Market Model Focused On the Order Book
: Evolutionary Game Theory
(板情報に着目した市場モデル
:進化ゲーム理論)

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THIS FILE IS AVAILABLE AT http://kikkawa.cyber-ninja.jp/

This Talk (本報告)

- This talk constructs the financial market model with Evolutionary Game Theory. (本報告は進化ゲーム理論を用いて市場のモデルを作った。)

- 各主体がどの価格にAsk/Bidするのか？またこの各個人の行動を集計した板情報から価格の変動の源泉を探り、潜在的な約定価格を予測する。

OUTLINE

Modeling → Analysis/Regression → Practice

1. Introduction (Motivation)
2. Review
3. Model (Empirical Evidence, Practice)
4. Dynamical System (Risk Attitude)
5. Time Series Analysis
6. Summary/Future works

1. INTRODUCTION

Motivation (動機)

- Financial Theory

  Probability Theory → Microeconomic Theory.

1. Data (Execution Price, Order book)
2. Method (Double Auction, Game Theory)
3. Practical Use (Excel)

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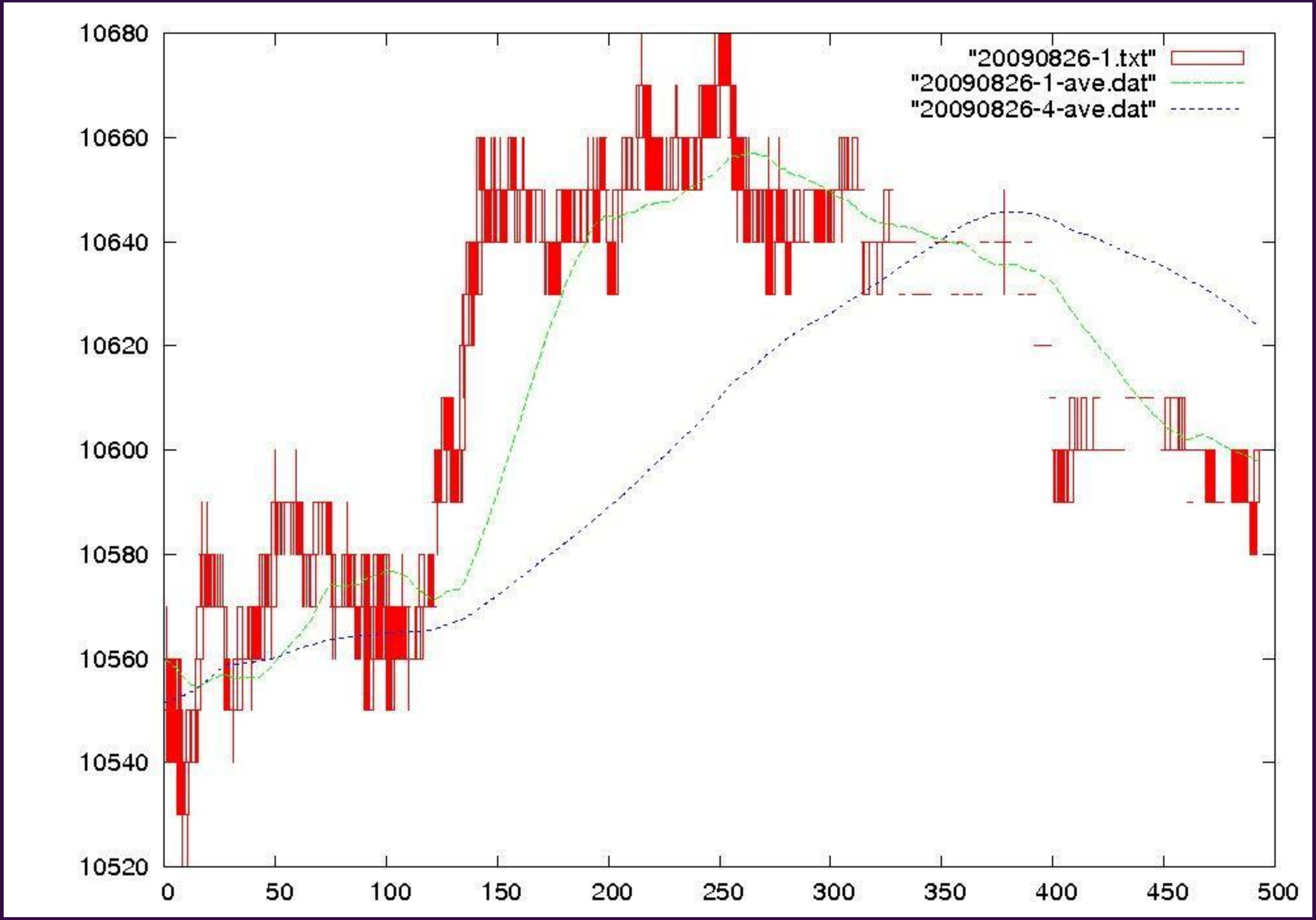

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500

The Order Book (板情報)

（Offer(sell)）	Price	（Bid (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

In this case, opening price is 500 or 501.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

【MOVIE】

Research Fields (this study)

WHAT IS THE "GAME" ? (Non-cooperative Game)

There are two interacting players (Player 1, Player 2).

If player 1 chooses strategy 1 and player 2 chooses strategy 1, player 1's payoff is *a* , player 2's payoff is *b*.

In this situation, which strategy does each player choose ?

(The game is played only once.)

→This game's solution is **Nash Equilibrium**.

player 2

player1

	S1	S2
S1	a,b	0,0
S2	0,0	c,d

Nash equilibrium depends on the signs: *a,b,c,d*.

Can we compare own utility with an another player's utility ?

- In micro economics, this answer is "**No**".

  Utility : **cardinal** (基数的, measurable)

  ordinal (序数的, not measurable, only order).

- In game theory, this answer is "**YES**".

  von Neumann-Morgenstern's utility function

- One of fundamental issues in Game Theory.

→Estimate the utility function which can't be observed with "**data**".

Double　Auction

- 売り手と買い手がある財についてそれぞれ自分の評価について私的情報を持ち、価格を戦略として取引を行う。

- 次の3つの条件を同時に満たすような取引制度は存在しない。

  i) Individual Rationality (個人合理性)

  ii) Pareto Efficient (パレート効率性)

  iii) Incentive Compatible (誘因両立性)

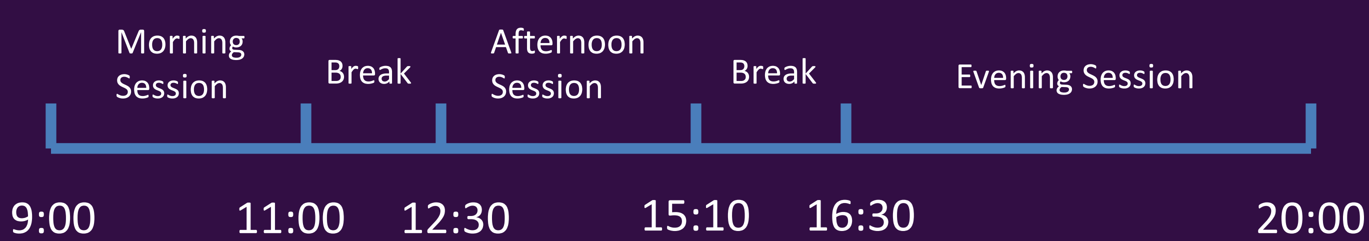
2. RELATED LITERATURES AND PRELIMINARIES

How are stock prices determined ?

- Stock prices are determined by two methods, the *Itayose(板寄せ)* and *Zaraba(ザラバ)* methods. The *Itayose* method is mainly used to decide opening and closing prices; the *Zaraba* method is used during continuous auction trading for the rest of the trading session.

→ The stock price are determined by **Rule**.

[Nikkei 225 Future Market(日経225先物)] [1day]



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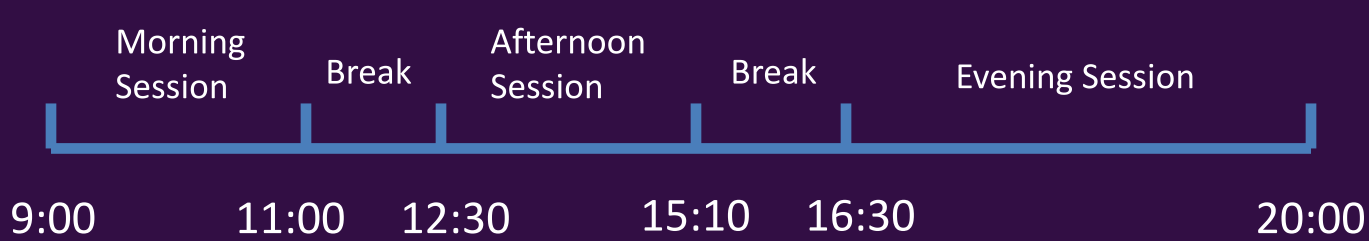

Two Principles (2つの原則)

1) **Price Priority** (価格優先の原則)

means that the lowest sell and highest buy orders take precedence over other orders.

2) **Time Priority** (時間優先の原則)

means that among orders at the same price, the order placed earliest takes precedence.

Offer(sell)	Price	Bid (buy)
A 3000(5), C 4000(4)	502	early ← → late
D 10000(3), E 9000(2), F 5000(1)	501	
	500	H 80000(1), B 1000(2), J 4000(3)
late← → early	499	H 1000(4), B 150000(5)

The Order Book (板情報)

（Offer(sell)）	Price	（Bid (buy)）
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8000	502	1000
20000	501	7000
4000	500	10000
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4000	498	30000

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

In this case, opening price is 500 or 501.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

【MOVIE】

Assume: opening price is 500.

（Offer(sell)）	Price	（Bid (buy)）
6000	Market orders	4000
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

- The market orders of 4000 shares to buy and 6000 shares to sell are matched, leaving sell orders of 2000 shares.

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

Second Step

（Offer(sell)）	Price	（Bid (buy)）
2000	Market orders	
8000	502	1000
20000	501	7000
4000	500	10000
2000	499	8000
4000	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- The market sell orders of 2000 shares and sell orders 6000 shares at limit prices of 499 or less are matched with the buy orders of 8000 shares at limit prices of 501 or more. Thus far, 12000 shares have been matched in total.

Third Step

（Offer(sell)）	Price	（Bid (buy)）
	Market orders	
8000	502	
20000	501	
4000	500	10000
	499	8000
	498	30000

Source : Tokyo Stock Exchange: Guide to TSE Trading Methodology

- Finally, the sell orders of 4000 shares at a limit price of 500 are matched with the buy orders of 10000 shares at a limit price of 500. Although this still leaves buy orders of 6000 shares at 500.

Fourth Step

（Offer(sell)）	Price	（Bid (buy)）
	Market orders	
8000	502	
20000	501	
	500	6000
	499	8000
	498	30000

- Thus the opening price is determined at 500 and transactions of 16000 shares are completed at 500.

The stock price and the trade depend on the **order book**. (価格や取引の可否は板情報によって決定する。)

3. MODEL

Model

- Ref. Chatterjee and Samuelson (1983).
- **Players**… large population : seller and buyer, potentially (大人数の潜在的な売り手と買い手)

  Seller and Buyer trade an asset.

- **Goods** (財) … 1財
- **Strategy** (戦略)… **n** (<∞) 個、p_s, p_b

Here, the strike price : how much do you buy or sell an asset. (ここでは購入、売却価格)

- **Payoff (利得) …**

  Buyer : $\max[v_b - p_b]$ Prob(OB),

  Seller : $\max[p_s - v_s]$ Prob(OB).

where Prob(OB) $\propto$ Prob ($p_b \geqq p_s(v_s)$) × Prob(OA)

(取引が起こる条件)　(取引への積極度)

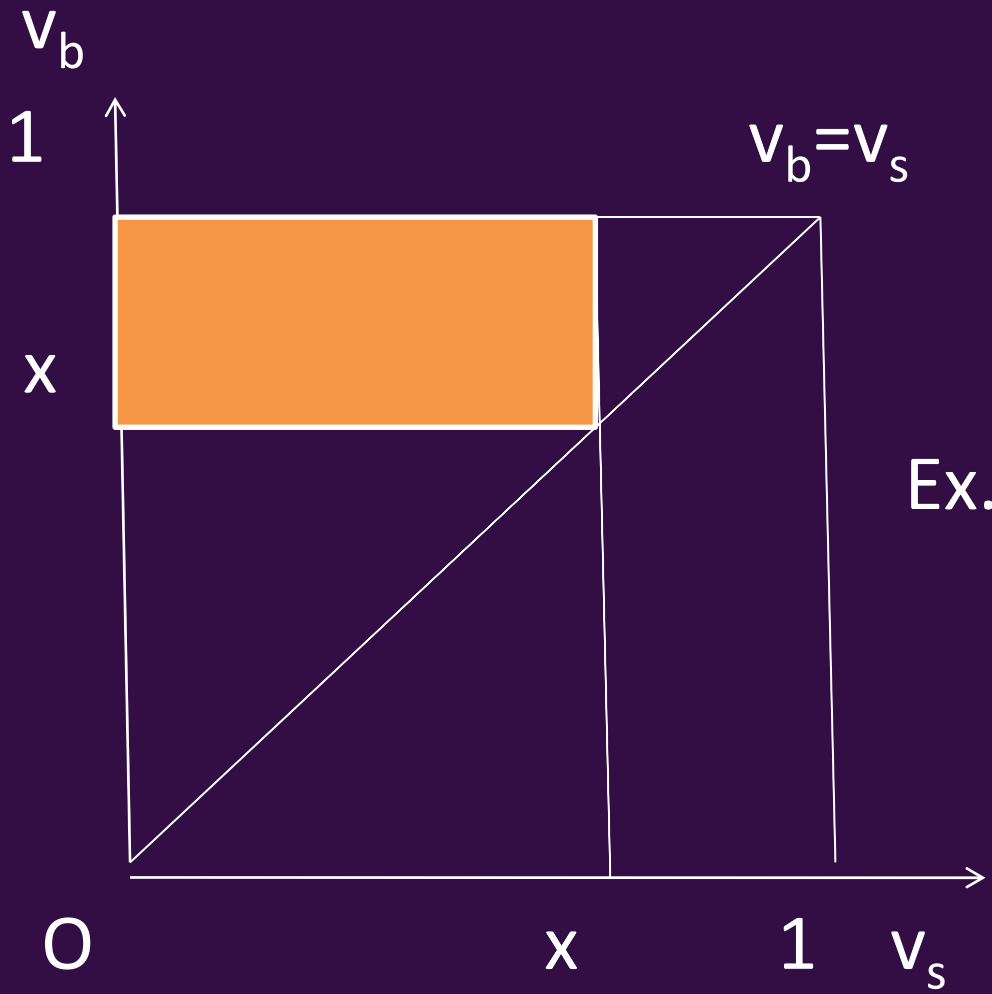
EXAMPLE: The Order Book (板情報)

（Offer(sell)）	Price	（Bid (buy)）
0	Market orders	0
492	9840	-----
506	9830	-----
444	9820	-----
530	9810	----
784	9800	-----
-----	9790	197
-----	9780	734
-----	9770	640
------	9760	643
-----	9750	598

This order book is on Nikkei Future Market(9:03, 5th, November, 2009. MOVIE (avi))

The center column gives the prices, the second column from the left shows the volume of individual offers (sell). The right hand side of the table represents the bid side (buy).

単一価格均衡



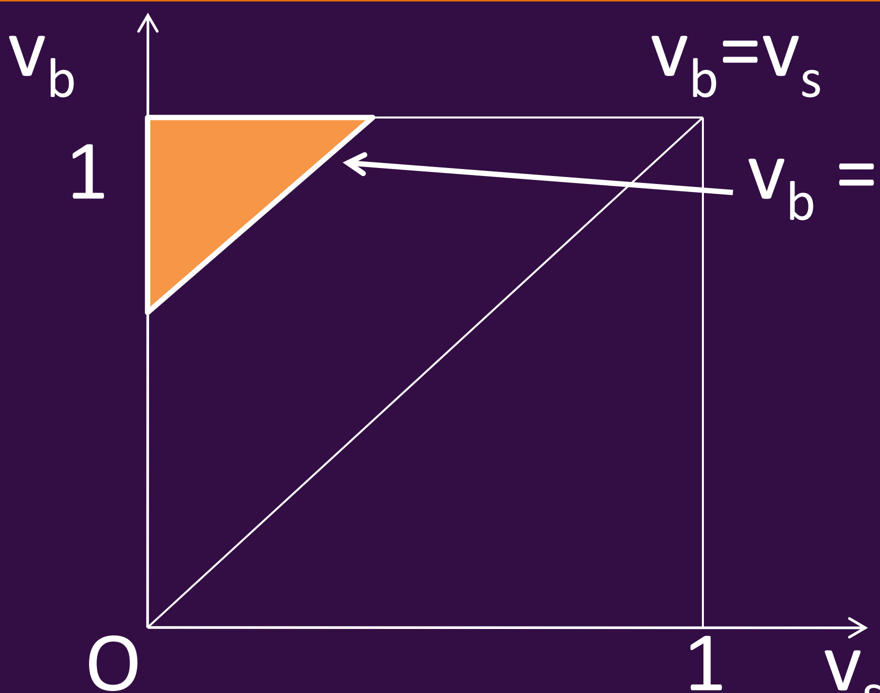
Ex. Itayose Method

- 面積が出来高に対応

Linear Equilibrium

- Seller's Strategy : $p_s(v_s)=a_s+c_sv_s$、p_s : uniform distribition on $[a_s,a_s+c_s]$ → $p_s=(a_b+c_b+v_s)/2$
- Buyer's Strategy : $p_b(v_b)=a_b+c_bv_b$、p_b : uniform distribution on $[a_b,a_b+c_b]$ → $p_b=(v_b+a_s)/2$

⇒ $p_s(v_s)=2/3+v_s/2$, $p_b(v_b)=1/3 + v_b/2$.



Myerson and Satterthwaite (1983)
→線形均衡の方が高い利得を得られる。

Kikkawa (2009) (related : Logit model)

- **Many players** play the game simultaneously.

- Kikkawa (2009) formulates this situation with statistical mechanics (統計力学).

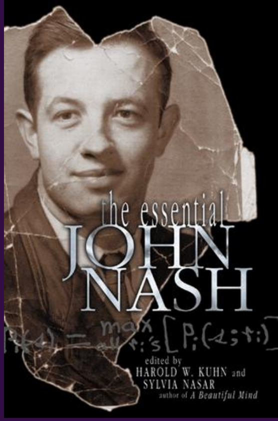
Prop. We obtain the probability distribution of actions, $\{S_i\}$, $i=1,\ldots,N$, and the player's payoff from the outcome is f,

$$P(\{S_i\})=Z^{-1}\exp(\gamma f),$$

$\{S_i\}$:a player i's action, γ: non-negative constant, f: the player's payoff from outcome $\{S_i\}$, Z: normalization parameter.

- Kikkawa (2009) is similar to **Quantal Response Equilibrium**. (Mckelvey and Palfrey (1995, 1996))

Interpretation of Nash Equilibrium (J.F.Nash’s Ph D. Thesis)



- 1. “**Rationality**” ･･･ the players are perceived as rational and they have complete information about the structure of the game, including all of the players’ preferences regarding possible outcomes, where this information about each other’s strategic alternatives and preferences, they can also compute each other’s optimal choice of strategy for each set of expectations. If all of the players expect the same Nash equilibrium, then there are no incentives for anyone to change his strategy.
- 2. “**Statistical Populations**” ･･･ is useful in so-called evolutionary games. This type of game has also been developed in biology in order to understand how the principles of natural selection operate in strategic interaction within among species.(→ **Mass Action**)

PROPOSITION (Kikkawa, 2009)

PRO.: $x \in \Delta$ is an evolutionary stable strategy in an evolutionary game with statistical mechanics, if there exists some *m* such that the inequality (*) holds for all m^*.

$$u(y,x) \leqq u(x,x), \quad \forall y,$$

(*) $$|m - m^*| < \varepsilon$$ ← Lyapunov Stable Condition

where, m^* stands for the index of the strategy

Multinominal Logit Model

- From Kikkawa (2009), we can know the probability of choosing the strategy for each player.

＋

- Data (the probability of choosing the strategy for each player)

- Regression analysis(回帰分析)

$Y_i = \alpha + \gamma f + u$, u : logistic distribution.

- We can estimate optimal parameters in this model with **Least Squares Method** (最小2乗法)

How to analyze the order book

Step 1) Logit Model (Derive the probability of choosing the strategy and transform this into log function.)

Step 2) Regression analysis.

OA: $Y=-0.65307+94079.26X_1-9.59255X_2$,

$Y=-0.66468+74928.44X_1-7.6642X_2$.

Step 3) Derive v_s, v_b. $v_s=9776$, $v_b=9807.53$.

~~**OA**~~**:**

Step 3') Derive the Demand and Supply function.

$Y=583.93-146.27X$, $Y=-237.14+59.57X$

Step 4') In equilibrium, we know that the quantity demanded is equal to the quantity supplied.

Step 5') Derive the Nash equilibrium.

$X^*=9740$.

Practice Use

- **Excel**
- **Realtime Spread Sheet** (provided by Rakuten Securities, Inc. (楽天証券))

- **[MOVIE] (YouTube)**

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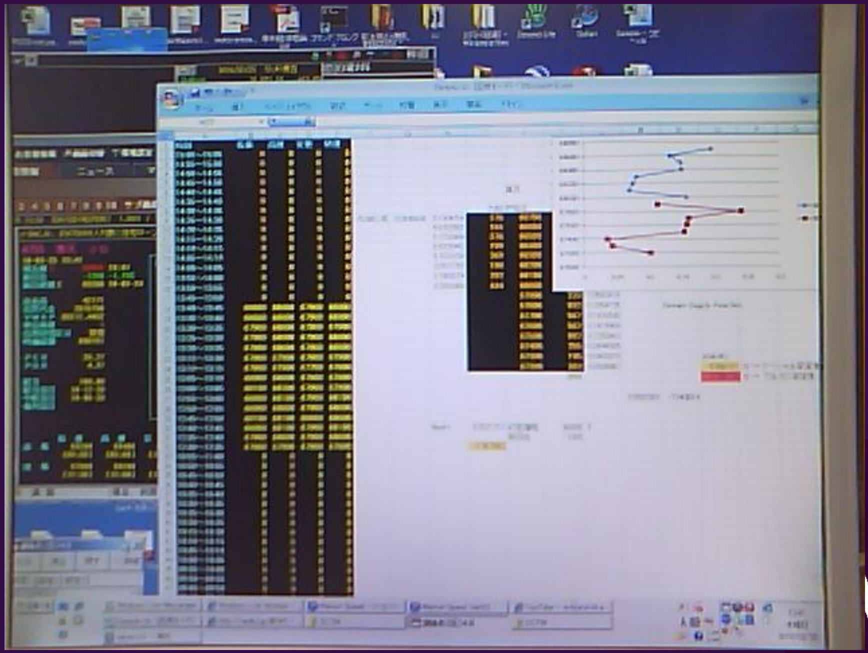


4. DYNAMICAL SYSTEM

Replicator Equation

- **Assumption** : Monotonic (単調性) : 各主体は期待利得が高くなる戦略を採用する

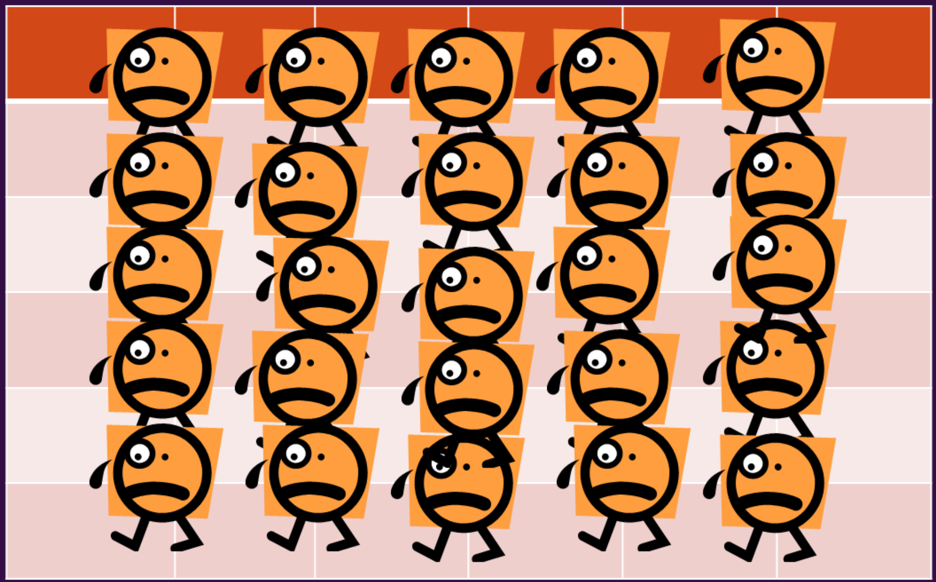
+ α

→ Replicator 方程式が導出される。

$$dx_i(t)/dt = x_i\,(Ax_i - x\,Ax),\ i = 1,\ldots, n.$$

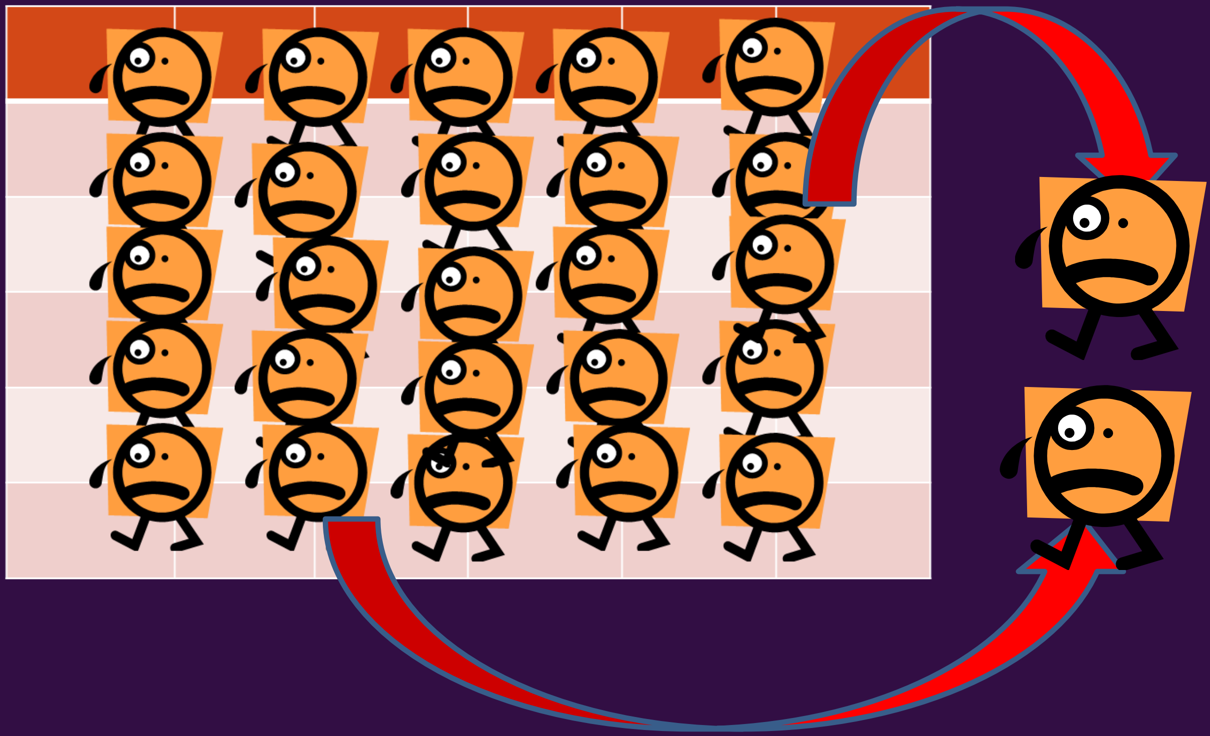
- x_i: 戦略i を採用する確率、Ax_i : 戦略 i を採用した時の利得、x Ax : 平均期待利得。

Situation (Traditional Evolutionary Game Theory)



Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



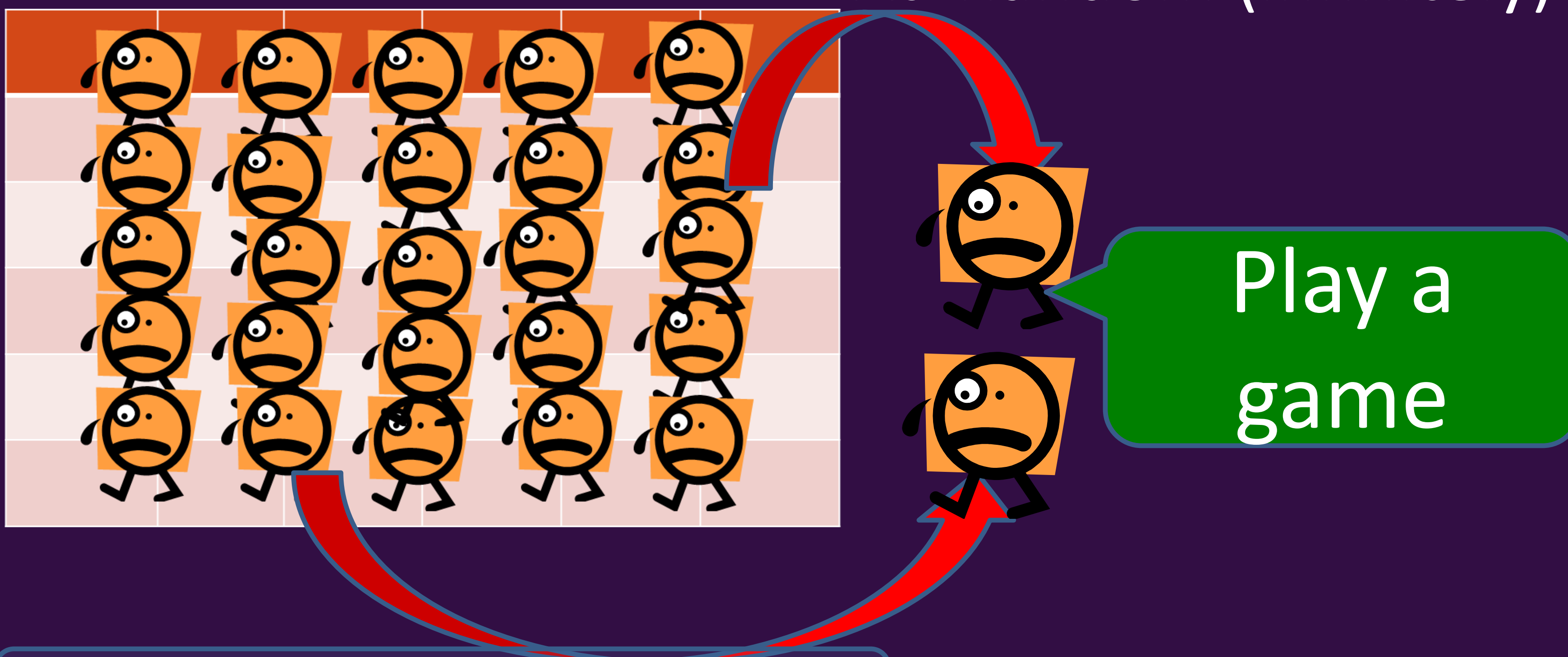
Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)

Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

At Random (infinitely)



Another players look at the game.

Situation (Traditional Evolutionary Game Theory)

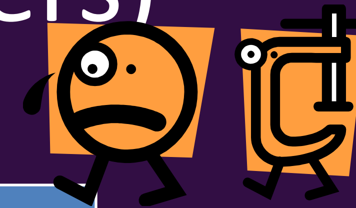
At Random (infinitely)

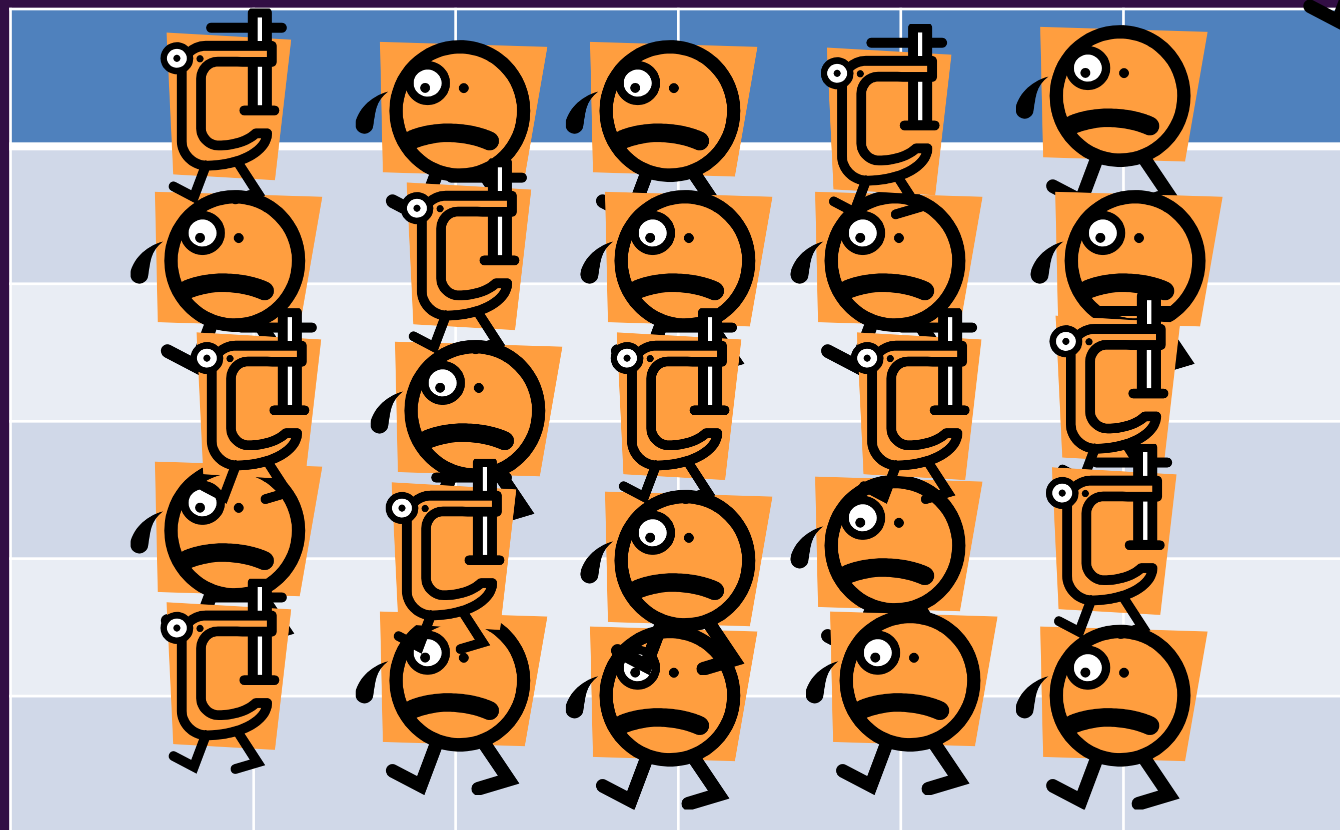
Play a game

Another players look at the game.

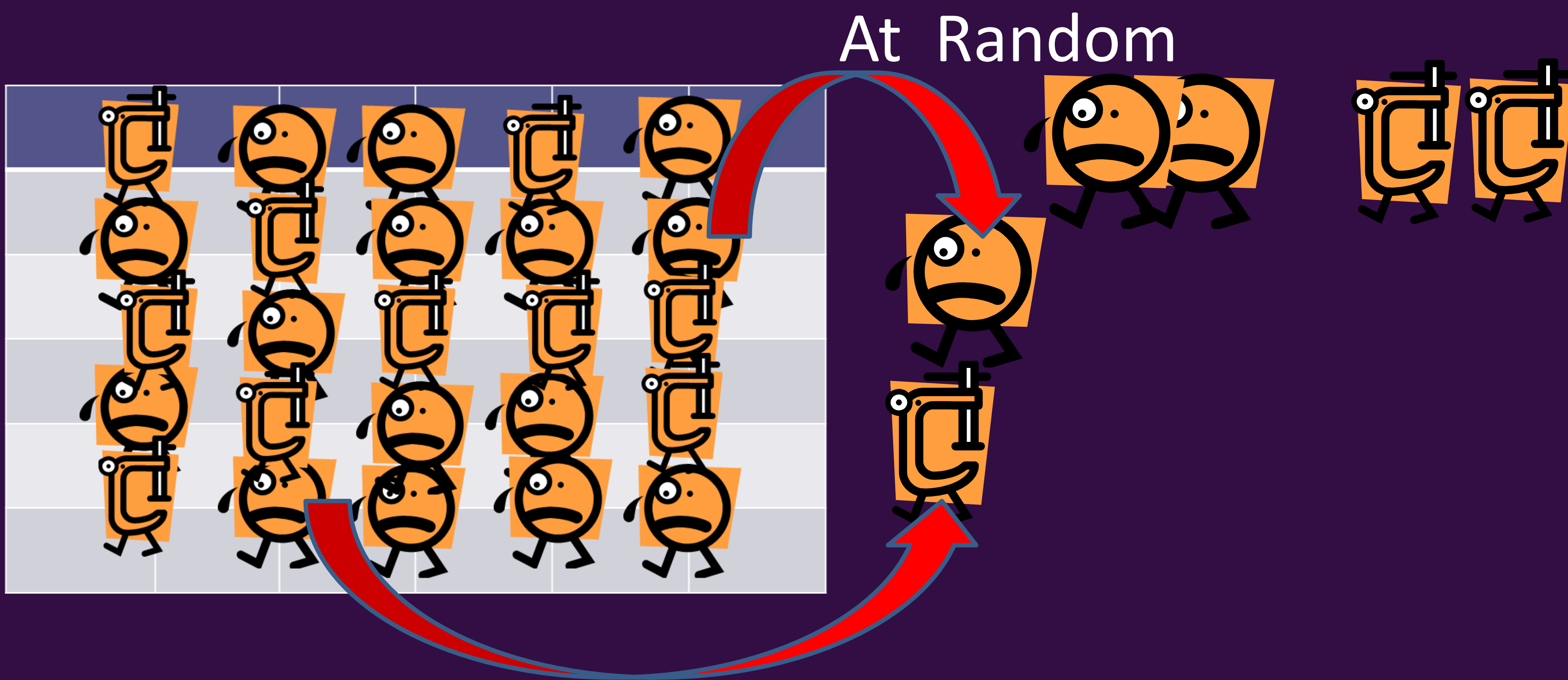
Replicator Equation

Situation (two types players)

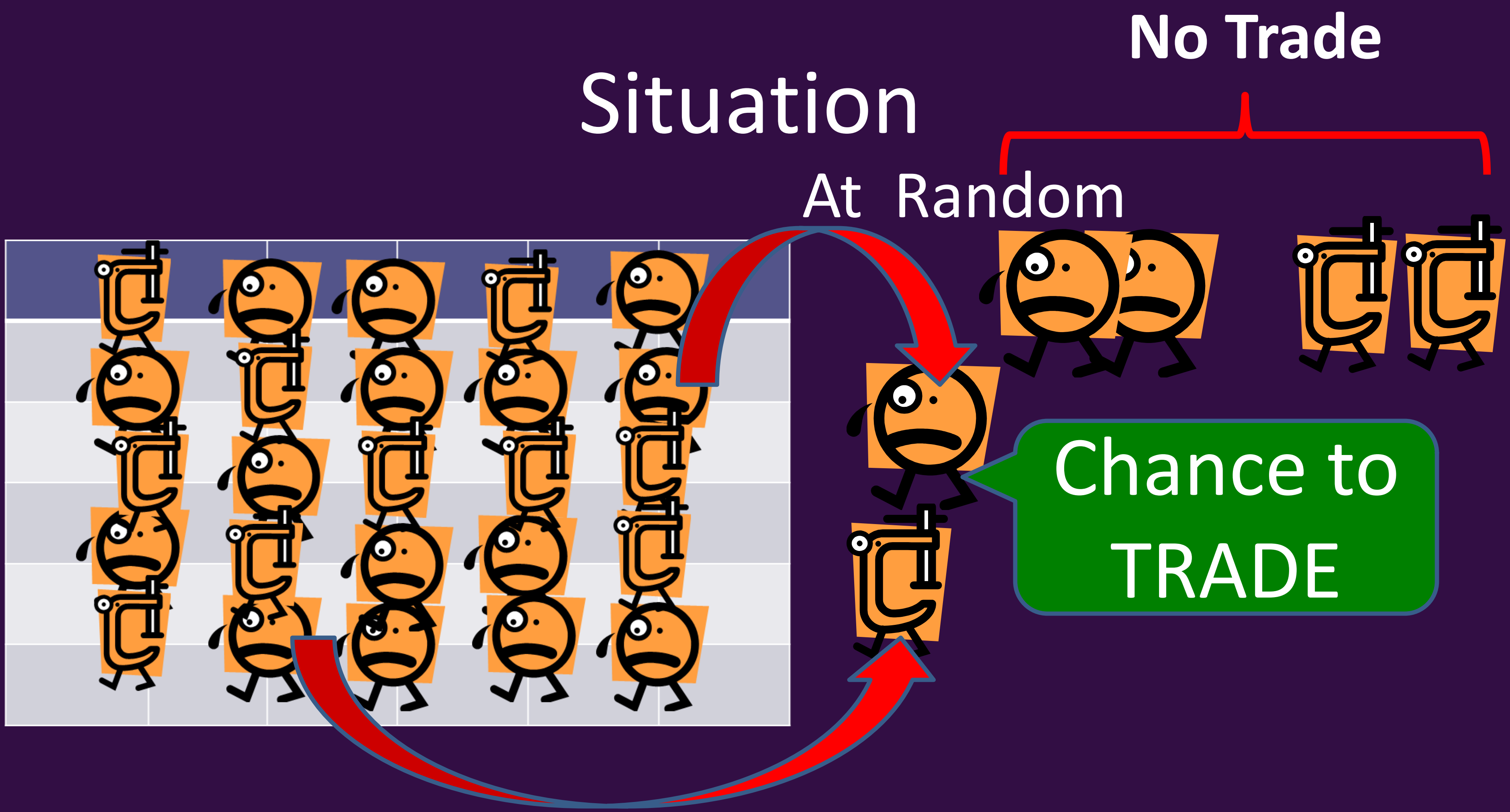




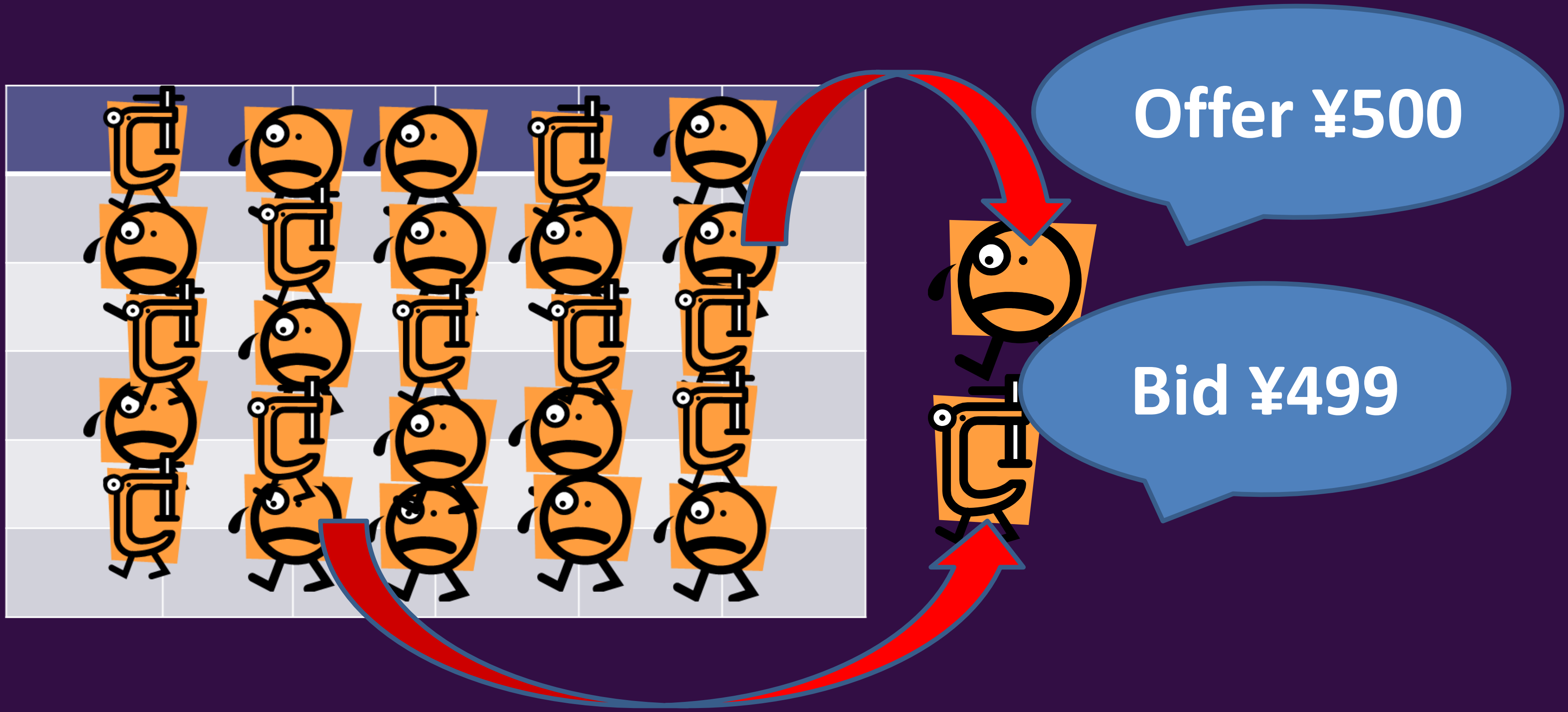
Situation



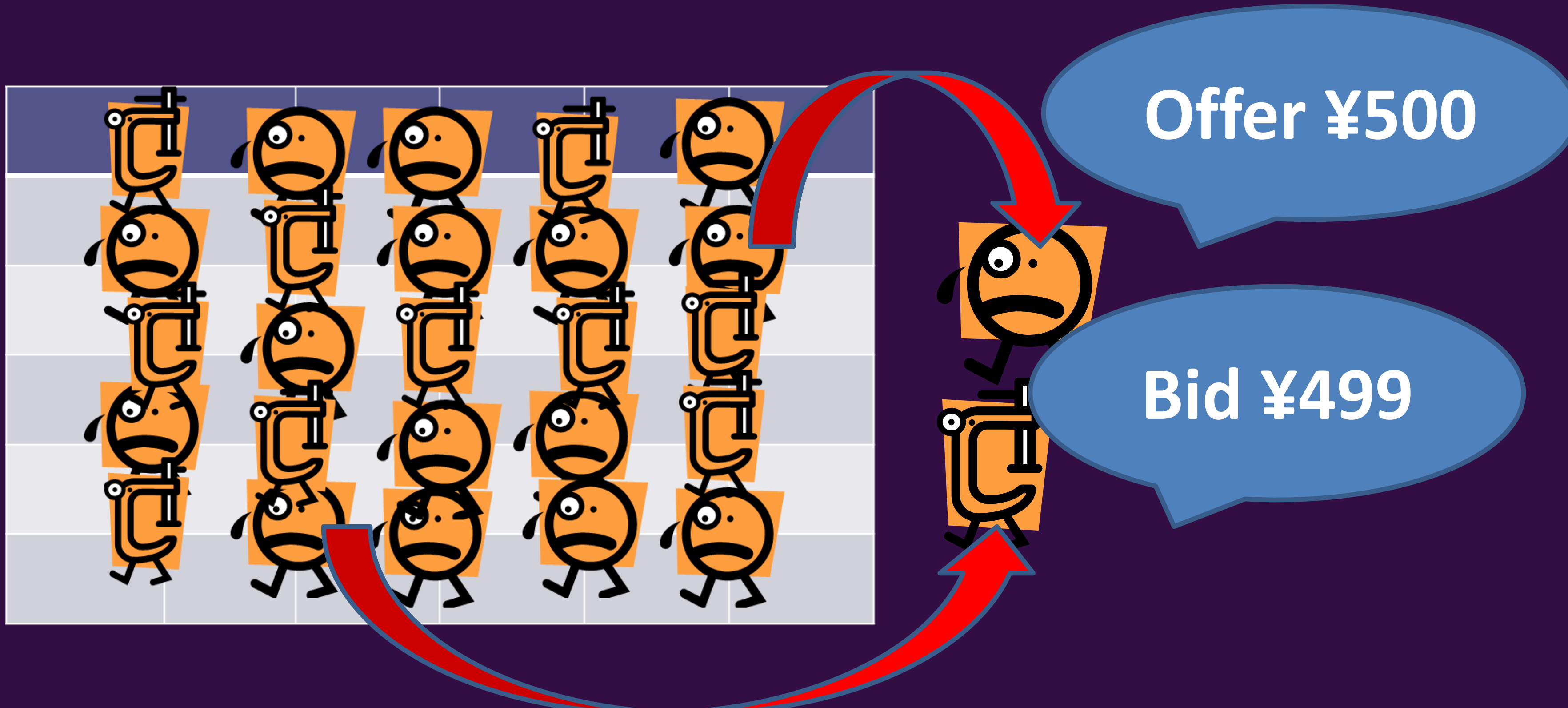
Situation



Situation

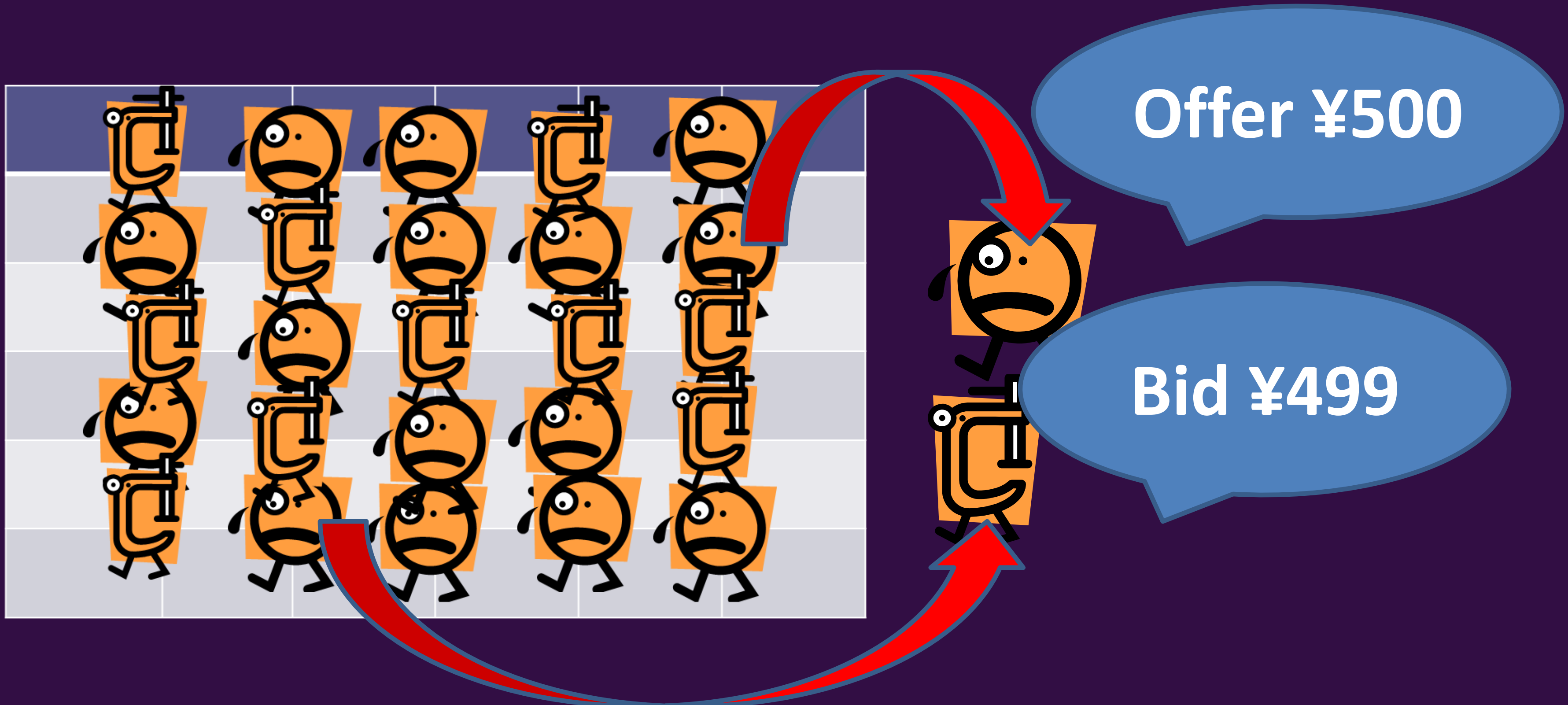


Situation



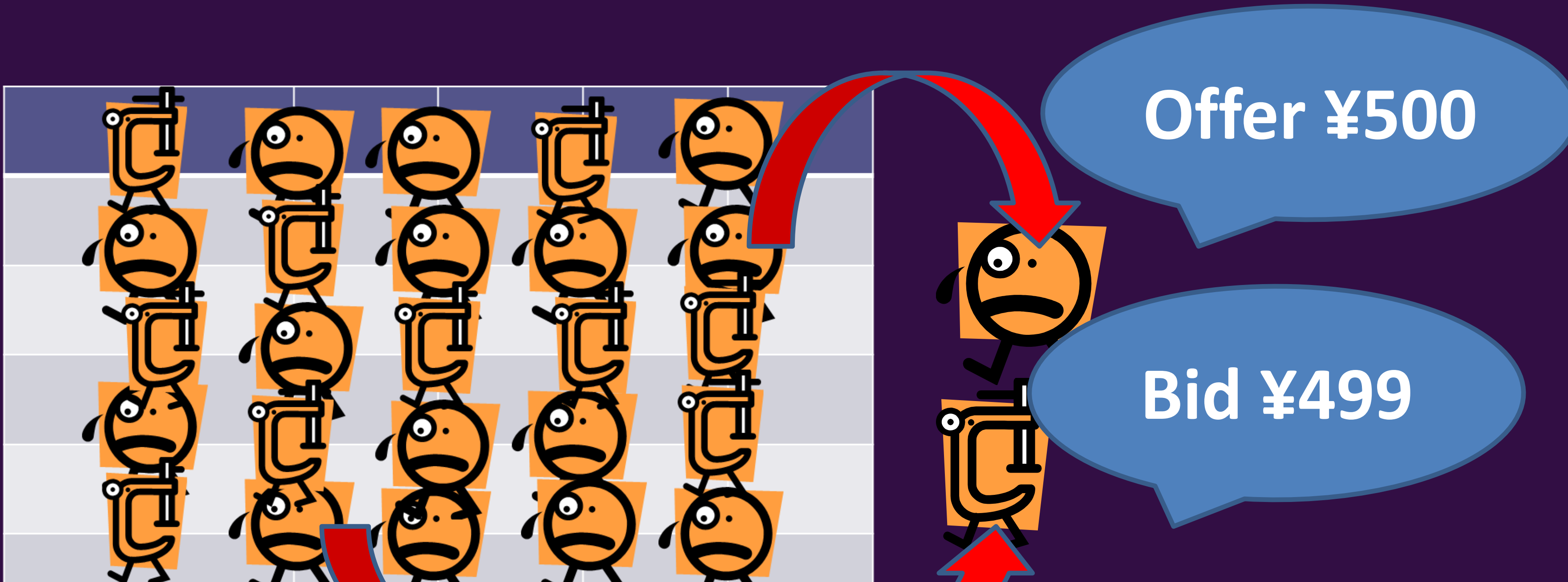
Stock Exchange which take account of the order book decides the trade's contract. (取引所が板情報をもとに、売買契約を決定する)

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Situation



Another players look at the order book (他のプレイヤーは板情報を見ている).

Which strategy is Nash Equilibrium,
if this game is played at infinite ?
(このゲームを無限回仮想的に行うと、どの戦略が均衡となるのか？)

Model (モデル)

- **Replicator Equation**

$$\frac{dx_i(t)}{dt} = x_i(t)\left(g_i(t) - \bar{g}(t)\right)$$

$$\frac{dy_i(t)}{dt} = y_i(t)\left(h_i(t) - \bar{h}(t)\right)$$

where x_i, y_i : the probability of choosing the strategy 1 for each player. g_i, h_i : the payoff when each player chooses the strategy 1.

Two Strategies Case
(戦略の数が2つ) :

- Replicator equation (see next slide)

$$\dot{x} = x(1-x)\{-b(t) + (a(t)+b(t))y\},$$

$$\dot{y} = y(1-y)\{b(t) - (a(t)+b(t))x\},$$

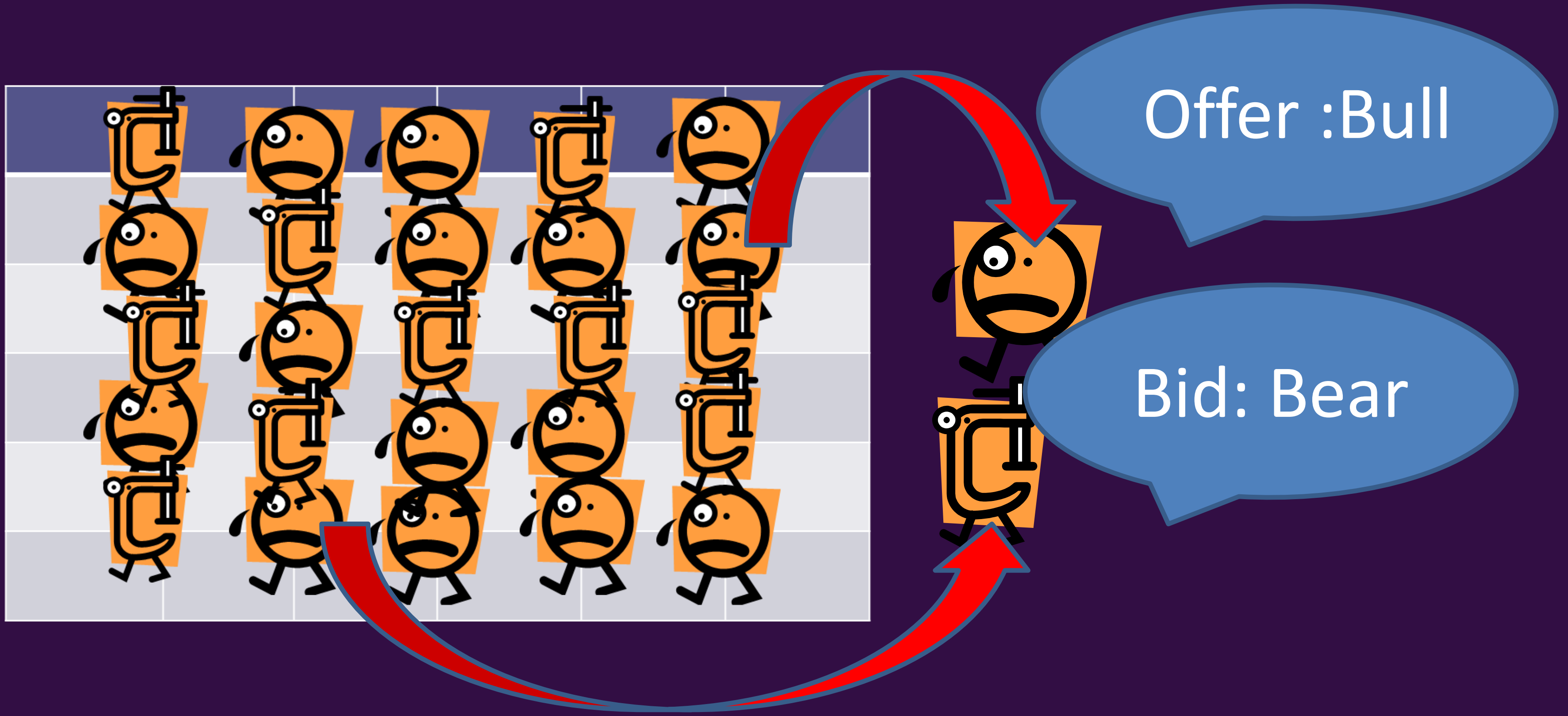
where x, y is the probability of choosing the strategy 1, 2 for each player.

Player 2

player1

	S1	S2
S1	a(t),-a(t)	0,0
S2	0,0	b(t),-b(t)

Situation



Prediction (予測)

- Replicator equation divided by xy(1-x)(1-y) :

$$\dot{x} = -\frac{b(t)}{y} + \frac{a(t)}{1-y}, \dot{y} = \frac{b(t)}{x} - \frac{a(t)}{1-x}.$$

- Discrete the above equations:

$$x(t+\varepsilon) = x(t) - \left(\frac{b(t)}{y} + \frac{a(t)}{1-y} \right) \varepsilon,$$

$$y(t+\varepsilon) = y(t) + \left(\frac{b(t)}{x} - \frac{a(t)}{1-x} \right) \varepsilon.$$

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

Buyer / Seller

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	+, -	0,0
S 2(BEAR)	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	+,+	0,0
S 2(BEAR)	0,0	-,+

iii) → (No change)

N.E. Mixed Strategy.

	S 1(BEAR)	S 2(BULL)
S 1(BULL)	-,+	0,0
S 2(BEAR)	0,0	-,+

Payoff Matrix (利

i) ↑ (UP)

N.E. (s2,s2)

ii) ↓ (Down)

N.E. (s1,s1)

iii) → (No change)

N.E. Mixed Strategy.

Seller

		LL)
S 1(BULL)	+, -	0,0
S 2(BEAR)	0,0	+ +

価格上昇時、売り手は約定価格よりも**強気に**高い売り、買い手は**弱気**で高い価格で購入

	S 1(B	
S 1(BULL)	+,+	
S 2(BEAR)	0,0	- +

価格下落時、売り手は約定価格よりも**弱気に**安く売り、買い手は**強気**で安い価格で購入

	S 1(B	
S 1(BULL)	-,+	
S 2(BEAR)	0,0	-,+

価格変化しない時、売り手は約定価格よりも**弱気に**安く売り、買い手は**強気**で安い価格で購入

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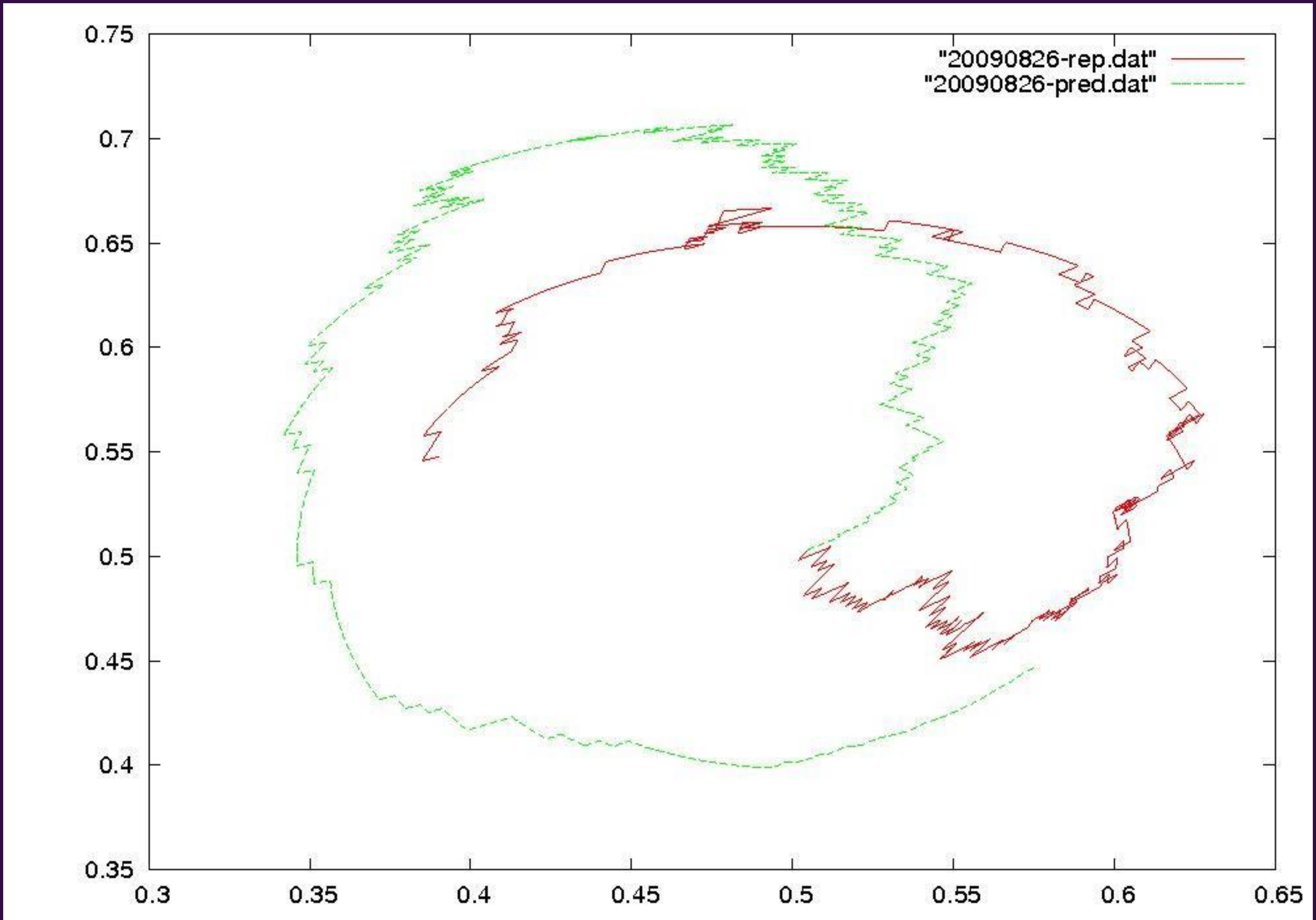

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0.4
0.35
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65

4. EXTENSION:

RISK ATTITUDE

RISK ATTITUDE

- We assume that the own utility is linear function.(今まで主体の効用は線形であると仮定してきた。)
- Each player has the non-linear utility.(そこで非線形の場合をも考慮に入れる。)

- We examine the equilibrium selection with this nonlinear-utility. (そこでこの非線形効用関数を用いて、均衡選択の問題を考察する。)

- Utility function : $g(x)$, z : payoff
- Taylor Expansion:
- $g(x+z)-g(x)=g'(x)z+0.5g''(x)z^2+O(z^3)$ .... (*)

Def. Given a (twice-differentiable) Bernoulli utility function u(.) for money, the ***Arrow-Pratt coefficient of absolute risk aversion*** at x is defined as $r_A(x)=-u''(x)/u'(x)$.

- (*) $g(x+z)-g(x) = zg'(x)(1-0.5zr_A(x))$
- (In economics, we assume $g'(x)>0$, $g''(x)<0$)

Payoff Matrix (利得表)

i) ↑ (UP)

N.E. (s2,s2)

	S 1	S 2
S 1	-, -	0,0
S 2	0,0	+,+

ii) ↓ (Down)

N.E. (s1,s1)

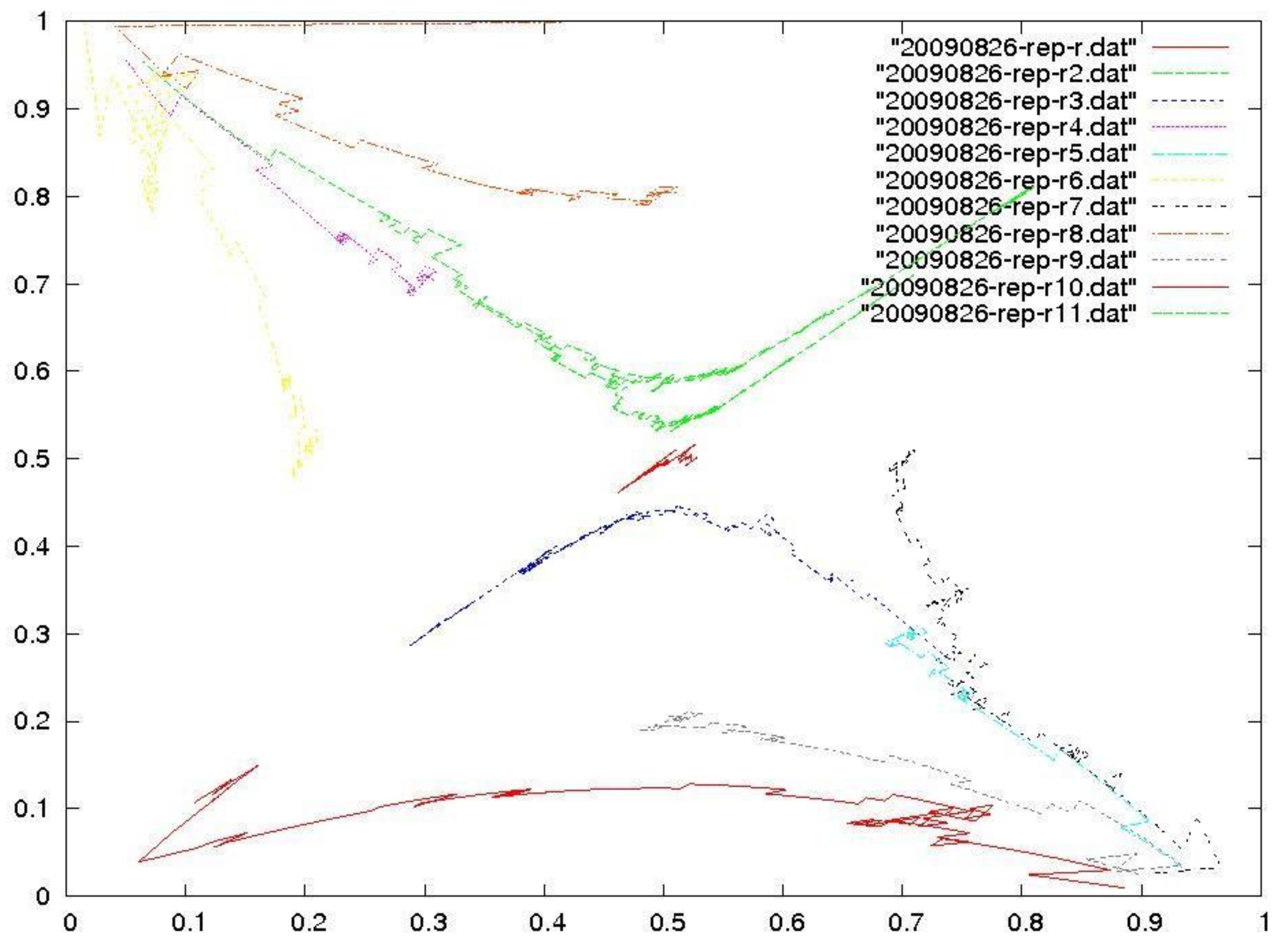
	S 1	S 2
S 1	+,+	0,0
S 2	0,0	-,-

iii) → (No change)

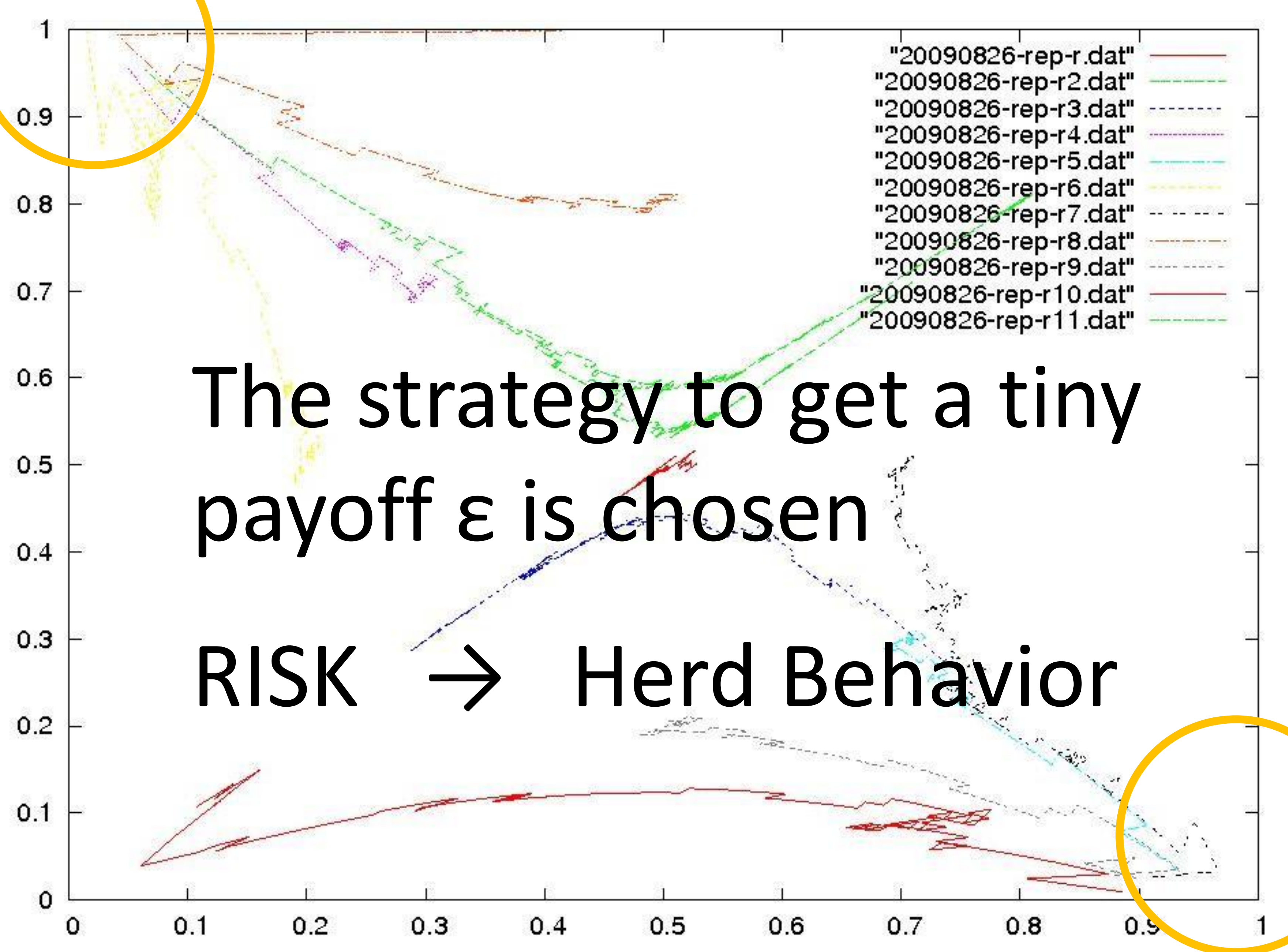
N.E. Mixed Strategy.

	S 1	S 2
S 1	-,-	0,0
S 2	0,0	-,-

EX: 20090826 (RISK)



EX: 20090826 (RISK)



The strategy to get a tiny payoff ε is chosen

RISK → Herd Behavior

5. TIME SERIESE ANALYSIS

Time Series Analysis (Particle Filter)

- Extending this game theory as a particle filter.(Kitagawa (1996))

- State-Space model (状態空間モデル)

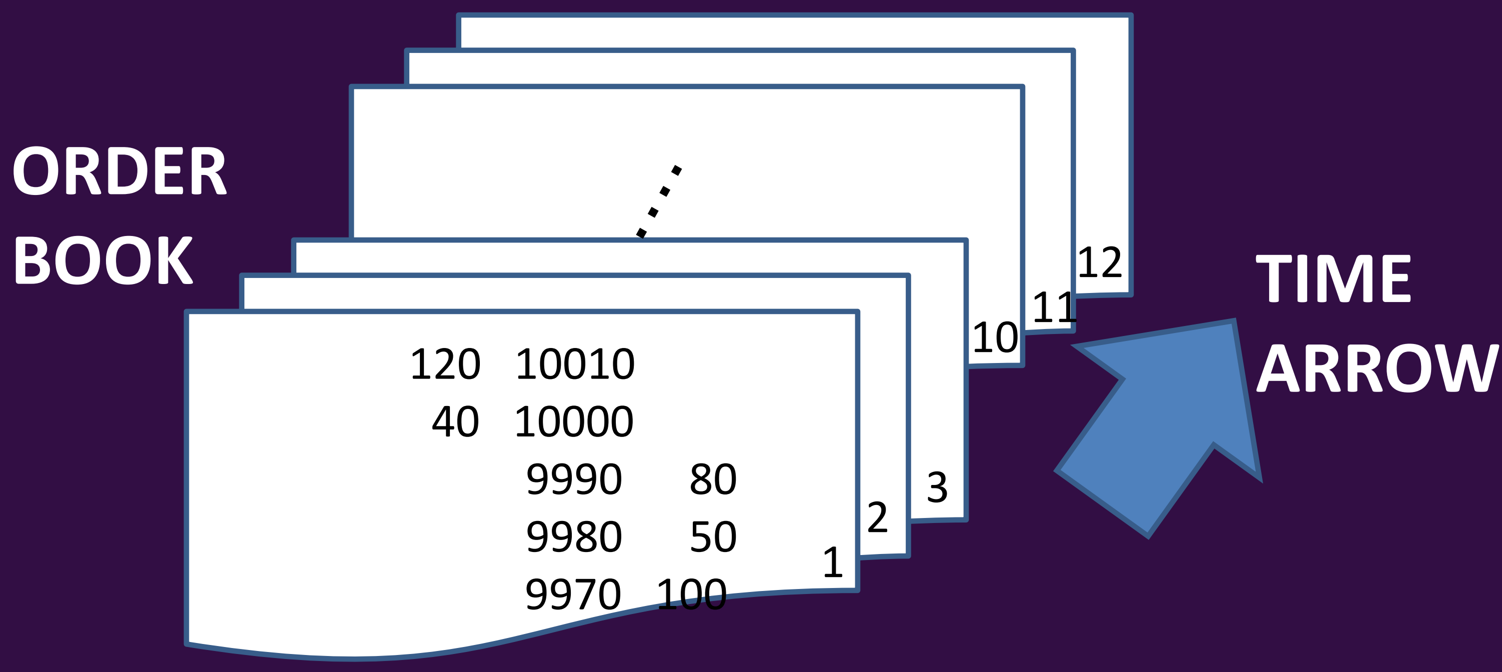
  (Observation model) $P_i = Z^{-1} \exp(\gamma(t) f_i(t)), \; i=1,\ldots,k.$

  (System model) $z_{t+1} = F z_t + G v_t.$

- We can analyze the dynamical model with this method.

- γ: 時変パラメータ.
- $\gamma_{t+1} = \gamma_t + v_t$, v_t: Normal distribution.
- v_s, v_b : constant (short time).

- Maximum likelihood estimation(最尤法)



WORKING

- 問題点：
- １．観測状態が複数存在する。そのため尤度が複数出てくるが、ここでγは共通。

6. SUMMARY AND FUTURE WORKS

Summary

- **MODELING** the Financial Market.(金融市場をモデリングした)

- **ANALYZING** the impact of each player’s Risk Attitude.(各主体にリスクに対する態度がある場合を考察した)

- **ANALYZING** the Order book with Multi-Logit Model and **DERIVING** how to forecast the next step’s price.(板情報を多項ロジットモデルを用いて分析し、次の約定価格を予想する新たな手法を導いた)

Future Works

- TIME SERIES ANALYSIS: Panel Data

- Victor Aguirregabiria and Pedro Mira ''Dynamic discrete choice structural models: A survey,'' Journal of Econometrics, Volume 156, Issue 1, May 2010, Pages 38-67. **[HP]**

- (Summary of Aguirregabiria and Mira (2010, JE)) **[PDF] (Adachiさん@Nagoya Univ.)**

Superstatistics (超統計)

- Compare with Superstatiscs and Particle Filter.

- Beck and Cohen (2003) により提案された Boltzman分布のパラメーターγが何らかの分布に従っている場合。
- exp (γf)=exp (γf)h(γ),

  h(γ):[a,a+b]上の一様分布 : h(γ)=1/b

$P(S_i)=\exp(\gamma_0 f)(1+b^2f^2/24+b^4f^4/1920 + \ldots)$

Thank You For Your Attention

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This File is available at

http://kikkawa.cyber-ninja.jp/

NEXT MY TALK 5/22 @Meiji Univ

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