Evolutionary Game Theory For Empirical Evidence
(実証分析のための進化ゲーム理論)

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OUTLINE

1. Introduction (motivation, review)

2. Main Contributions

FORMULATING the NEW GAME THEORY for Empirical Evidence. (Base: <u>Kikkawa (2009)</u>)

3. Example

GIVING some typical examples (Experimental Economics, Financial Market)

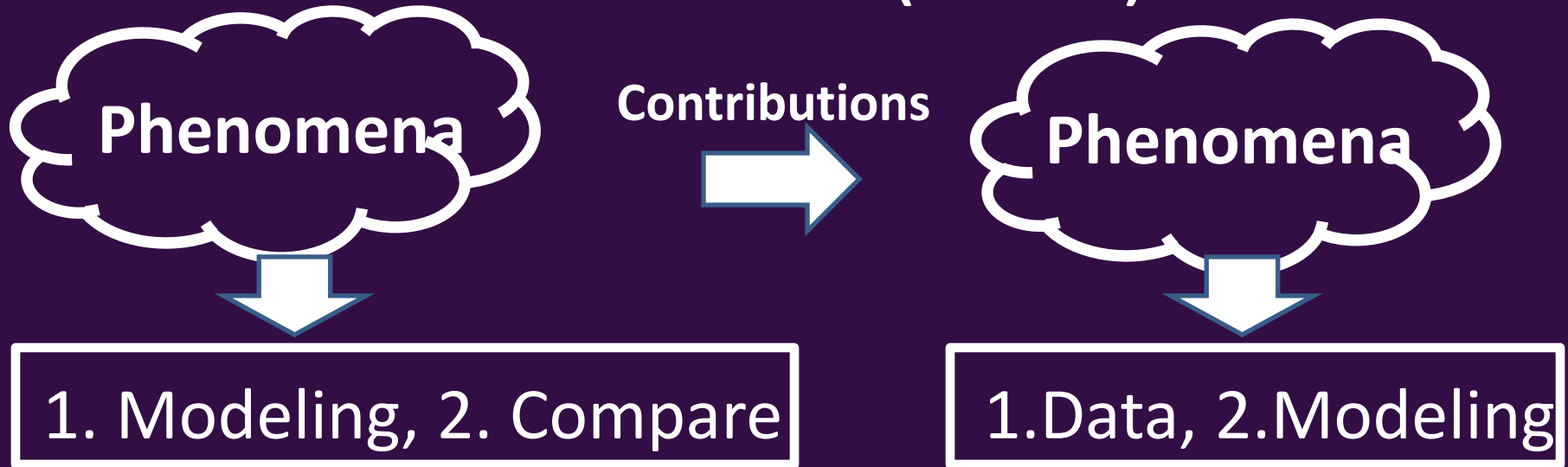**4. Extention** (as a time series analysis)

5. Summary and Future works

1. INTRODUCTION

Motivation (動機)

Phenomena → **Contributions** → **Phenomena**

| 1. Modeling, 2. Compare | 1.Data, 2.Modeling |

- To describe the complex social phenomena.

1. "Gap (real phenomena and model)"

+

2. "Data" (technological advances)

⇒ Formulate new game theory with **micro-econometrics's method**(experimentally).

2. STATISTICAL GAME THEORY

DEF. Best Response, Nash eq.

DEF. : A **best response** of the player i's strategy $q_i \in Q_i$

for another $n-1$ players' strategy sets $q_{-i} = (q_1, ..., q_{i-1}, q_{i+1}, ..., q_n)$ is

$$F_i\left(q_i, q_{-i}\right) = \max_{r_i \in Q_i} F_i\left(r_i, q_{-i}\right)$$.

The whole best response for player i is $B_i(q_{-i})$ for strategy set q_{-i}.

DEF. A **Nash equilibrium** of a strategic game n-person game G^* is a profile $\vec{q}^* = \left(q_1^*, \cdots, q_n^*\right)$ with the property that for every player $i=(1,...,n)$ we have the best response for another player's strategy set q^*_{-1}.

Kikkawa (2009)

- **Many players** play the game simultaneously.

- Kikkawa (2009) formulates this situation with statistical mechanics (統計力学).

> **Prop.** We obtain the probability distribution of actions, $\{S_i\}$, $i=1,...,N$, and the player's payoff from the outcome is f,
>
> $$P(\{S_i\})=Z^{-1} \exp(\gamma\, f),$$
>
> $\{S_i\}$: a player i's action, γ: non-negative constant, f: the player's payoff from outcome $\{S_i\}$, Z: normalization parameter.

- Kikkawa (2009) is similar to **Quantal Response Equilibrium**. (Mckelvey and Palfrey (1995, 1996))

Multinominal Logit Model

- From <u>Kikkawa (2009)</u>, we can know the probability of choosing the strategy for each player.

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- Data (the probability of choosing the strategy for each player)

- Regression analysis(回帰分析)

  $Y_i = \alpha + \gamma f + u,$ u : logistic distribution.

- We can estimate optimal parameters in this model with **Least Squares Method** (最小2乗法)

4. EXAMPLE

EXAMPLE 1 (Experimental Economics)

We consider the following game with the payoff.

	B_1	B_2	B_3
A_1	15,-15	0,0	-2,2
A_2	0,0	-15,15	-1,1
A_3	1,-1	2,-2	0,0

Payoff matrix 1 (<u>Lieberman (1960)</u>)

Nash equilibrium: (A_3, B_3)

+ Experimental Data.

We can obtain the following payoff matrix with our method.

	B_1	B_2	B_3
A_1	3.55,2.10	1.75,3.63	-1.14,-2.19
A_2	0.98,4.59	-2.31,1.18	-1.15,-1.47
A_3	-0.01,-0.624	-0.37,-0.257	-0.01,0.021

Payoff matrix 2

Nash equilibria:
(A_3, B_3), (A_1, B_2)

EXAMPLE 2 : The Order Book

（Offer(sell)）	Price	（Bid (buy)）
0	Market orders	0
492	9840	-----
506	9830	-----
444	9820	-----
530	9810	----
784	9800	-----
-----	9790	197
-----	9780	734
-----	9770	640
------	9760	643
-----	9750	598

This order book is on Nikkei Future Market(9:03, 5th, November, 2009.)

The center column gives the **prices**, the second column from the left shows the volume of individual **offers** (sell**)**. The right hand side of the table represents the **bid** side (buy).

11

How to derive the Nash equilibirum

Step 1) Logit Model (Derive the probability of choosing the strategy and transform this into log function.)

Step 2) Regression analysis.

Step 3) Derive the Demand and Supply function.

$$Y=583.93-146.27X, \quad Y=-237.14+59.57X$$

Step 4) In equilibrium, we know that the quantity demanded is equal to the quantity supplied.
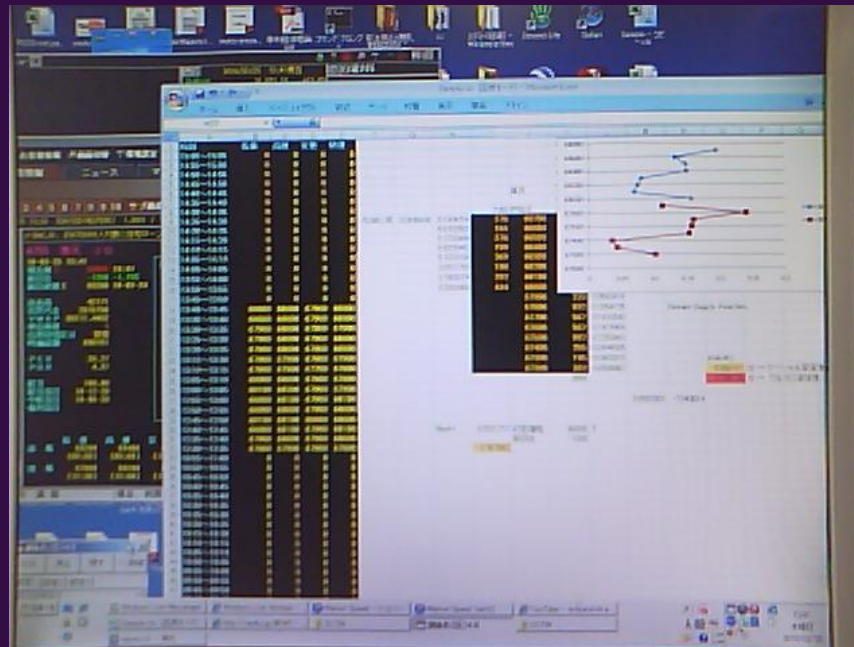
Step 5) Derive the Nash equilibrium.

$$X^*=9740.$$

Implementation

- **Excel**

- **Realtime Spread Sheet** (provided by Rakuten Securities, Inc. (楽天証券))

- **[MOVIE]**

5. EXTENSION

Time Series Analysis (Particle Filter)

- Extending this game theory as a particle filter.(<u>Kitagawa (1996)</u>)

- State-Space model

  (System model) $Y_i = \alpha + \gamma(t) f + v_n,$

  (Observation model) $y_n = Z^{-1} exp(\gamma (w_n) U_n)$

- We can analyze the dynamical model with this method.

6. SUMMARY AND FUTURE WORKS

Summary and Future Works

- FORMULATING new game theory.

- CONSIDERING the theory from data with micro-econometrics method.

- Merit : Complex Phenomena

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[3] Kitagawa, Genshiro: ''Monte Carlo Filter and Smoother for Non-Gaussian Nonlinear State Space Models,'' *Journal of Computational and Graphical Statistics*, Vol. **5**(1960), pp. 1-25. **[HP]**

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Text Book

For Detail, See my Website(**Bookguide   Readinglist**)

Classsic:

[1] Maynard Smith, John **Evolution and the Theory of Games** , Cambridge University Press, 1982/10. 日本語訳

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[4] Samuelson, Larry **Evolutionary Games and Equilibrium Selection (Mit Press Series on Economic Learning and Social Evolution, 1)** , MIT Press, 1997/04.

For Beginner :

[1] 石原英樹, 金井雅之 **進化的意思決定 (シリーズ意思決定の科学)** , 朝倉書店, 2002/04/05.

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