

Evolutionary Game Theory For Empirical Evidence

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OUTLINE

1. Introduction (motivation, review)

2. Main Contributions

FORMULATING the NEW GAME THEORY for Empirical Evidence. (Base: Kikkawa (2009))

3. Example

GIVING some typical example(Financial Market)

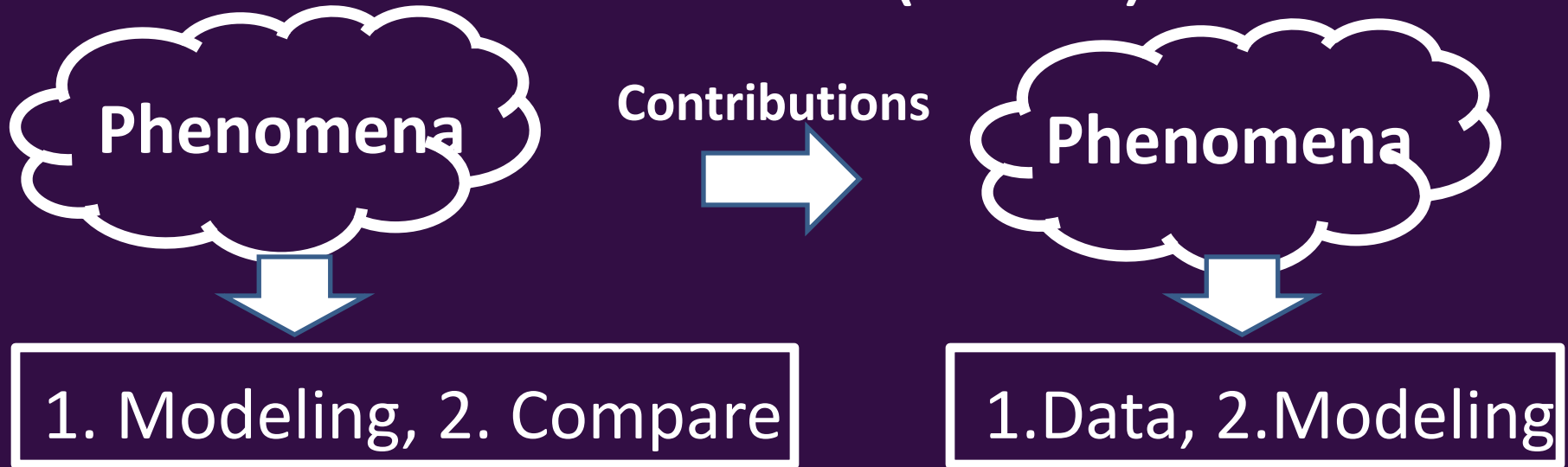
4. Summary and Future works



1. INTRODUCTION



Motivation (動機)



- To describe the complex social phenomena.

1. “Gap (real phenomena and model)”

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2. “Data” (technological advances)

⇒ Formulate new game theory with **micro-econometrics’s method**(experimentally).



WHAT IS THE “GAME” ?

(Non-cooperative Game)

There are two interacting players (Player 1, Player 2).

If player 1 chooses strategy 1 and player 2 chooses strategy 1, player 1's payoff is a , player 2's payoff b .

In this situation, which strategy does each player choose ?

(The game is played only once.)

→ This game's solution is **Nash Equilibrium**.

| | | player 2 | |
|---------|----|----------|-----|
| | | S1 | S2 |
| player1 | S1 | a,b | 0,0 |
| | S2 | 0,0 | c,d |

Nash equilibrium depends on the signs: a, b, c, d .



2. STATISTICAL GAME THEORY



DEF. Best Response, Nash eq.

DEF. : A **best response** of the player i 's strategy $q_i \in Q_i$ for another $n-1$ players' strategy sets $q_{-i} = (q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n)$ is

$$F_i(q_i, q_{-i}) = \max_{r_i \in Q_i} F_i(r_i, q_{-i}).$$

The whole best response for player i is $B_i(q_{-i})$ for strategy set q_{-i} .

DEF. A **Nash equilibrium** of a strategic game n -person game G^* is a profile $\vec{q}^* = (q_1^*, \dots, q_n^*)$ with the property that for every player $i = (1, \dots, n)$ we have the best response for another player's strategy set q_{-i}^* .



Kikkawa (2009)

- **Many players** play the game simultaneously.
- Kikkawa (2009) formulates this situation with statistical mechanics (統計力学).

Prop. We obtain the probability distribution of actions, $\{S_i\}$, $i=1,\dots,N$, and the player's payoff from the outcome is f ,

$$P(\{S_i\})=Z^{-1} \exp(\gamma f),$$

$\{S_i\}$: a player i 's action, γ : non-negative constant, f : the player's payoff from outcome $\{S_i\}$, Z : normalization parameter.

- Kikkawa (2009) is similar to **Quantal Response Equilibrium**. (McKelvey and Palfrey (1995, 1996))



Multinomial Logit Model

- From Kikkawa (2009), we can know the probability of choosing the strategy for each player.

+

- Data (the probability of choosing the strategy for each player)

- Regression analysis(回帰分析)

$$Y_i = \alpha + \gamma f + u, \quad u : \text{logistic distribution.}$$

- We can estimate optimal parameters in this model with **Least Squares Method (最小2乗法)**



3. EXAMPLE



EXAMPLE : The Order Book

(Offer(sell)) Price (Bid (buy))

| (Offer(sell)) | Price | (Bid (buy)) |
|---------------|---------------|-------------|
| 0 | Market orders | 0 |
| 492 | 9840 | ---- |
| 506 | 9830 | ---- |
| 444 | 9820 | ---- |
| 530 | 9810 | --- |
| 784 | 9800 | ---- |
| ---- | 9790 | 197 |
| ---- | 9780 | 734 |
| ---- | 9770 | 640 |
| ----- | 9760 | 643 |
| ---- | 9750 | 598 |

This order book is on Nikkei Future Market(9:03, 5th, November, 2009. [MOVIE \(avi\)](#))

The center column gives the **prices**, the second column from the left shows the volume of individual **offers (sell)**. The right hand side of the table represents the **bid side (buy)**.



How to derive the Nash equilibrium

Step 1) Logit Model (Derive the probability of choosing the strategy and transform this into log function.)

Step 2) Regression analysis.

Step 3) Derive the Demand and Supply function.

$$Y=583.93-146.27X, Y=-237.14+59.57X$$

Step 4) In equilibrium, we know that the quantity demanded is equal to the quantity supplied.

Step 5) Derive the Nash equilibrium.

$$X^*=9740.$$



4. SUMMARY AND FUTURE WORKS



Summary and Future Works

- FORMULATING new game theory.
(Simple, Extention : Time Series Analysis)
- CONSIDERING the theory from data with micro-econometrics method.
- Merit : Complex Phenomena
- PREPARING to trade the stock with this method. (automatically trading, etc.)



REFERENCE

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- [2] Kikkawa, Mitsuru : "Statistical Mechanics of Games — Evolutionary Game Theory —," *Progress of Theoretical Physics Supplement*, No. **179** (2009), pp.216-226. [\[HP\]](#)
- [3] Kitagawa, Genshiro: "Monte Carlo Filter and Smoother for Non-Gaussian Nonlinear State Space Models," *Journal of Computational and Graphical Statistics*, Vol. **5**(1960), pp. 1-25. [\[HP\]](#)
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- [6] McKelvey, Richard D. and Palfrey, Thomas R. : "A Statistical Theory of Equilibrium in Games," *Japanese Economic Review*, Vol. **47** (1996), No.2, pp.186-209. [\[HP\]](#)



Text Book

For Detail, See my Website([Bookguide](#) [Readinglist](#))

Classic:

- [1] Maynard Smith, John Evolution and the Theory of Games, Cambridge University Press, 1982/10. [日本語訳](#)
- [2] Axelrod, Robert The Evolution of Cooperation, Basic Books, 1984/03. [日本語訳](#)

Text Book:

- [1] Weibull, Jorgen W. Evolutionary Game Theory, MIT Press, 1995/08/14. [日本語訳](#)
- [2] Hofbauer, Josef and Sigmund, Karl Evolutionary Games and Population Dynamics, Cambridge University Press, 1998/07. [日本語訳](#)
- [3] Vega-Redondo, Fernando Evolution, Games and Economic Behaviour, Oxford University Press, 1997/01.
- [4] Samuelson, Larry Evolutionary Games and Equilibrium Selection (Mit Press Series on Economic Learning and Social Evolution, 1), MIT Press, 1997/04.

For Beginner :

- [1] 石原英樹, 金井雅之 進化的意思決定 (シリーズ意思決定の科学), 朝倉書店, 2002/04/05.
- [2] 大浦宏邦 社会科学者のための進化ゲーム理論—基礎から応用まで, 書房, 2008/09/25.



Thank You For Your Attention

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Next my talk: 3/27 @ Osaka

