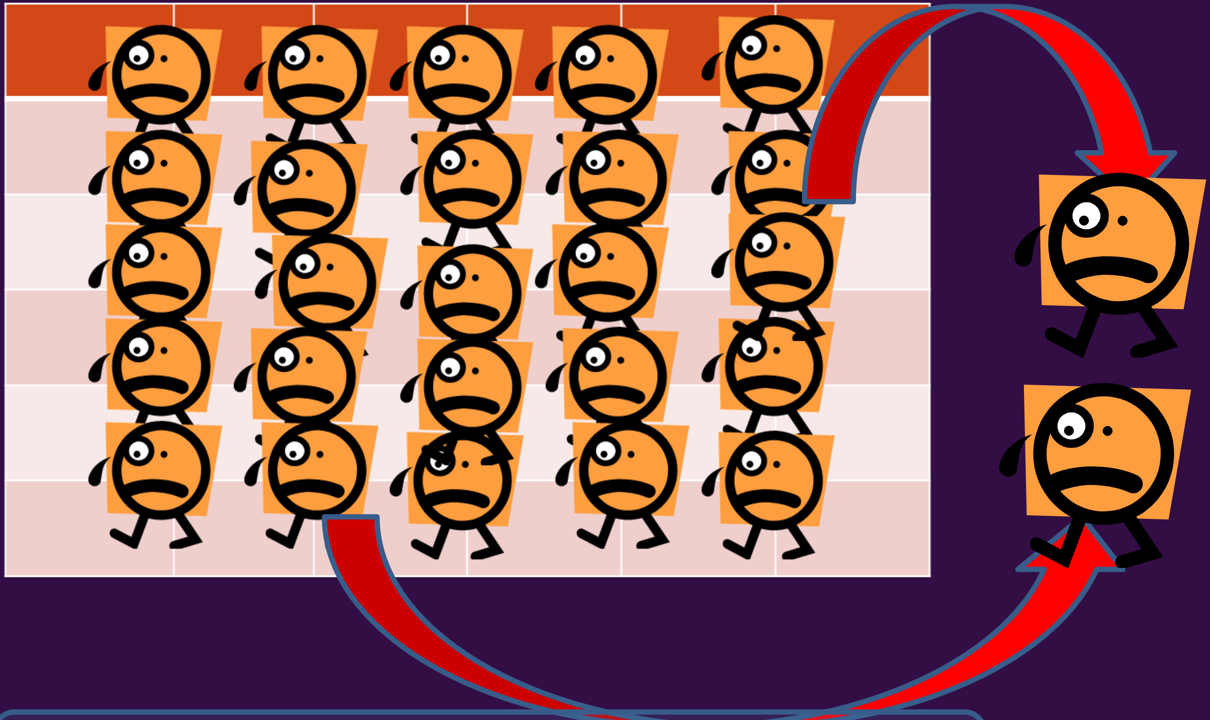
# Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



# Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



Another players look at the game.

# Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)

Play a game

Another players look at the game.

# Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)

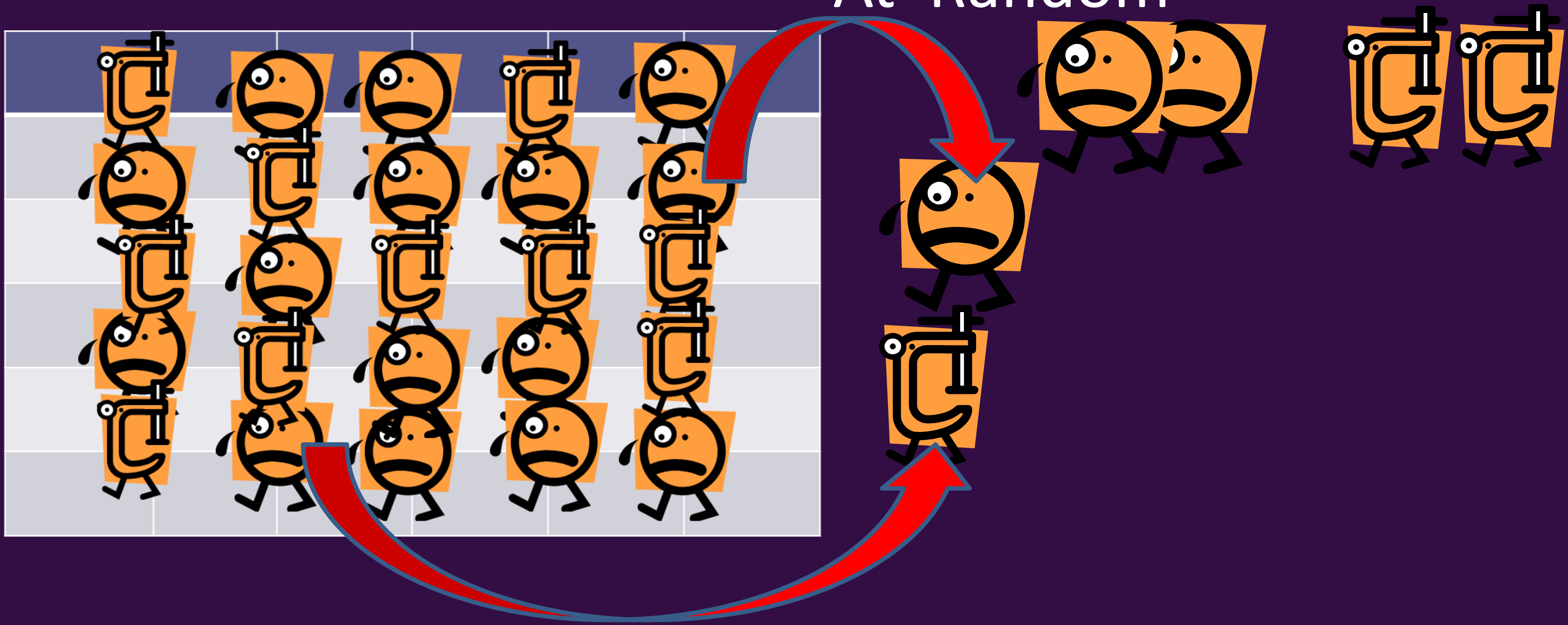
Play a game

Another players look at the game.

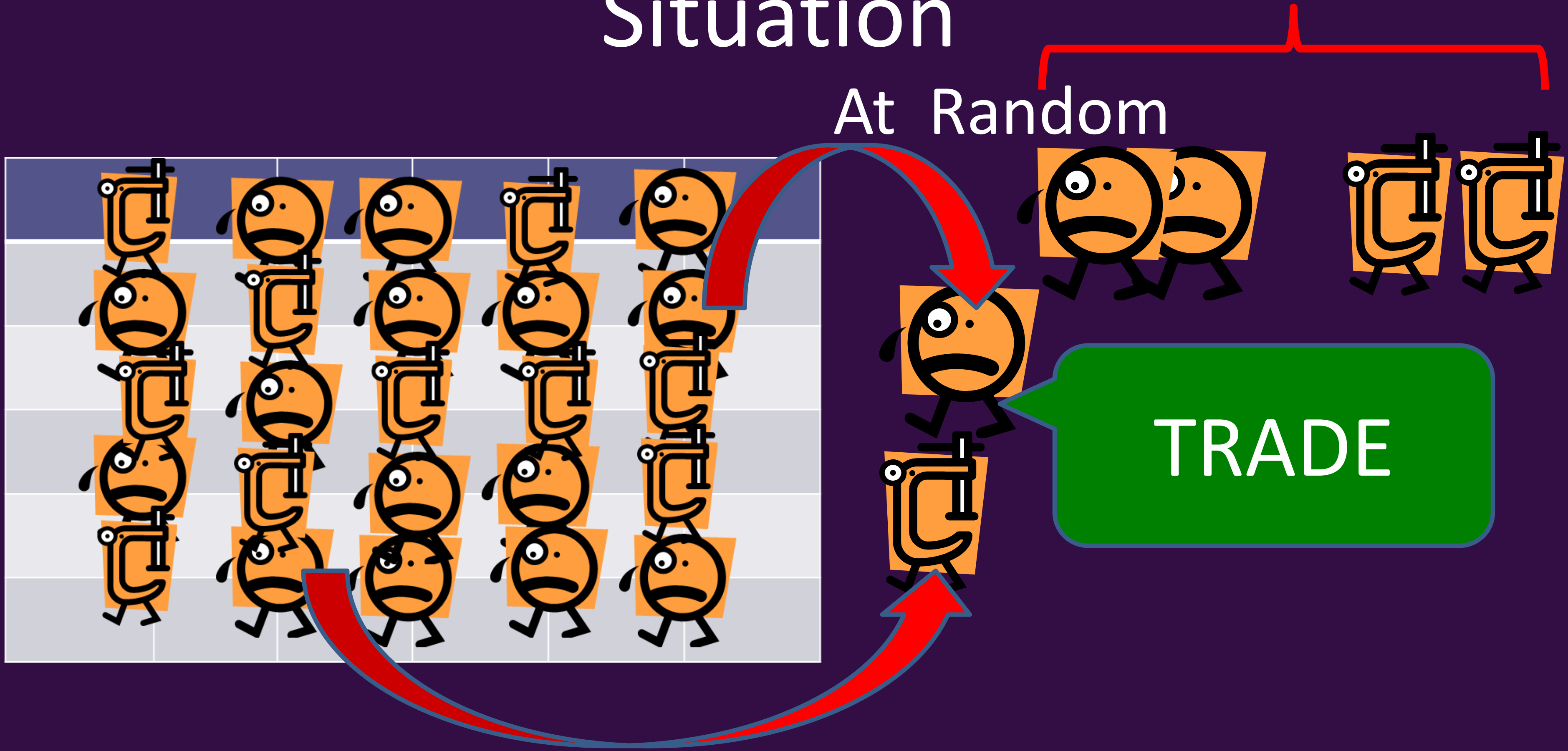
**Replicator Equation**

# Situation (two types players)

# Situation



# Situation

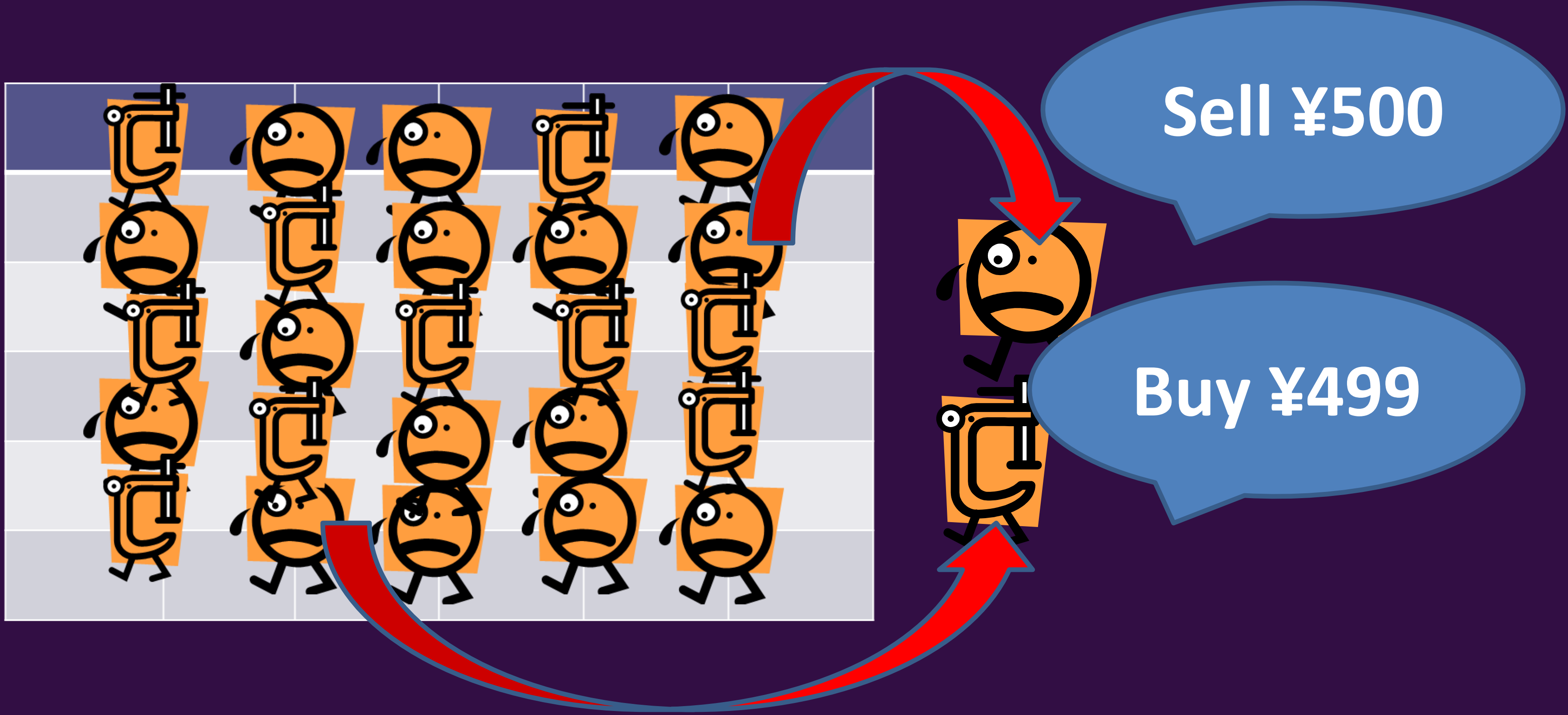


No Trade

At Random

TRADE

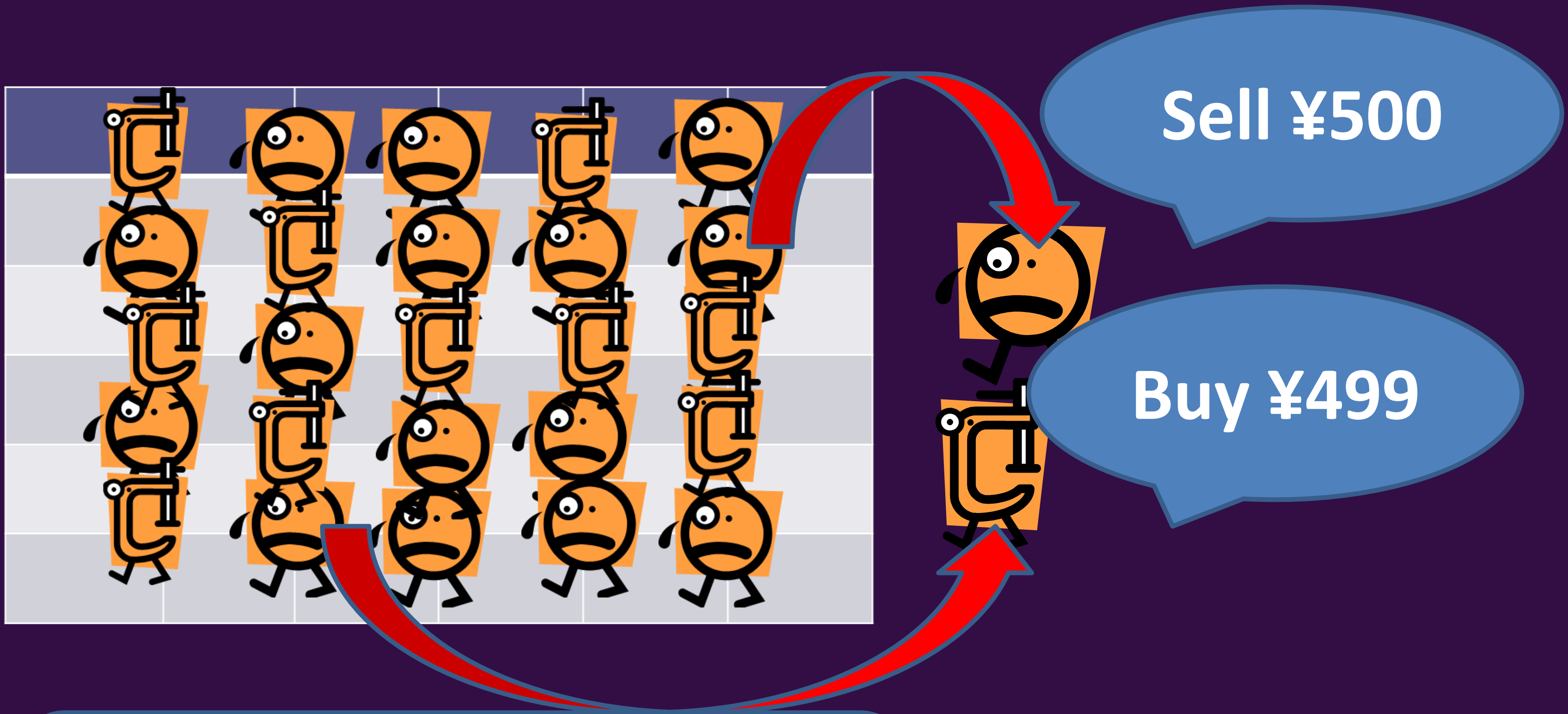
# Situation



Sell ¥500

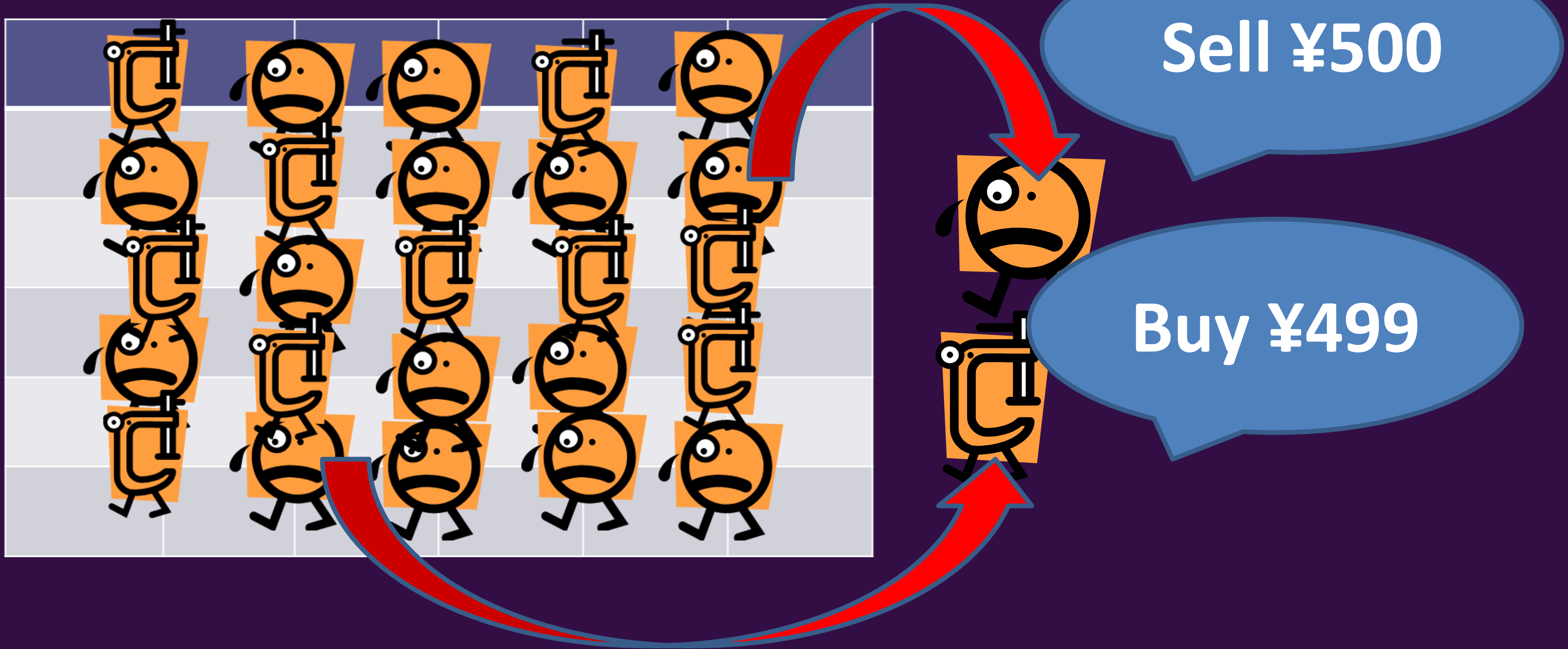
Buy ¥499

# Situation



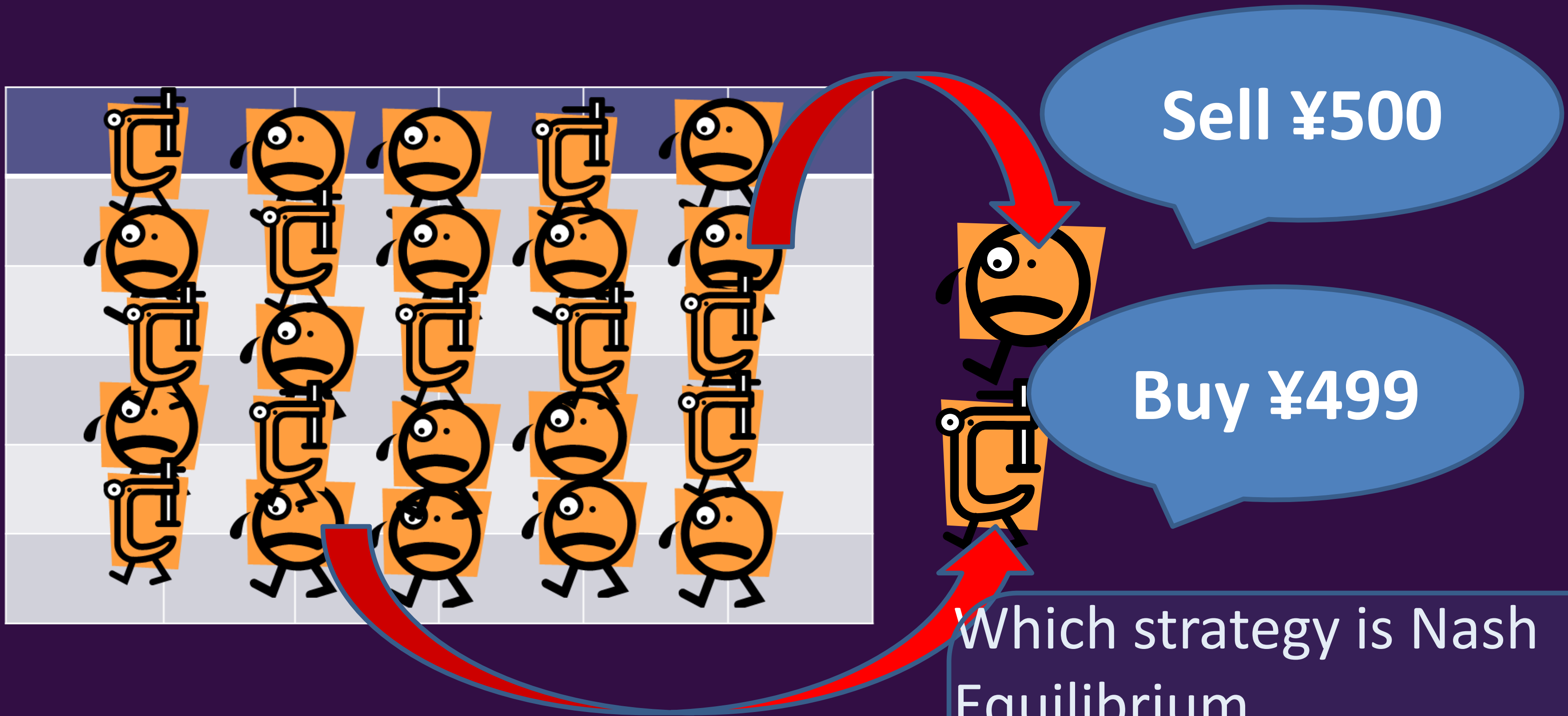
Stock Exchange which take account of the market depth decides the trade’s contract. (取引所が板情報をもとに、売買契約を決定する)

# Situation



Another players look at the market depth (他のプレイヤーは板情報を見ている).

# Situation



Which strategy is Nash Equilibrium,
if this game is played at infinite ?
(このゲームを無限回仮想的に行うと、どの戦略が均衡となるのか？)

Another players look at the market depth (他のプレイヤーは板情報を見ている).

# Model (モデル)

- **Payoff (利得) … Buyer : S(t)-K, Seller : K-S(t)**

where S(t) : Brownian Motion.

- **Replicator Equation**

$$\frac{dx_i(t)}{dt} = x_i(t)\left(g_i(t) - \overline{g}(t)\right), g_i(t) = g_i + \zeta(t)$$

$$\frac{dy_i(t)}{dt} = y_i(t)\left(h_i(t) - \overline{h}(t)\right), h_i(t) = h_i + \zeta'(t)$$

where $x_i$, $y_i$ : the probability of choosing the strategy 1 for each player. $g_i$, $h_i$ : the payoff when each player chooses the strategy 1.

# Two Strategies Case
# (戦略の数が2つ) :

- Replicator equation (see next slide)

$$\dot{x} = x(1-x)\{-b(t)+\left(a(t)+b(t)\right)y\},$$

$$\dot{y} = y(1-y)\{b(t)-\left(a(t)+b(t)\right)x\},$$

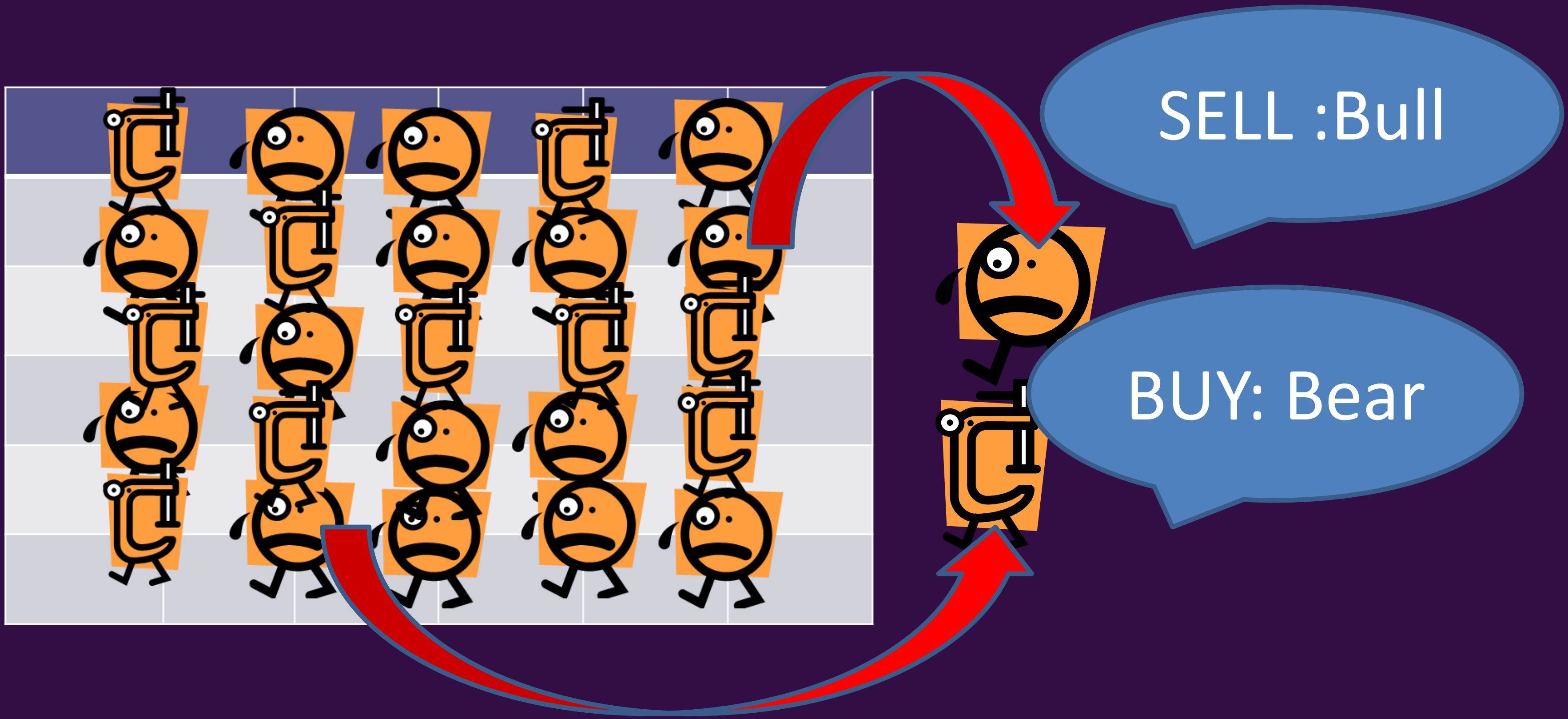
where x, y is the probability of choosing the strategy 1, 2 for each player.

Player 2

player1

|  | S1 | S2 |
|---|---|---|
| S1 | a(t),-a(t) | 0,0 |
| S2 | 0,0 | b(t),-b(t) |

# Situation



# Replicator Equation

REPLICATOR EQ.

$$\dot{x}_i = x_i\left(\underline{(Ax)_i - x\cdot Ax\,}\right), i = 1,\cdots,n.$$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before.

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If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before. And this equation shows that the probability of the action *i* chosen by another players is also higher than before (**externality**).

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# Replicator Equation

REPLICATOR EQ.

$$\dot{x}_i = x_i\left((Ax)_i - x \cdot Ax\right), i = 1, \cdots, n.$$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before. And this equation shows that the probability of the action *i* chosen by another players is also higher than before (**externality**). Furthermore, the equation is derived uniquely by the **monotonic** (that is if one type has increased its share in the population then all types with higher profit should also have increased their shares).

**Two Strategies**

$$\dot{x} = x(1-x)\{b-(a+b)x\} \quad \cdots(*)$$

## Classification

**(I) Non-dilemma**: $a > 0, b < 0$, ESS : one

**(II) Prisoner's dilemma :** $a < 0, b > 0$, ESS :one

**(III) Coordination** : $a>0,b>0$, ESS two

**(IV) Hawk-Dove** : $a<0,b < 0$, ESS one (mixed strategy)

2

1

| | S 1 | S 2 |
|---|---|---|
| S 1 | a,a | 0,0 |
| S 2 | 0,0 | b,b |

Payoff Matrix

# Prediction (予測)

- Replicator equation divided by xy(1-x)(1-y) :

$$\dot{x} = -\frac{b(t)}{y} + \frac{a(t)}{1-y}, \dot{y} = \frac{b(t)}{x} - \frac{a(t)}{1-x}.$$

- Discrete the above equations:

$$x(t+\varepsilon) = x(t) - \left( \frac{b(t)}{y} + \frac{a(t)}{1-y} \right) \varepsilon,$$

$$y(t+\varepsilon) = y(t) + \left( \frac{b(t)}{x} - \frac{a(t)}{1-x} \right) \varepsilon.$$

# Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s1,s2),(s2,s2)

|  | S 1 | S 2 |
|---|---|---|
| S 1 | +, - | 0,0 |
| S 2 | 0,0 | 0,0 |

ii) ↓ (Down)

N.E. (s1,s1),(s1,s2)

|  | S 1 | S 2 |
|---|---|---|
| S 1 | 0,0 | 0,0 |
| S 2 | 0,0 | -,+ |

iii) → (No change)

N.E. (s1,s2)

|  | S 1 | S 2 |
|---|---|---|
| S 1 | +,- | 0,0 |
| S 2 | 0,0 | -,+ |

i),ii),iii) → (s1,s2) (x→1, y→0)

# Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s1,s2),(s2,s2)

| | S 1 | S 2 |
|---|---|---|
| S 1 | +, - | 0,0 |
| S 2 | 0,0 | 0,0 |

ii) ↓ (Down)

N.E. (s1,s1),(s1,s2)

**Is This OK?**

| | S 1 | S 2 |
|---|---|---|
| S 1 | 0,0 | 0,0 |
| S 2 | 0,0 | -,+ |

iii) → (No change)

N.E. (s1,s2)

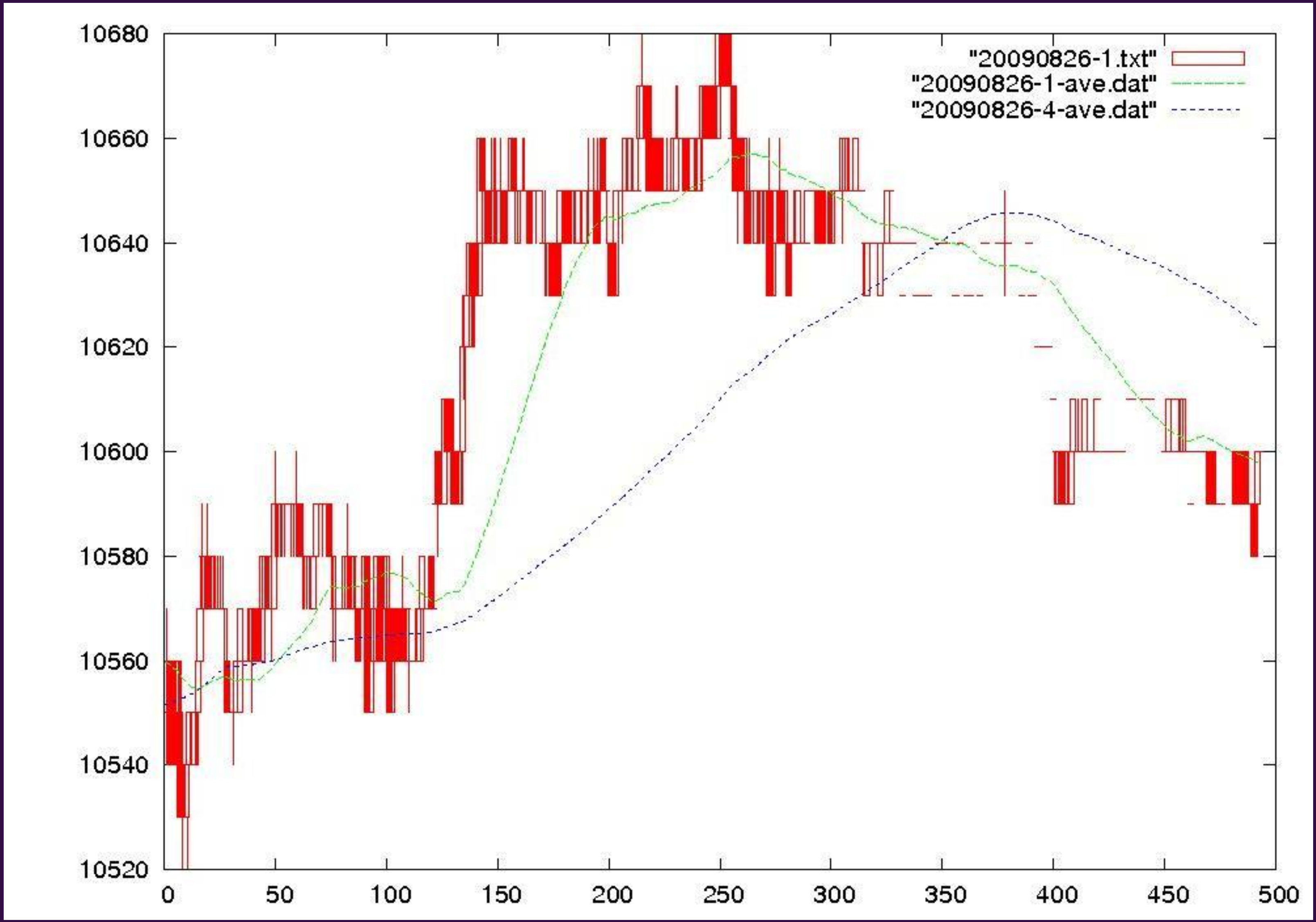
| | S 1 | S 2 |
|---|---|---|
| S 1 | +,- | 0,0 |
| S 2 | 0,0 | -,+ |

i),ii),iii) → (s1,s2) (x→1, y→0)

# 4. Application:

# NIKKEI 225 FUTURE MARKET (日経225先物市場)

# EX: 20090826


10680
10660
10640
10620
10600
10580
10560
10540
10520
0
50
100
150
200
250
300
350
400
450
500
"20090826-1.txt"
"20090826-1-ave.dat"
"20090826-4-ave.dat"

# Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

|  | S 1 | S 2 |
|---|---|---|
| S 1 | +, - | 0,0 |
| S 2 | 0,0 | +,+ |

ii) ↓ (Down)

N.E. (s1,s1)

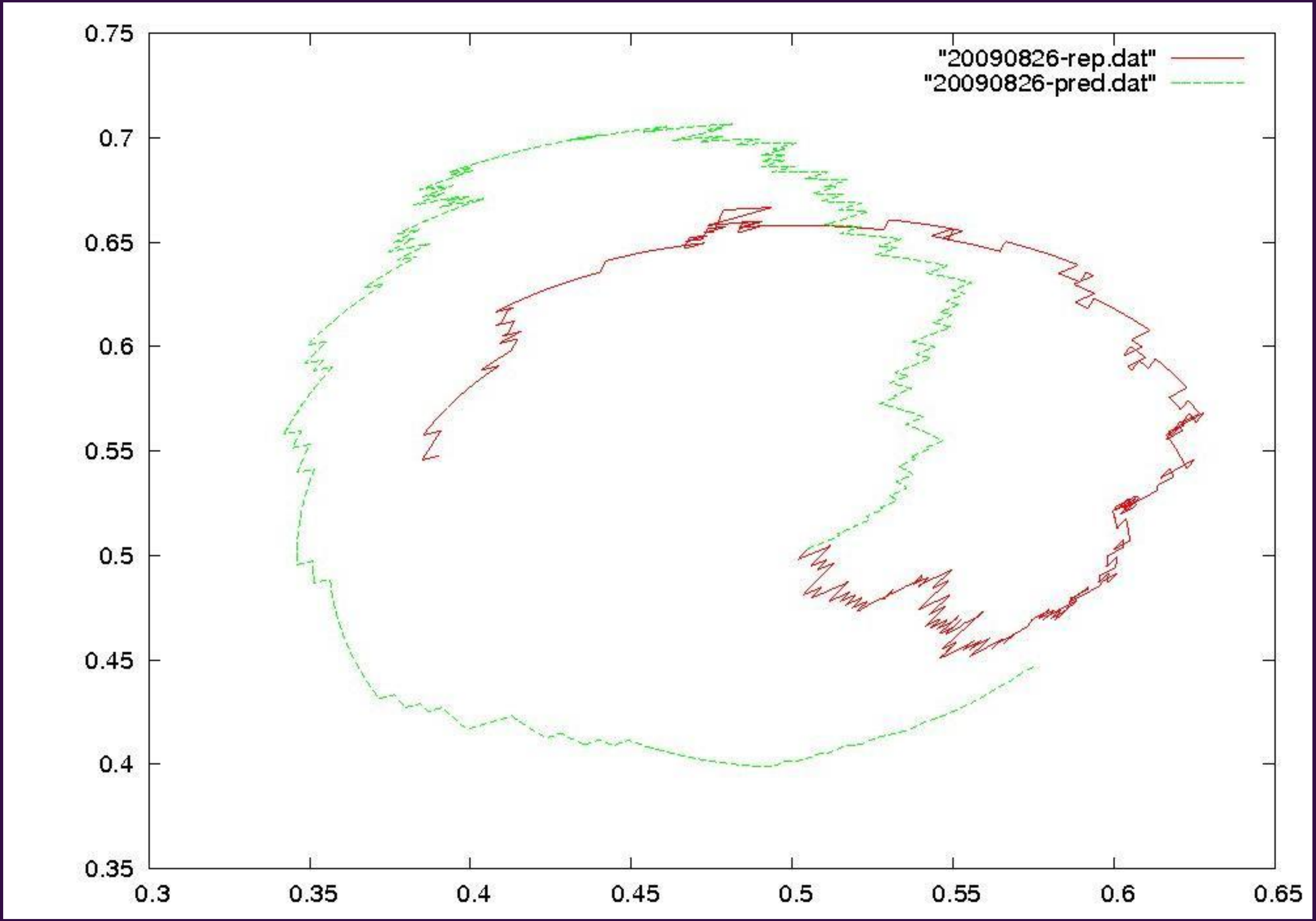
|  | S 1 | S 2 |
|---|---|---|
| S 1 | +,+ | 0,0 |
| S 2 | 0,0 | -,+ |

iii) → (No change)

N.E. Mixed Strategy.

|  | S 1 | S 2 |
|---|---|---|
| S 1 | -,+ | 0,0 |
| S 2 | 0,0 | -,+ |

# EX: 20090826


"20090826-rep.dat"
"20090826-pred.dat"
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65

5. Application:

# OPTION MARKET
# (オプション市場)

# この場合のBlack-Sholesの公式

- Black-Sholesモデルにおいて、行使価格の影響があるのは、**境界条件**を使用するとき。
- よって、K := $\overline{K}$ とすればよい. つまり

$$f(S,t)=S\cdot N\left(\frac{u}{\sigma\sqrt{\tau}}+\sigma\sqrt{\tau}\right)-\enclose{circle}{\overline{K}}\,e^{-r\tau}\cdot N\left(\frac{u}{\sigma\sqrt{\tau}}\right).$$

ただし $\overline{K} =$ 平衡時の戦略1における行使価格 $\cdot s_1^*$
$+$ 平衡時の戦略2における行使価格 $\cdot (1-s_1^*)$

$s_1^*$ は混合戦略を採用する場合の確率.

# 6. SUMMARY AND FUTURE WORKS

# Summary and Future Works

**Summary**

- **MODELING** the Financial Market.
- **DERIVE** the payoff matrix for each player.
- **APPLY** the Real Market.
- **DERIVE** the Optimal Behavior for each player.

**Future Works**

- **GET** the Online Financial Data, **CALCULATE** and **DISPLAY**. (オンラインでデータを入手し、計算し、それを表示する)
- **MAKE** the software like a PUCK based on the Evolutionary Game Theory. (PUCKの進化ゲーム理論版の構築)

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# Artificial Market (人工市場)

- 和泉潔(2003)「人工市場」**[amazon]**
- 塩沢, 中島, 松井, 小山, 谷口, 橋本(2006)「人工市場で学ぶマーケットメカニズム」経済学編**[amazon]**,
- 喜多, 森, 小野, 佐藤, 小山, 秋元(2009)「人工市場で学ぶマーケットメカニズム」工学編 **[amazon]**
- 鳥海不二夫(2007)「株ロボを作ろう！」**[amazon]**

etc

# Thank You For Your Attention

Mitsuru KIKKAWA (mitsurukikkawa@hotmail.co.jp)

This File is available at

**http://kikkawa.cyber-ninja.jp/**

# REFERENCE

[1] Black, Fischer and Scholes, Myron: ”The Pricing of Options and Corporate Liabilities,” The Journal of Political Economy, Vol. 81, pp. 637-654 (1973) **[HP]**

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[3] 川西 諭: ノイズのある合理的期待均衡モデルにおける投資情報獲得戦略の多様性について, 現代経済学の潮流2008, 第4章, pp.105--141 (2008) **[amazon]**

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[5] 吉川 満:「オプションの戦略的な価格付け : Black-Sholes 方程式の周辺」, 北海道大学数学講究録, #140, pp. 142-146 (2009) **[HP]**

[6] Takayasu, et al., (2006): Potential force observed in market dynamics, Physica A 370, 91-97.**[HP]**

[7] Yamada, et al., (2008): The grounds for time dependent market potentials from dealer’s dynamics, Eur.Phys.J. B 63. 529-532.**[HP]**

[8] Yamada, et al. (2009): Solvable stochastic dealer models for financial markets, Physical Review E 79, 051120.**[HP]**

# EVOLUTIONARY STABLE STRATEGY（ESS)

**DEF.**： Weibull(1995): $x \in \Delta$ is an ***evolutionary stable strategy (ESS)*** if for every strategy $y \neq x$ there exists some $\overline{\varepsilon_y} \in (0,1)$ such that the following inequality holds for all $\varepsilon \in (0, \overline{\varepsilon_y})$.

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x].$$

**INTERPRETATION**：incumbent payoff (fitness) is higher

than that of the post-entry strategy

（ESS : ①the solution of the Replicator equation + ② asymptotic stable.）

- 本研究の一部は，平成20年度採択, 文部科学省 グローバルCOEプログラム**「現象数理学の形成と発展」** 現象数理若手プロジェクト「人間特有の現象に対する学習の影響 - 進化ゲーム理論による分析 -」に関する研究拠点形成費の助成を受けて行われた.