SIG-FIN, 23 Jan. 2010, 11:00~11:25 (AIST)

Market Model Focused On the Order Book : Evolutionary Game Theory
(板情報に着目した市場モデル : 進化ゲーム理論)

Mitsuru KIKKAWA (吉川満)

（**Department of Science and Technology, Meiji University**)

THIS FILE IS AVAILABLE AT http://kikkawa.cyber-ninja.jp/

This Talk (本報告)

- This talk is based on my talk at 3rd SIG-FIN and extends the basic model. (本報告は第3回のSIG-FINの報告を基礎として、そのモデルを拡張した)

1 . **RISK ATTITUDE** (リスクに対する態度).

2. **ANALYZE** the Order Book with Micro-Econometric Method(Multi-Logit Model) (ミクロ計量経済の分析手法を用いて、板情報を分析)

- MOVIE (avi)

OUTLINE

1. Introduction (Motivation)
2. Review
3. Model
4. Extension (Risk Attitude)
5. Empirical Evidence (Multi-Norminal Logit Model)
6. Summary (Future works)

1. INTRODUCTION

Motivation　(動機)

- For **Practical Use** (実務への応用を目指して)

More **Detail** (より具体的で), More **Useful** (より役に立つ)

→We construct the market from the **order book**. (板情報に着目)

- ＋Use the ''Real Data'' (実際のデータを取り扱う)

Motivation　(動機)

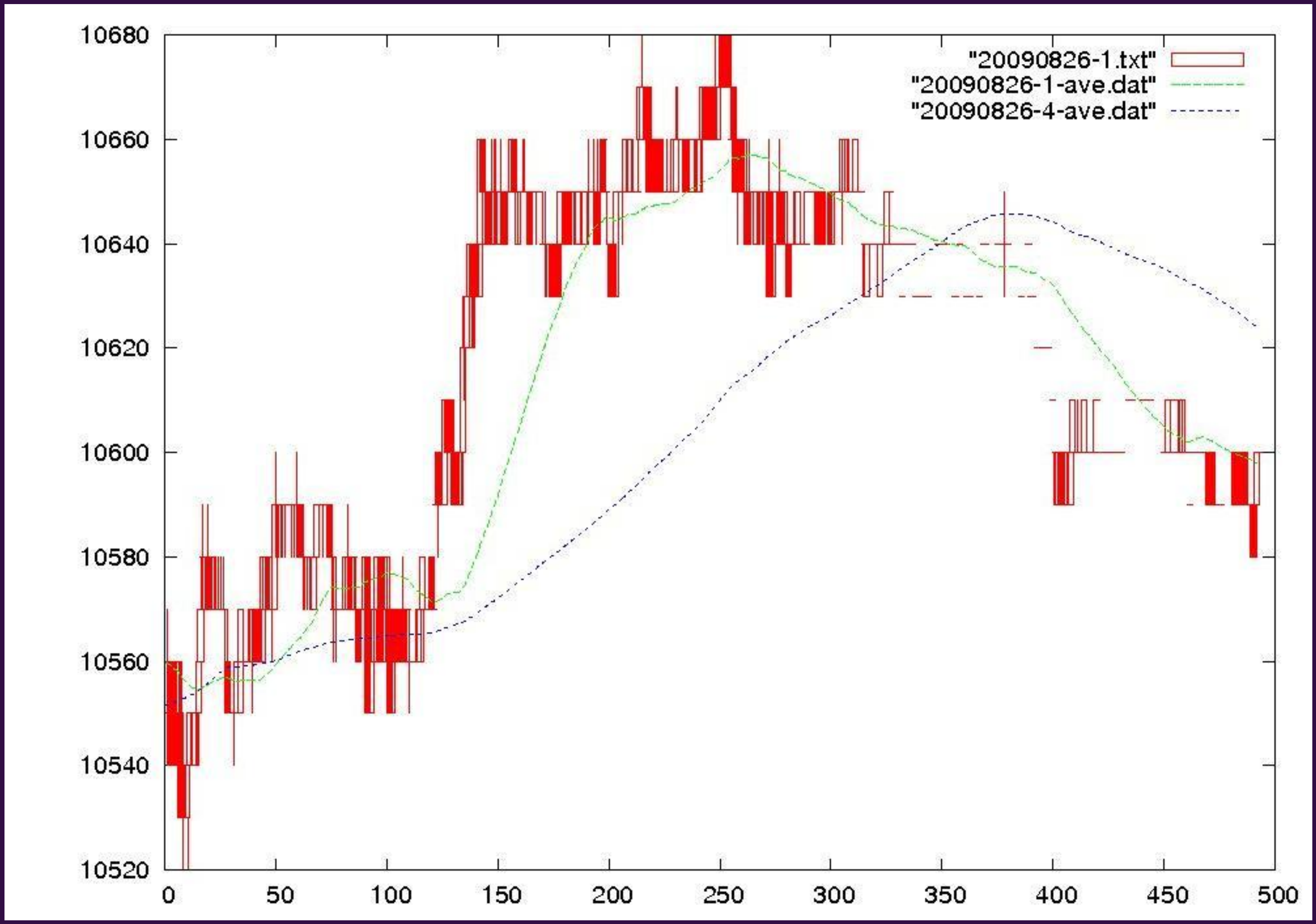
- For **Practical Use** (実務への応用を目指して)

More **Detail** (より具体的で), More **Useful** (より役に立つ)

→We construct the market from the **order book**. (板情報に着目)

- ＋Use the ”Real Data” (実際のデータを取り扱う)
- MOVIE (avi)

EX: 20090826



10680
10660
10640
10620
10600
10580
10560
10540
10520
0
50
100
150
200
250
300
350
400
450
500
"20090826-1.txt"
"20090826-1-ave.dat"
"20090826-4-ave.dat"

Research Fields (this study)

Market (市場)

Research Fields (this study)

Market (市場)

General Equilibrium (一般均衡理論)

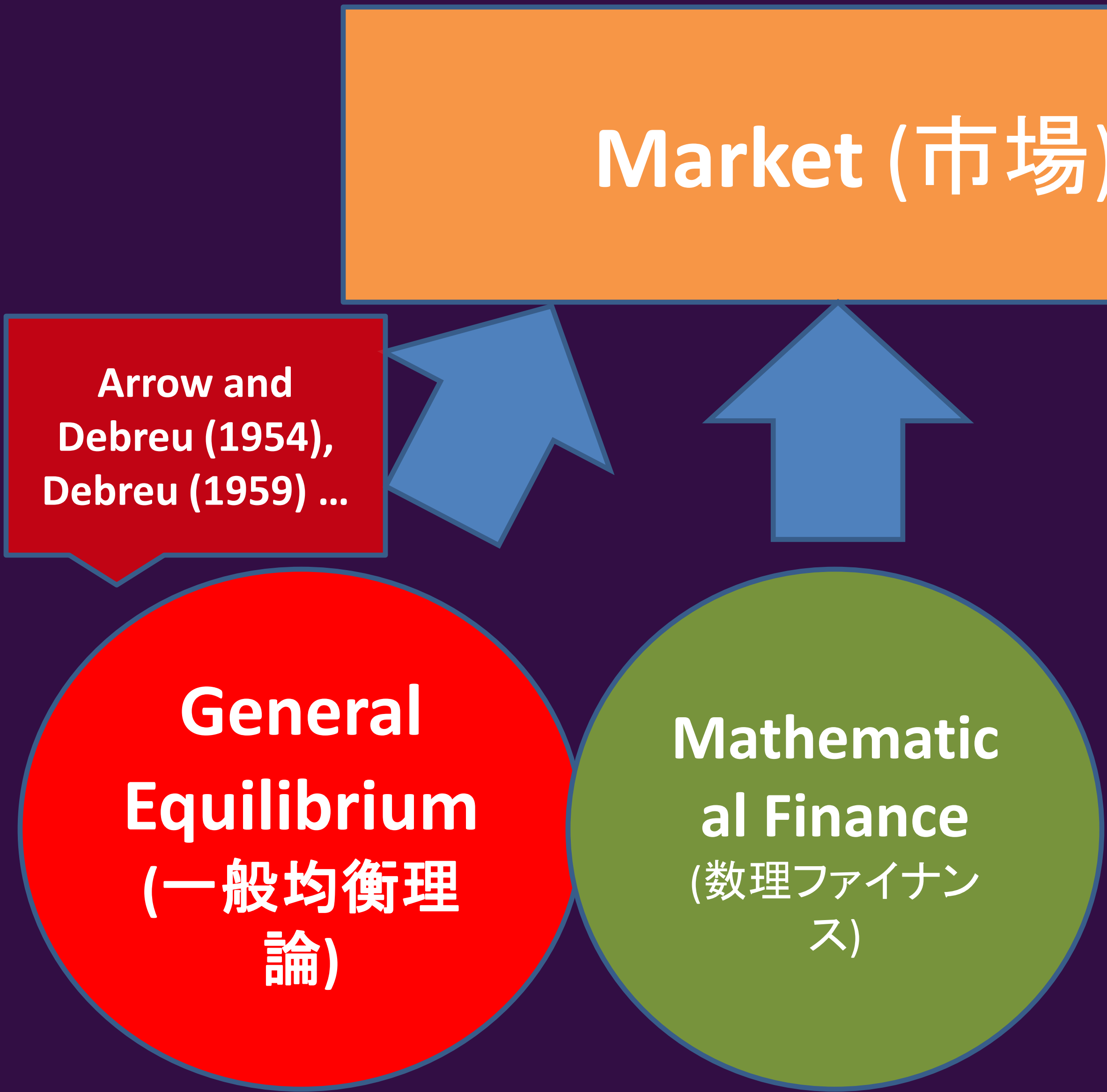
Research Fields (this study)

Market (市場)

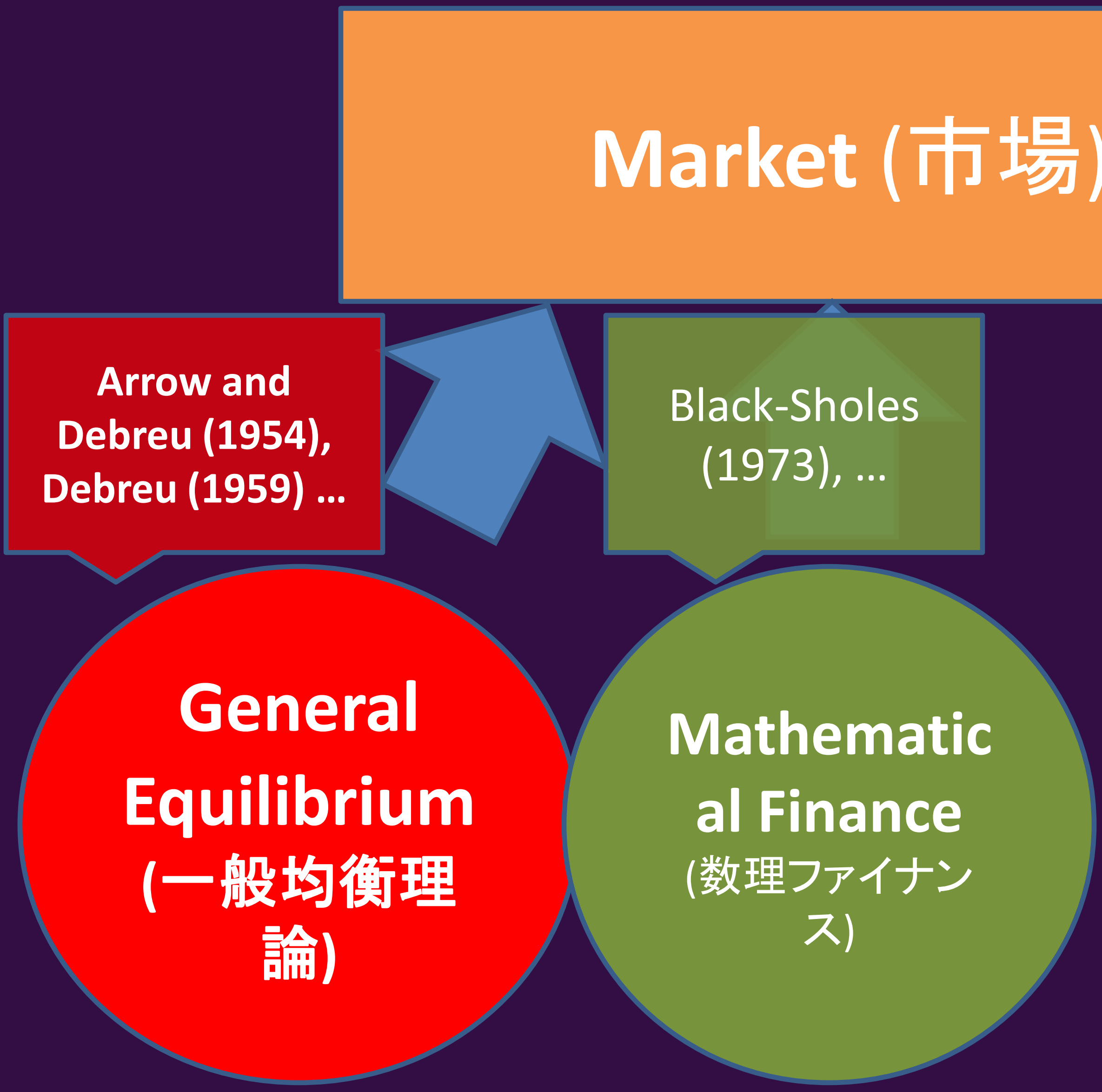
Arrow and Debreu (1954), Debreu (1959) …

General Equilibrium (一般均衡理論)

Research Fields (this study)



Research Fields (this study)



Research Fields (this study)

Invisible Hand (神の見えざる手) ？

Market (市場)

Arrow and Debreu (1954), Debreu (1959) …

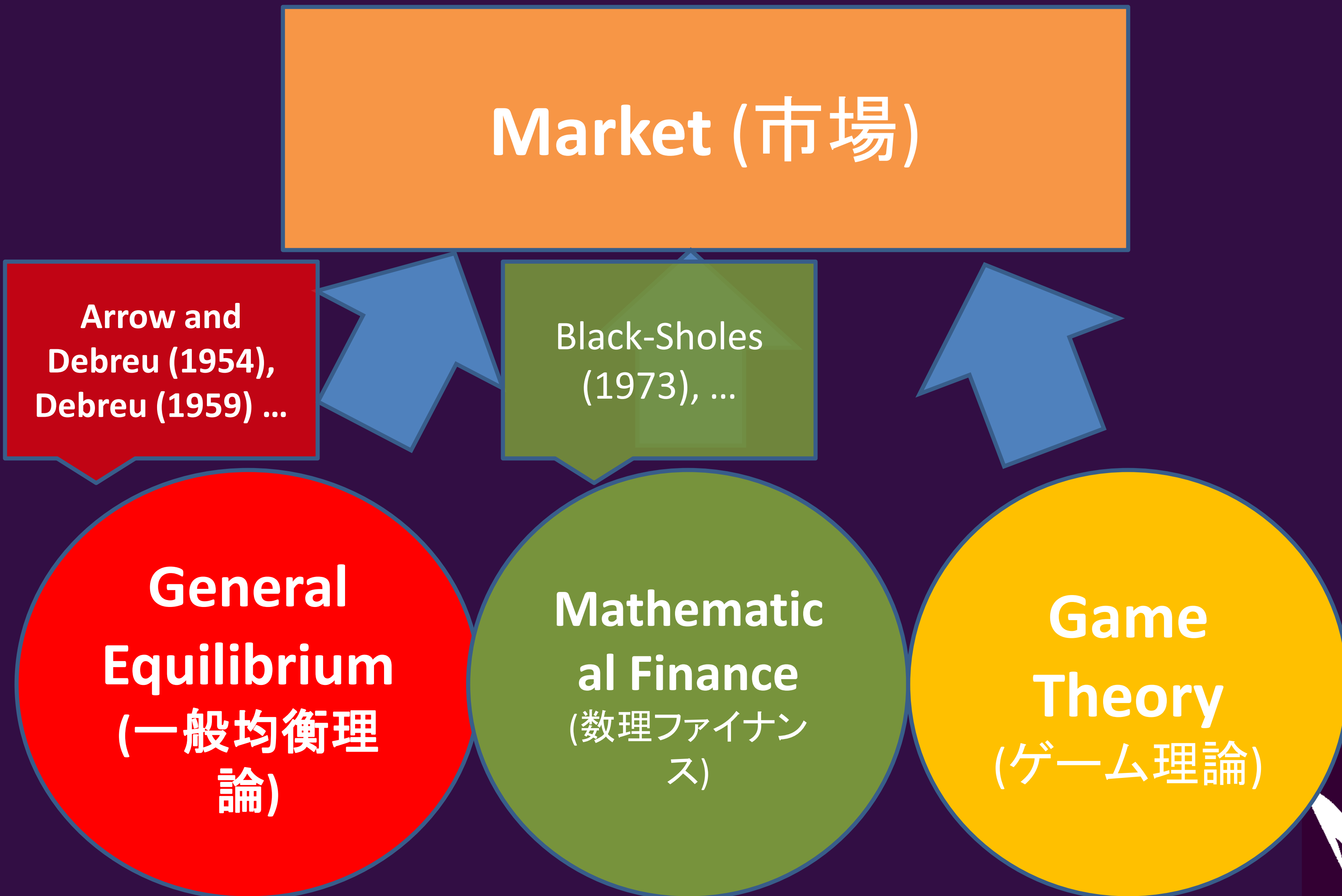
Black-Sholes (1973), …

Heat Equation (熱方程式)?, Micro-Foundation ?

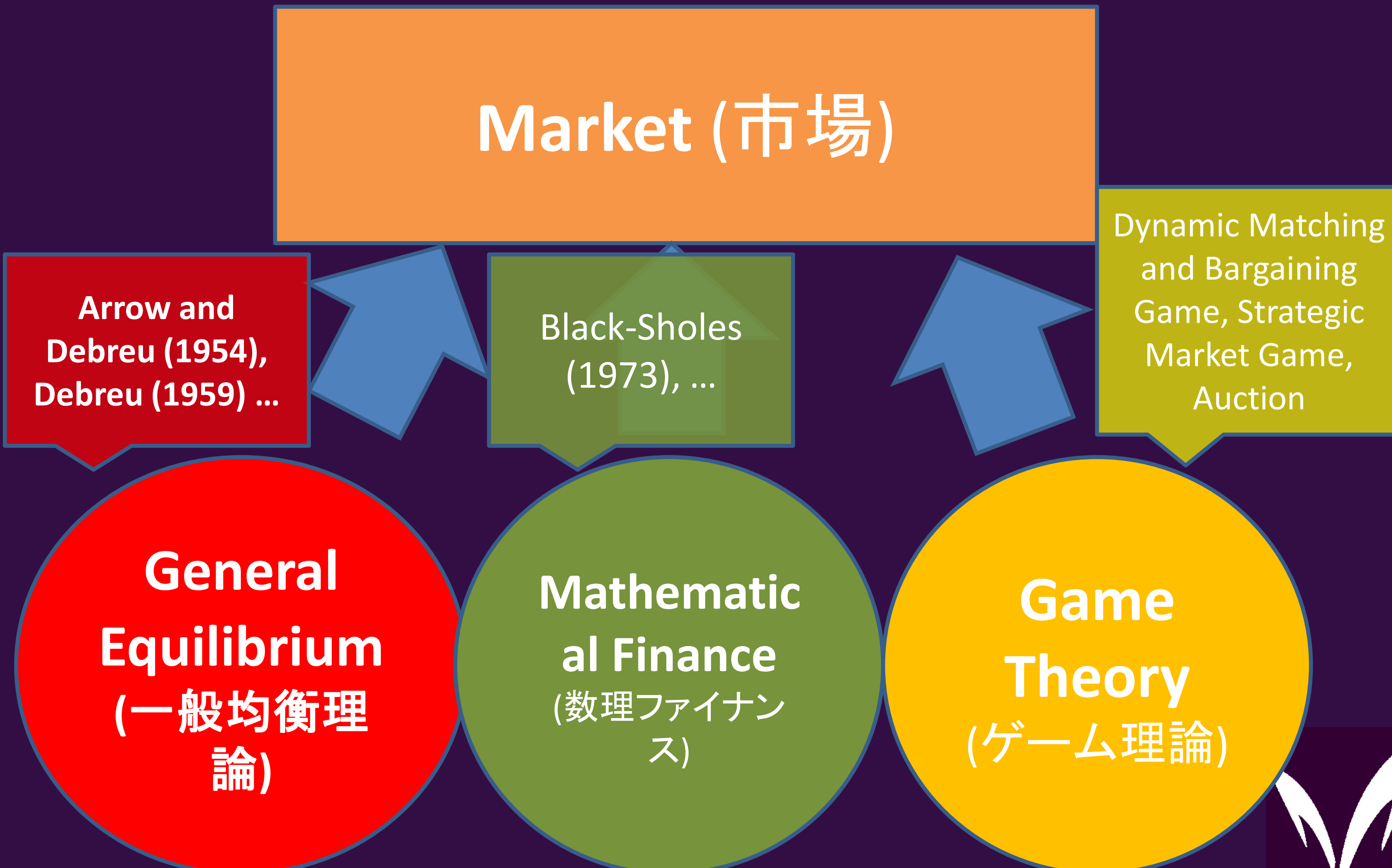
General Equilibrium (一般均衡理論)

Mathematical Finance (数理ファイナンス)

Research Fields (this study)



Research Fields (this study)



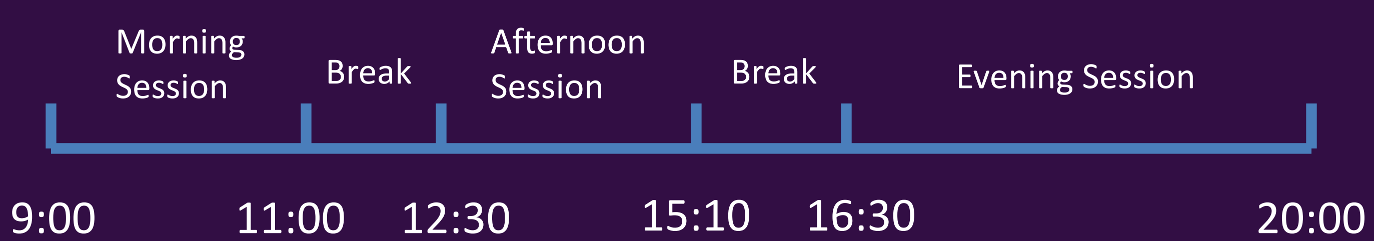
2. RELATED LITERATURES AND PRELIMINARIES

How are stock prices determined ?

- Stock prices are determined by two methods, the *Itayose(板寄せ)* and *Zaraba(ザラバ)* methods. The *Itayose* method is mainly used to decide opening and closing prices; the *Zaraba* method is used during continuous auction trading for the rest of the trading session.

→ The stock price are determined by **Rule**.

[Nikkei 225 Future Market(日経225先物)] [1day]

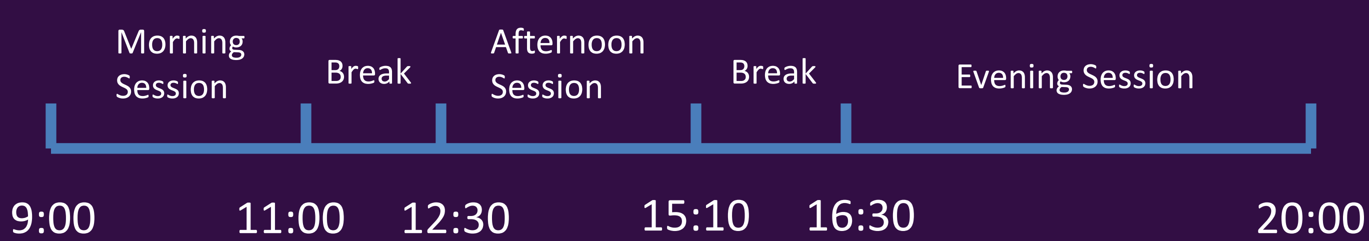


How are stock prices determined ?

- Stock prices are determined by two methods, the *Itayose(板寄せ)* and *Zaraba(ザラバ)* methods. The *Itayose* method is mainly used to decide opening and closing prices; the *Zaraba* method is used during continuous auction trading for the rest of the trading session.

→ The stock price are determined by **Rule**.

[Nikkei 225 Future Market(日経225先物)] [1day]



Two Principles (2つの原則)

1) **Price Priority** (価格優先の原則)

means that the lowest sell and highest buy orders take precedence over other orders.

2) **Time Priority** (時間優先の原則)

means that among orders at the same price, the order placed earliest takes precedence.

Offer(sell)	Price	Bid (buy)
		early ← → late
A 3000(5), C 4000(4)	502	
D 10000(3), E 9000(2), F 5000(1)	501	
	500	H 80000(1), B 1000(2), J 4000(3)
late← → early	499	H 1000(4), B 150000(5)

The Order Book (板情報)

（Offer(sell)）	Price	（Bid (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

In this case, opening price is 500 or 501.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

Assume: opening price is 500.

（Offer(sell)）	Price	（Bid (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

- The market orders of 4000 shares to buy and 6000 shares to sell are matched, leaving sell orders of 2000 shares.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

Second Step

（Offer(sell)）	Price	（Bid (buy)）
2000	Market orders	
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- The market sell orders of 2000 shares and sell orders 6000 shares at limit prices of 499 or less are matched with the buy orders of 8000 shares at limit prices of 501 or more. Thus far, 12000 shares have been matched in total.

Third Step

（Offer(sell)）	Price	（Bid (buy)）
	Market orders	
8000	502	
20000	501	
4000	500	10000
	499	8000
	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- Finally, the sell orders of 4000 shares at a limit price of 500 are matched with the buy orders of 10000 shares at a limit price of 500. Although this still leaves buy orders of 6000 shares at 500.

Fourth Step

（Offer(sell)）	Price	（Bid (buy)）
	Market orders	
8000	502	
20000	501	
	500	6000
	499	8000
	498	30000

- Thus the opening price is determined at 500 and transactions of 16000 shares are completed at 500.

The stock price and the trade depend on the **order book**. (価格や取引の可否は板情報によって決定する。)

3. MODEL

Model (モデル)

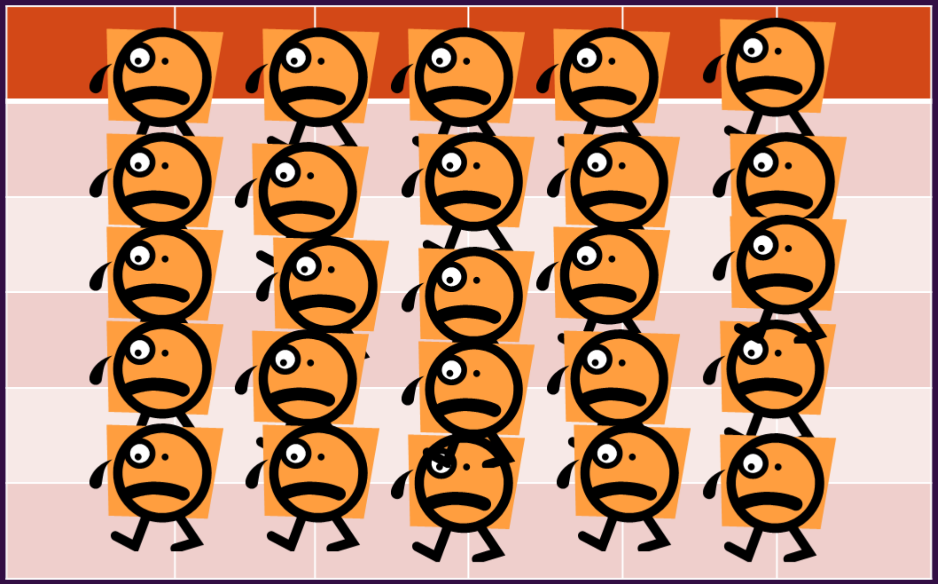
- **Players**… large population : seller and buyer, potentially (大人数の潜在的な売り手と買い手)

Seller and Buyer trade an asset.

- **Goods** (財) … 1財
- **Strategy** (戦略)… **n** (<∞) 個

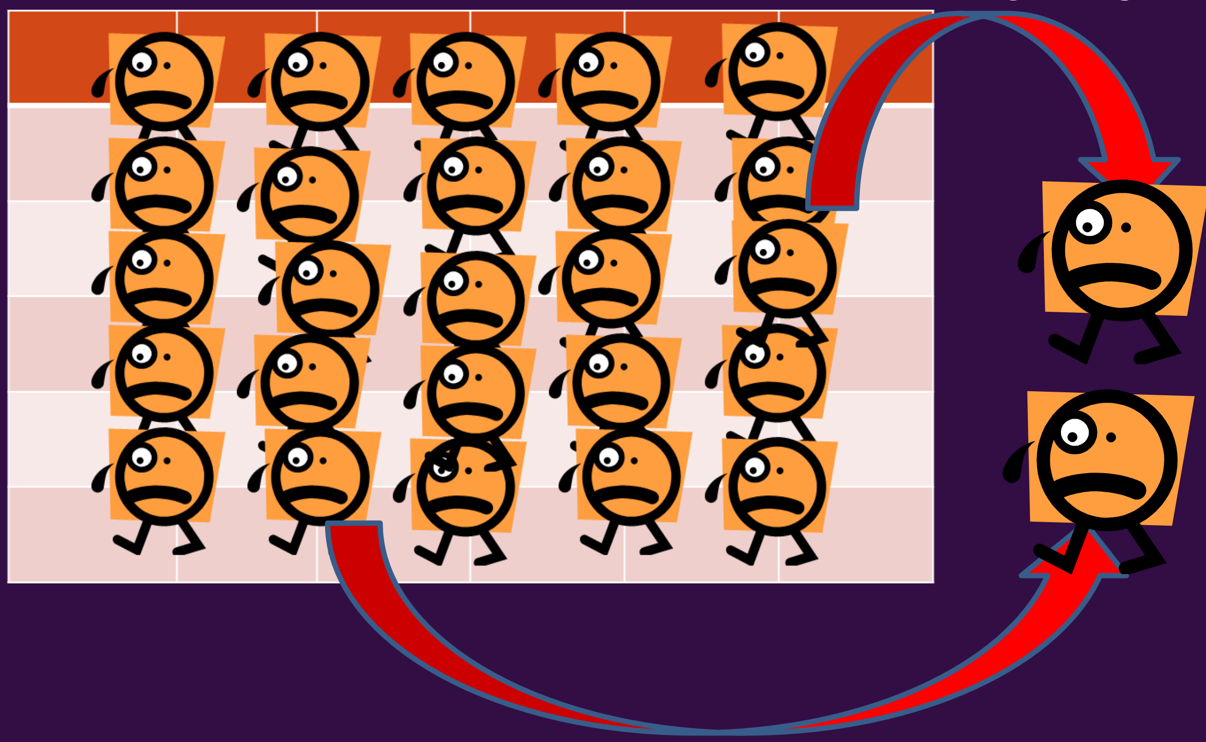
Here, the strike price : how much do you buy or sell an asset. (ここでは購入、売却価格)

Situation (Traditional Evolutionary Game Theory)



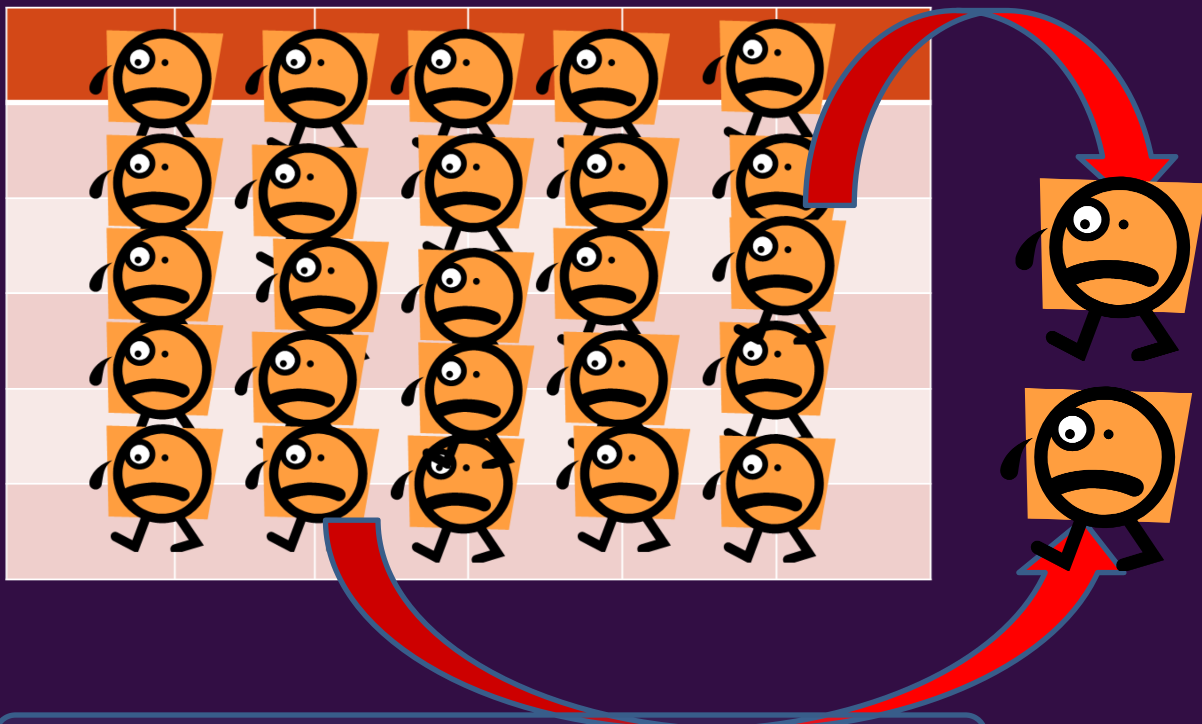
Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



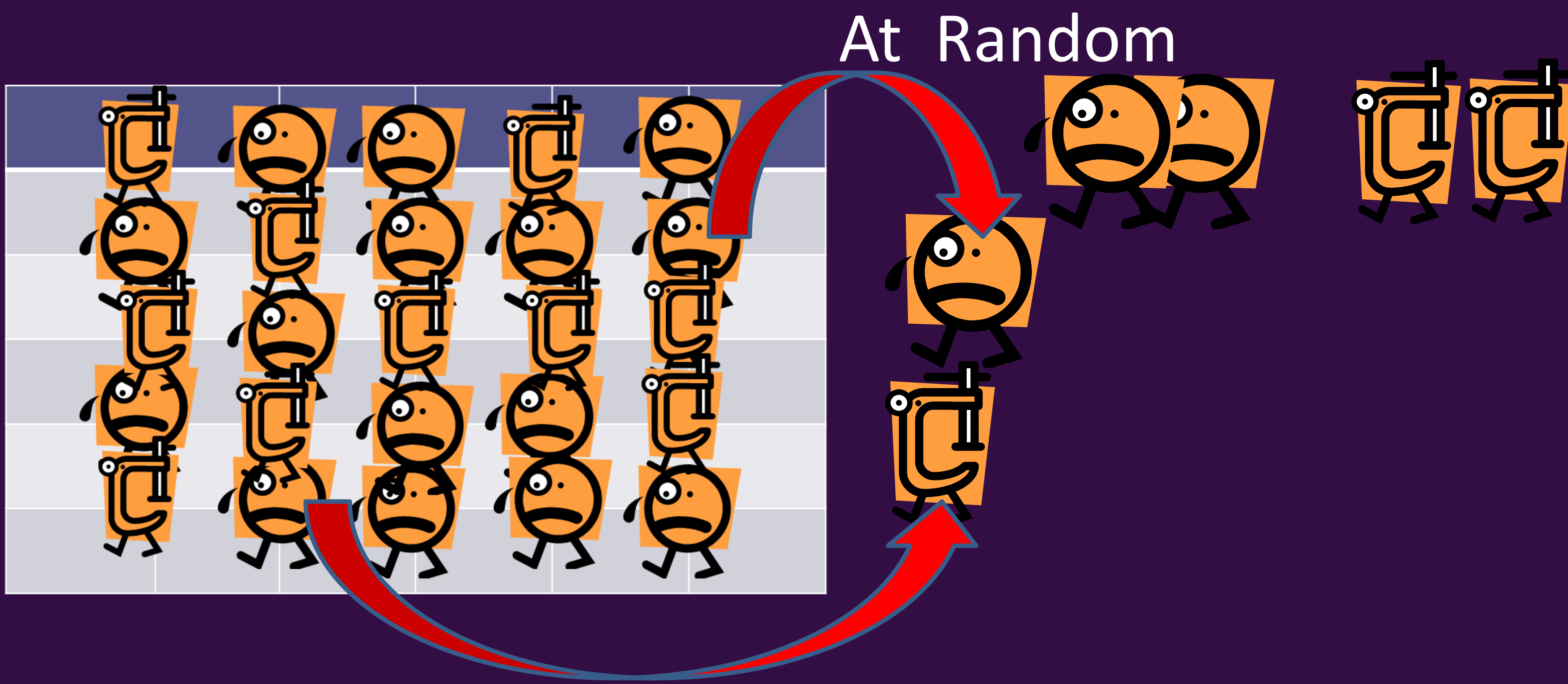
Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

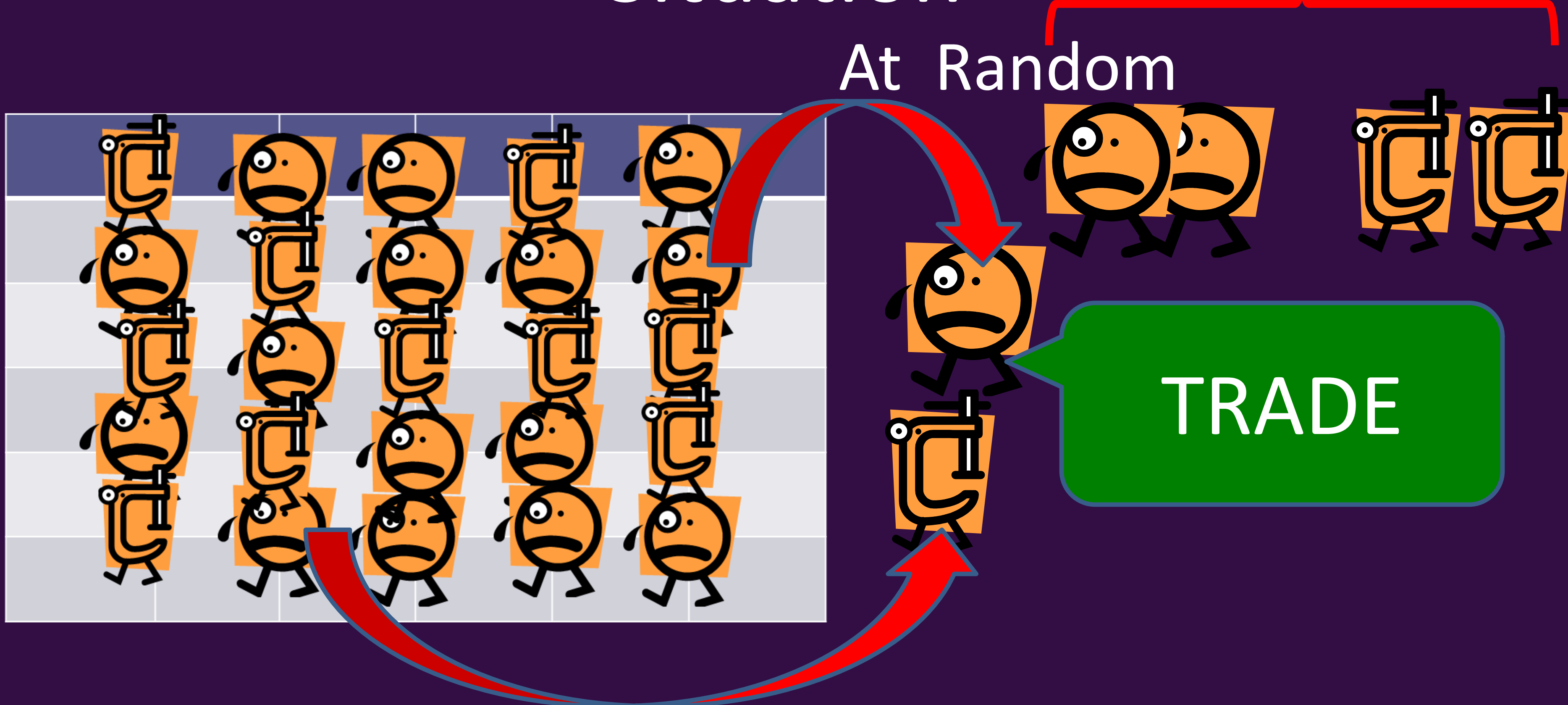
Situation (Traditional Evolutionary Game Theory)

Situation (two types players)

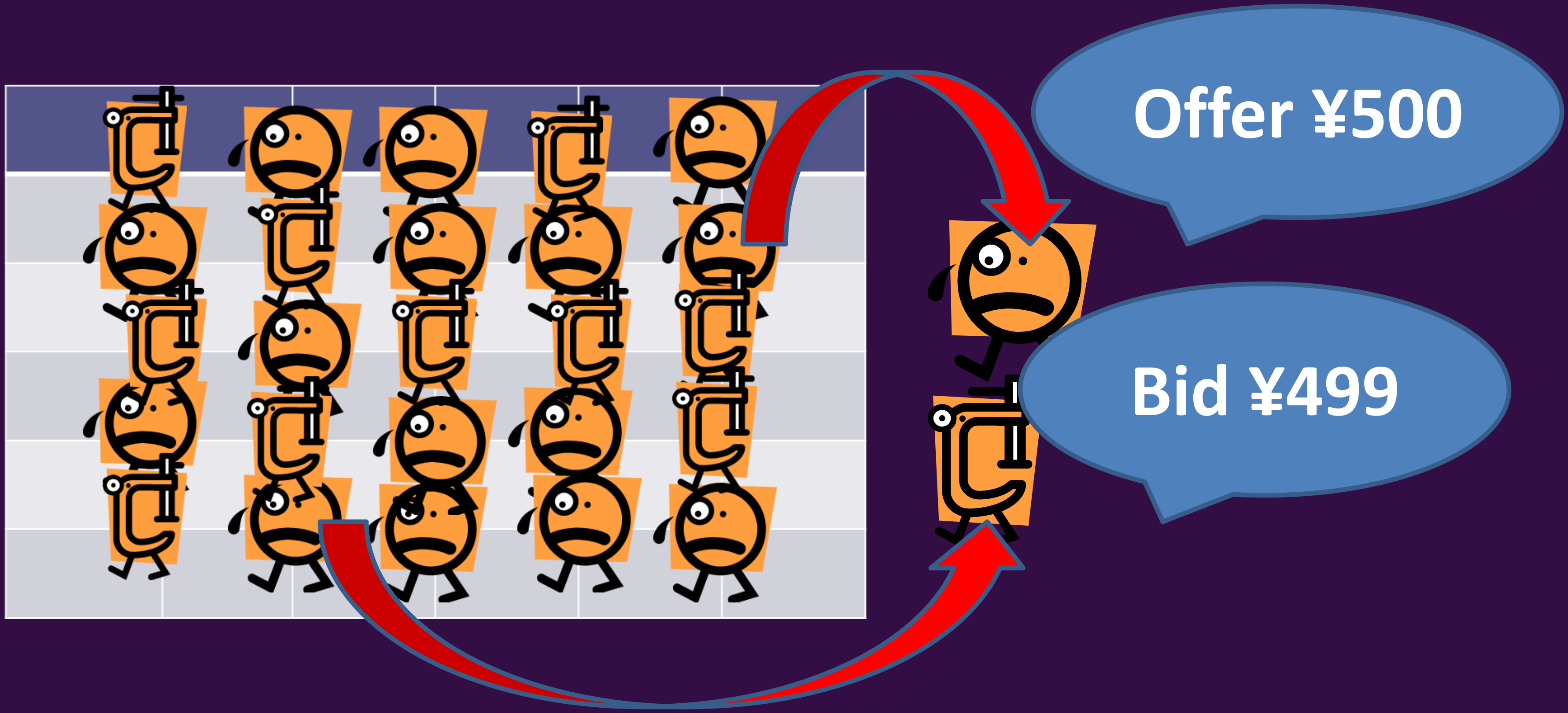
Situation



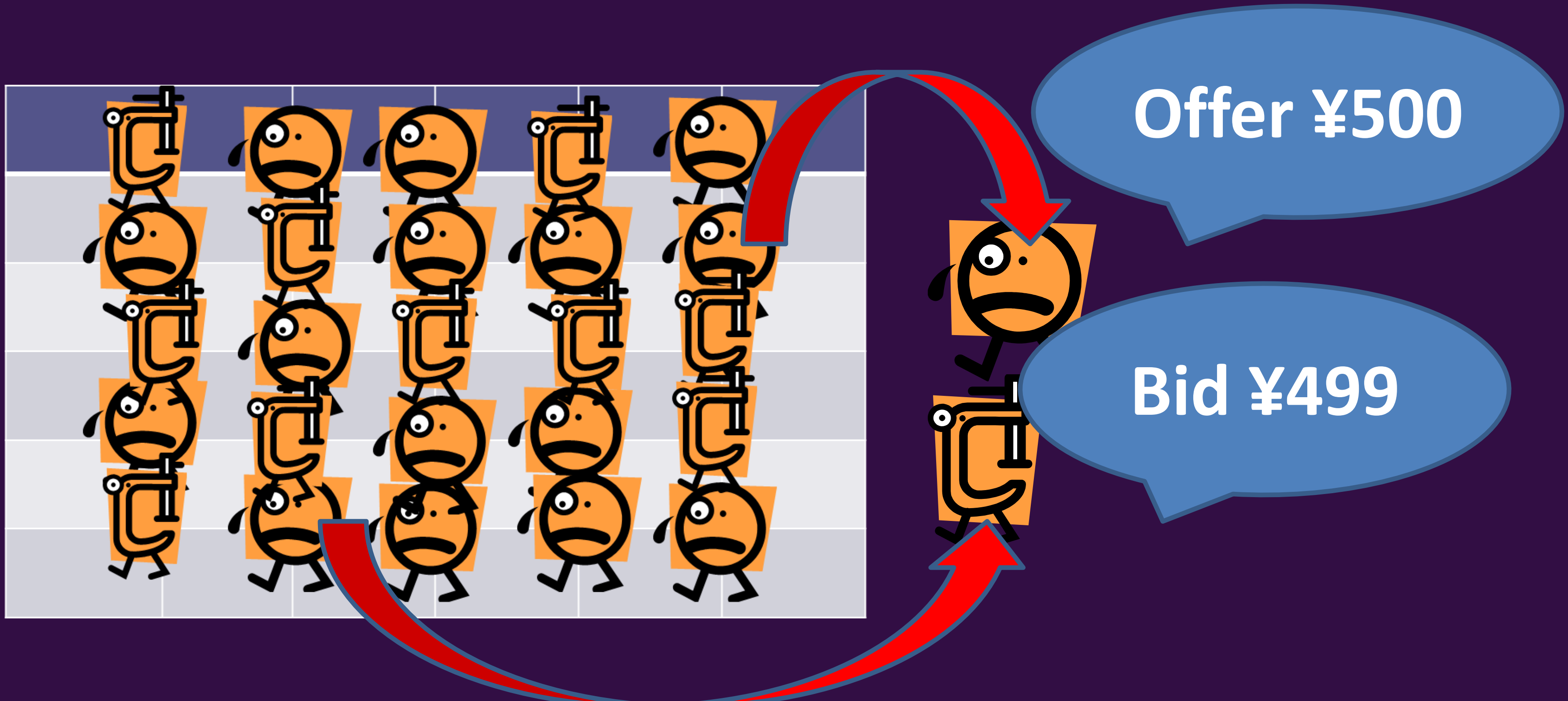
Situation



Situation

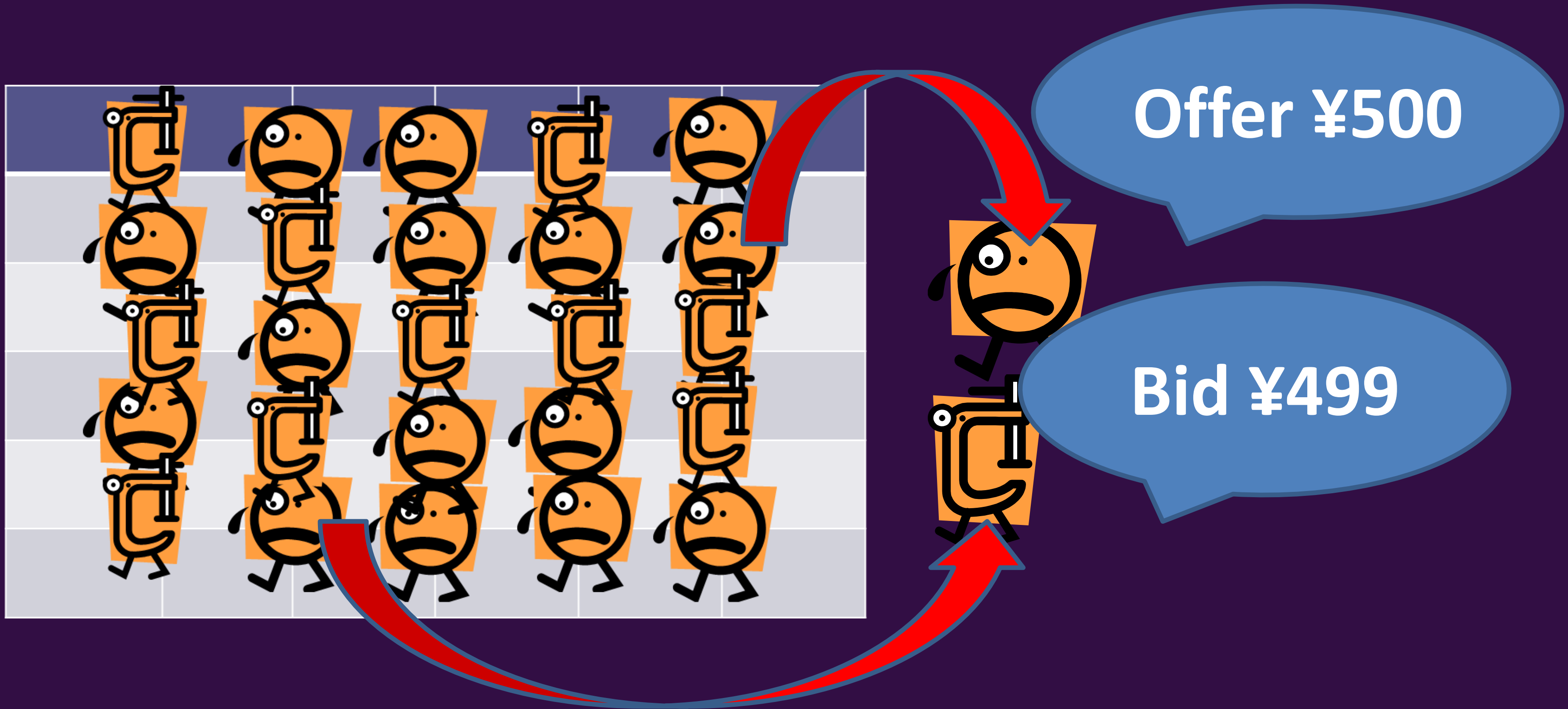


Situation



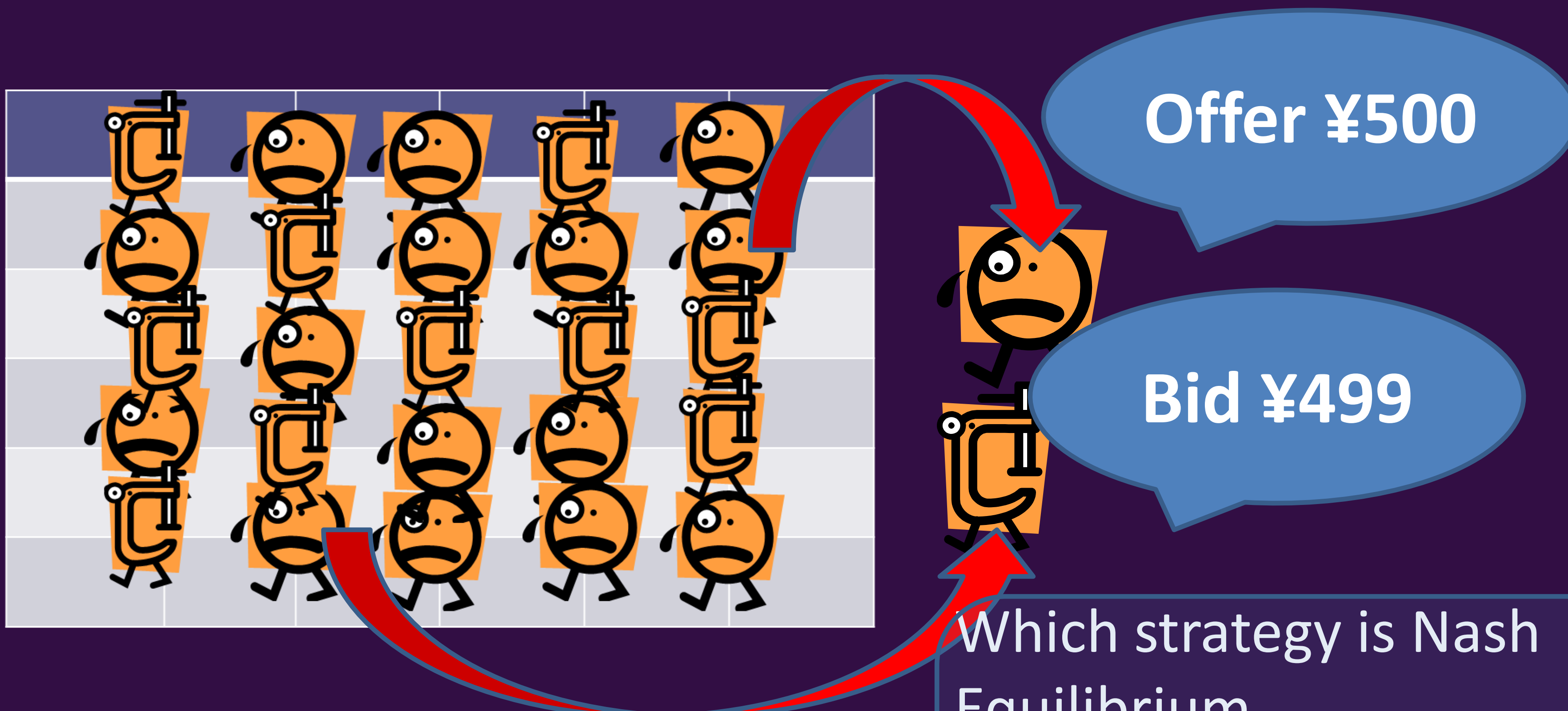
Stock Exchange which take account of the order book decides the trade's contract. (取引所が板情報をもとに、売買契約を決定する)

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Which strategy is Nash Equilibrium,
if this game is played at infinite ?
(このゲームを無限回仮想的に行うと、どの戦略が均衡となるのか？)

Model (モデル)

- **Payoff (利得) … Buyer : S(t)-K, Seller : K-S(t)**

where S(t) : current stock price, Brownian Motion, K: strike price (行使価格)

- **Replicator Equation**

$$\frac{dx_i(t)}{dt} = x_i(t)\left(g_i(t) - \bar{g}(t)\right)$$

$$\frac{dy_i(t)}{dt} = y_i(t)\left(h_i(t) - \bar{h}(t)\right)$$

where x_i, y_i : the probability of choosing the strategy 1 for each player. g_i, h_i : the payoff when each player chooses the strategy 1.

Two Strategies Case (戦略の数が2つ) :

- Replicator equation (see next slide)

$$\dot{x} = x(1-x)\{-b(t) + (a(t)+b(t))y\},$$

$$\dot{y} = y(1-y)\{b(t) - (a(t)+b(t))x\},$$

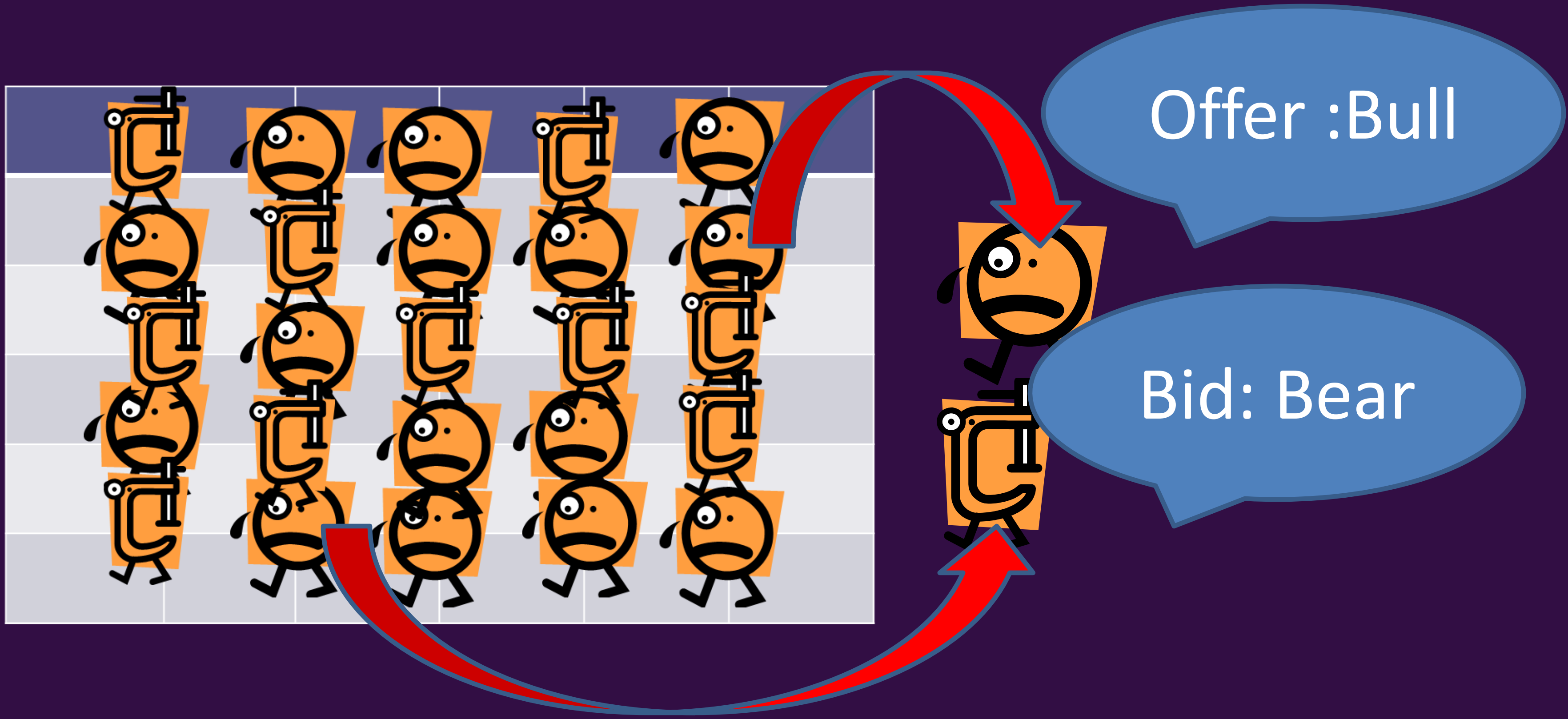
where x, y is the probability of choosing the strategy 1, 2 for each player.

Player 2

player1

	S1	S2
S1	a(t),-a(t)	0,0
S2	0,0	b(t),-b(t)

Situation



Prediction (予測)

- Replicator equation divided by xy(1-x)(1-y) :

$$\dot{x} = -\frac{b(t)}{y} + \frac{a(t)}{1-y}, \dot{y} = \frac{b(t)}{x} - \frac{a(t)}{1-x}.$$

- Discrete the above equations:

$$x(t+\varepsilon) = x(t) - \left(\frac{b(t)}{y} + \frac{a(t)}{1-y}\right)\varepsilon,$$

$$y(t+\varepsilon) = y(t) + \left(\frac{b(t)}{x} - \frac{a(t)}{1-x}\right)\varepsilon.$$

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

Buyer

Seller

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	+, -	0,0
S 2(BEAR)	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	+,+	0,0
S 2(BEAR)	0,0	-,+

iii) → (No change)

N.E. Mixed Strategy.

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	-,+	0,0
S 2(BEAR)	0,0	-,+

Payoff Matrix (利

i) ↑ (UP)

N.E. (s2,s2)

Seller

		LL)
S 1(BULL)	+, -	0,0
S 2(BEAR)	0,0	+,+

価格上昇時、売り手は約定価格よりも**強気に**高い売り、買い手は**弱気**で高い価格で購入

ii) ↓ (Down)

N.E. (s1,s1)

	S 1(B	
S 1(BULL)	+,+	
S 2(BEAR)	0,0	-,+

価格下落時、売り手は約定価格よりも**弱気に**安く売り、買い手は**強気**で安い価格で購入

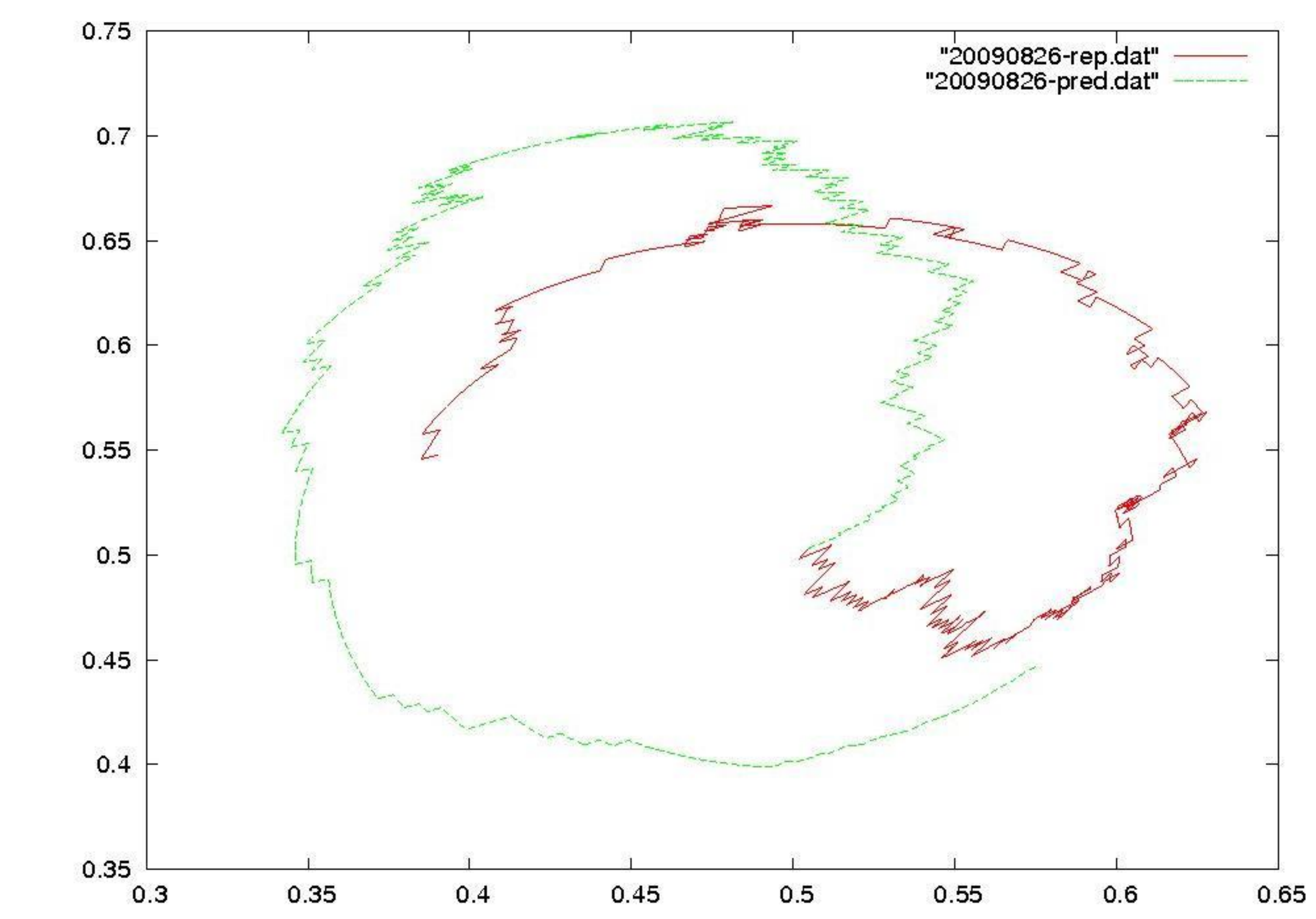
iii) → (No change)

N.E. Mixed Strategy.

	S 1(B	
S 1(BULL)	-,+	
S 2(BEAR)	0,0	-,+

価格変化しない時、売り手は約定価格よりも**弱気に**安く売り、買い手は**強気**で安い価格で購入

EX: 20090826


"20090826-rep.dat"
"20090826-pred.dat"
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65

4. EXTENSION:
RISK ATTITUDE

RISK ATTITUDE

- We assume that the own utility is linear function.(今まで主体の効用は線形であると仮定してきた。)
- Each player has the non-linear utility.(そこで非線形の場合をも考慮に入れる。)

- We examine the equilibrium selection with this nonlinear-utility. (そこでこの非線形効用関数を用いて、均衡選択の問題を考察する。)

EXAMPLE(Allais Paradox)

- Which lotteries do you like ?
- There are three possible monetary prizes.

First Prize	Second Prize	Third Prize
2500 000 USD	500 000 USD	0 USD

- The decision maker is subjected to two choice tests. The first consists of a choice between the lotteries L_1 and L'_1:
- $L_1=(0,1,0)$ $L'_1=(0.10,0.89,0.01)$.
- The second consists of a choice between the lotteries L_2 and L'_2:
- $L_2=(0,0.11,0.89)$ $L'_2=(0.10,0,0.90)$.
- ⇒ $L_1 \succ L'_1$ and $L'_2 \succ L_2$.

EXAMPLE(Allais Paradox)

- Which lotteries do you like ?
- There are three possible monetary prizes.

	First Prize	Second Prize	Third Prize
	2500 000 USD	500 000 USD	0 USD

- The decision maker is subjected to two choice tests. The first consists of a choice between the lotteries L_1 and L'_1:
- $L_1=(0,1,0)$ $L'_1=(0.10,0.89,0.01)$.
- The second consists of a choice between the lotteries L_2 and L'_2:
- $L_2=(0,0.11,0.89)$ $L'_2=(0.10,0,0.90)$.
- $\Rightarrow$ $L_1 \succ L'_1$ and $L'_2 \succ L_2$.

- Utility function : g(x) , z : payoff
- Taylor Expansion:
- $g(x+z)-g(x)=g'(x)z+0.5g''(x)z^2+O(z^3)$ .... (*)

Def. Given a (twice-differentiable) Bernoulli utility function u(.) for money, the ***Arrow-Pratt coefficient of absolute risk aversion*** at x is defined as $r_A(x)=-u''(x)/u'(x)$.

- (*) $g(x+z)-g(x) = zg'(x)(1-0.5zr_A(x))$
- (In economics, we assume $g'(x)>0$, $g''(x)<0$)

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

	S 1	S 2
S 1	-, -	0,0
S 2	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

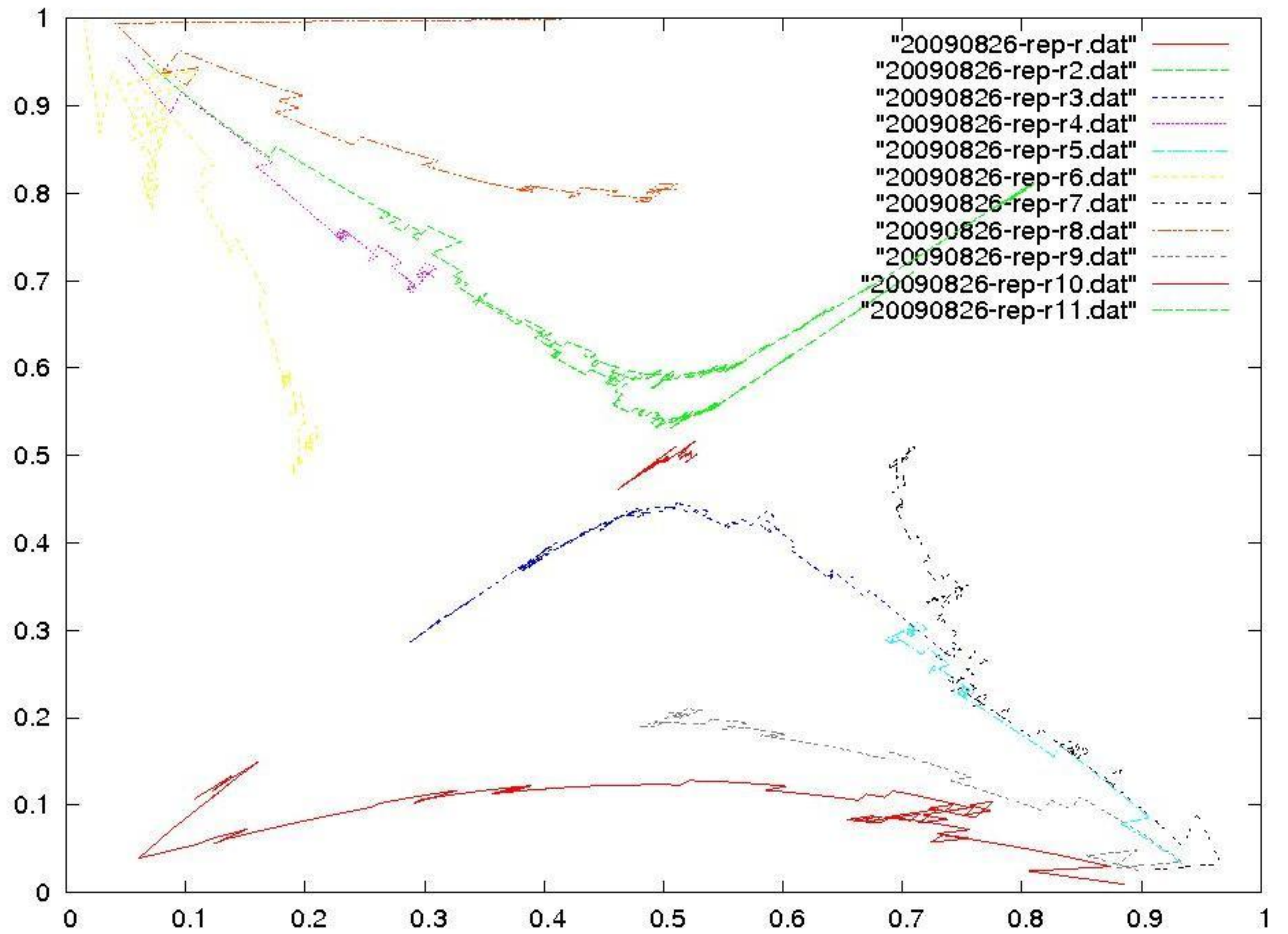
	S 1	S 2
S 1	+,+	0,0
S 2	0,0	-,-

iii) → (No change)

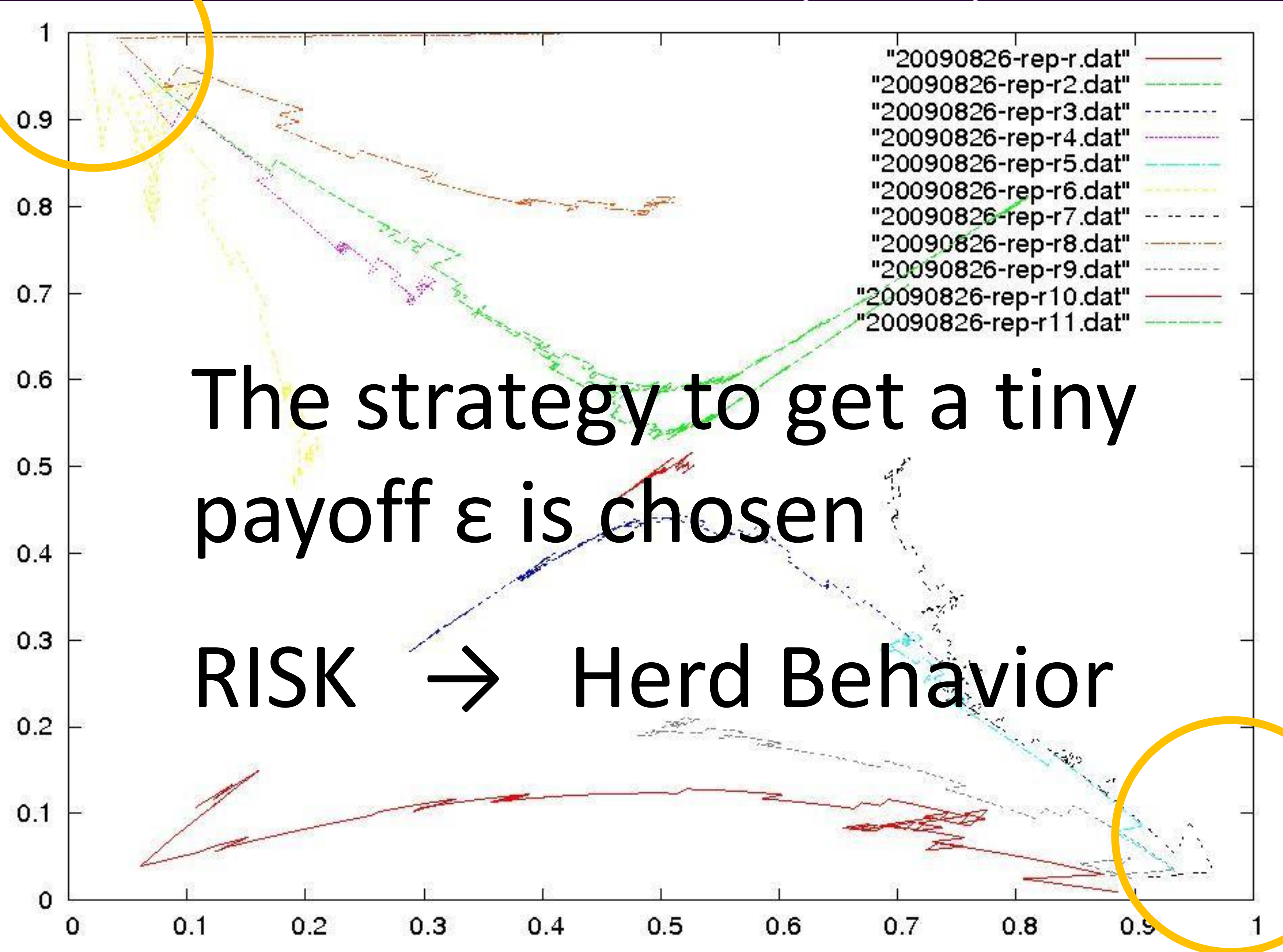
N.E. Mixed Strategy.

	S 1	S 2
S 1	-,-	0,0
S 2	0,0	-,-

EX: 20090826 (RISK)



EX: 20090826 (RISK)



The strategy to get a tiny payoff ε is chosen

RISK → Herd Behavior

5. Empirical Evidence:

MICRO ECONOMETRICS

Multinominal Logit Model (多項選択モデル)

- Order Book (板情報) is expressing that the players choose the strategy/price at that time.(ある時間におけるどの価格(戦略)を採用しているのかを表している.)
- Strategy/Price : J (<∞) 個,
- The probability of choosing the strategy j with x_i (x_iという属性を持った個人*i* が選択肢 *j* を選ぶ確率) : π_{ij}, ($P(y_i=j \mid x_i)=\pi_{ij}$)

$$\pi_{ij} = \frac{\exp(x'_i \beta_j)}{\sum_{r=i}^{J} \exp(x'_i \beta_j)} \qquad j = 1, \cdots, J.$$

EXAMPLE: The Order Book (板情報)

（Offer(sell)）	Price	（Bid (buy)）
0	Market orders	0
492	9840	-----
506	9830	-----
444	9820	-----
530	9810	----
784	9800	-----
-----	9790	197
-----	9780	734
-----	9770	640
------	9760	643
-----	9750	598

This order book is on Nikkei Future Market(9:03, 5th, November, 2009. MOVIE (avi))

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

How to analyze the order book

Step 1) Logit Model (Derive the probability of choosing the strategy and transform this into log function.)

Step 2) Regression analysis.

Step 3) Derive the Demand and Supply function.

$$Y=583.93-146.27X, \quad Y=-237.14+59.57X$$

Step 4) In equilibrium, we know that the quantity demanded is equal to the quantity supplied.

Step 5) Derive the Nash equilibrium.

$$X^{*}=9740.$$

6. SUMMARY AND FUTURE WORKS

Summary

- **MODELING** the Financial Market.(金融市場をモデリングした)

- **ANALYZING** the impact of each player’s Risk Attitude.(各主体にリスクに対する態度がある場合を考察した)

- **ANALYZING** the Order book with Multi-Logit Model and **DERIVING** how to forecast the next step’s price.(板情報を多項ロジットモデルを用いて分析し、次の約定価格を予想する新たな手法を導いた)

Future Works

- **EMPIRICAL EVIDENCE** (実証研究) **: Particle Filter** (粒子フィルタ)
- **GET** the Online Financial Data, **CALCULATE** and **DISPLAY**. (オンラインでデータを入手し、計算し、それを表示する)
- **MAKE** the software like a PUCK based on the Evolutionary Game Theory. (PUCKの進化ゲーム理論版の構築)

PANEL ANALYSIS(パネル分析):

- TIME SERIES(時系列): **PARTICLE FILTER**(粒子フィルタ) (Kikkawa[4]の動学に対応)
- State Space Model (状態空間モデル)
- System model: UTILITY (ex. $Y_i=\alpha+\beta X_i+u_i$)
- Observation model:

$$\pi_{ij} = \frac{\exp(x'_i\,\beta_j)}{\sum_{r=i}^{J}\exp(x'_i\,\beta_j)} \qquad j = 1, \cdots, J.$$

- We examine empirical evidence : why the player chooses this strategy. (なぜこの戦略を採用したのか？を時系列に実証分析)

Thank You For Your Attention

Mitsuru KIKKAWA (mitsurukikkawa@hotmail.co.jp)

This File is available at

http://kikkawa.cyber-ninja.jp/

NEXT MY TALK 2/16@HOKKAIDO UNIV.

REFERENCE

[1] Black, Fischer and Scholes, Myron: ''The Pricing of Options and Corporate Liabilities,'' The Journal of Political Economy, Vol. 81, pp. 637-654 (1973) **[HP]**

[2] Esaley and O'hara : Time and the Process of Security Price Adjustment, The Journal of Finance, Vol. 47, No.2, pp. 577-605.(1992) **[HP]**

[3] 川西 諭: ノイズのある合理的期待均衡モデルにおける投資情報獲得戦略の多様性について, 現代経済学の潮流2008, 第4章, pp.105--141 (2008) **[amazon]**

[4] Kikkawa, Mitsuru : "Statistical Mechanics of Games — Evolutionary Game Theory —," Progress of Theoretical Physics Supplement, No. 179 (2009), pp.216-226. **[HP]**

[5] 吉川 満:「オプションの戦略的な価格付け : Black-Sholes 方程式の周辺」, 北海道大学数学講究録, #140, pp. 142-146 (2009) **[HP]**

[6] Kikkawa, Mitsuru: ''Co-evolution and Diversity in Evolutionary Game Theory : Stochastic Environment,'' 京都大学数理解析研究所講究録, **1663**号 (2009), pp. 147-152.

[7]吉川 満:「進化ゲーム理論を用いたオプション市場分析」, 人工知能学会研究会資料, SIG-FIN-003, pp. 23-28 (2009) **[HP]**

[8] 西岡寛兼, 鳥海不二夫, 石井健一郎「板情報を用いた市場変化の分析」, 人工知能学会研究会資料, SIG-FIN-007, pp. 58-63 (2009) **[HP]**

APPENDIX

Replicator Equation

REPLICATOR EQ.

$$\dot{x}_i = x_i\left((Ax)_i - x \cdot Ax\right), i = 1, \cdots, n.$$

If the player's payoff from the outcome *i* is greater than the expected utility *x Ax*, the probability of the action *i* is higher than before. And this equation shows that the probability of the action *i* chosen by another players is also higher than before (**externality**). Furthermore, the equation is derived uniquely by the **monotonic** (that is if one type has increased its share in the population then all types with higher profit should also have increased their shares).

Two Strategies

$$\dot{x} = x(1-x)\{b-(a+b)x\} \quad \cdots(*)$$

Classification

(I) Non-dilemma: a > 0, b < 0, ESS : one

(II) Prisoner's dilemma : a < 0, b > 0, ESS :one

(III) Coordination : a>0,b>0, ESS two

(IV) Hawk-Dove : a<0,b < 0, ESS one (mixed strategy)

Player 2 across the top, player 1 down the side:

	S 1	S 2
S 1	a,a	0,0
S 2	0,0	b,b

Payoff Matrix

Replicator Eq. and Payoff Matrix

- Strategy : Two, Player : Two
- Payoff

$$P^1 = \begin{pmatrix} f_1 & f_3 \\ f_2 & f_4 \end{pmatrix}, P^2 = \begin{pmatrix} g_1 & g_3 \\ g_2 & g_4 \end{pmatrix}$$

- Replicator Equation

$$\dot{y} = y(1-y)\{f_1 - f_2 + x(f_3 - f_4 - f_1 + f_2)\}$$

$$\dot{x} = x(1-x)\{g_4 - g_2 + y(g_3 - g_4 - g_1 + g_2)\}$$

x is the probability of the type 2 player chooses the strategy 2.
y is the probability of the type 1 player chooses the strategy 1.

$$f_1 - f_2 := a,\ f_4 - f_3 := c,\ g_4 - g_2 := d,\ g_1 - g_3 := b$$

Derive

$$\dot{y} = y(1-y)\{a-(a+c)x\},\ \dot{x} = x(1-x)\{d-(b+d)y\}$$

Classification

(I) Non-Dilemma, **Prisoner's Dilemma :**

ac < 0, bd >0, ESS :1

(II) Coordination :

a>0,b>0,c>0,d>0, ESS :2

(III) Chicken :

a<0,b<0,c<0,d<0, ESS :2

(IV) Matching Pennie:

ab<0,cd<0, ac>0,bd<0, ESS:Mixed

	S1	S2
S1	a,b	0,0
S2	0,0	c,d

- 本研究の一部は，平成20年度採択, 文部科学省 グローバルCOEプログラム**<u>「現象数理学の形成と発展」</u>** 現象数理若手プロジェクト「人間特有の現象に対する学習の影響 - 進化ゲーム理論による分析 -」に関する研究拠点形成費の助成を受けて行われた. (This research was supported in part by <u>Meiji University Global COE Program (Formation and Development of Mathematical Sciences Based on Modeling and Analysis)</u> of the Japan Society for the Promotion of Science.